Review of Algebra

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C O N N E X I O N S

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Table of Contents

Chapter 1

Arithmetic Review: Factors, Products, and Exponents[1]

1.1 Overview

- Factors
- Exponential Notation

1.2 Factors

Let's begin our review of arithmetic by recalling the meaning of multiplication for whole numbers (the counting numbers and zero).

Multiplication

Multiplication is a description of repeated addition.

In the addition

$7 + 7 + 7 + 7$

the number 7 is repeated as an **addend*** 4 **times.** Therefore, we say we have **four times seven** and describe it by writing

$4 \cdot 7$

The raised dot between the numbers 4 and 7 indicates multiplication. The dot directs us to multiply the two numbers that it separates. In algebra, the dot is preferred over the symbol $\times$ to denote multiplication because the letter x is often used to represent a number. Thus,

$4 \cdot 7 = 7 + 7 + 7 + 7$

Factors and Products

In a multiplication, the numbers being multiplied are called **factors.** The result of a multiplication is called the **product.** For example, in the multiplication

$4 \cdot 7 = 28$

[1] This content is available online at <http://cnx.org/content/m18882/1.5/>.

the numbers 4 and 7 are factors, and the number 28 is the product. We say that 4 and 7 are factors of 28. (They are not the only factors of 28. Can you think of others?)

Now we know that

$$(\text{factor}) \cdot (\text{factor}) = \text{product}$$

This indicates that a first number is a factor of a second number if the first number divides into the second number with no remainder. For example, since

$$4 \cdot 7 = 28$$

both 4 and 7 are factors of 28 since both 4 and 7 divide into 28 with no remainder.

1.3 Exponential Notation

Quite often, a particular number will be repeated as a factor in a multiplication. For example, in the multiplication

$$7 \cdot 7 \cdot 7 \cdot 7$$

the number 7 is repeated as a factor 4 times. We describe this by writing 7^4. Thus,

$$7 \cdot 7 \cdot 7 \cdot 7 = 7^4$$

The repeated factor is the lower number (the base), and the number recording how many times the factor is repeated is the higher number (the superscript). The superscript number is called an **exponent.**

Exponent

An **exponent** is a number that records how many times the number to which it is attached occurs as a factor in a multiplication.

1.4 Sample Set A

For Examples 1, 2, and 3, express each product using exponents.

Example 1.1

$3 \cdot 3 \cdot 3 \cdot 3 \cdot 3 \cdot 3$. Since 3 occurs as a factor 6 times,

$$3 \cdot 3 \cdot 3 \cdot 3 \cdot 3 \cdot 3 = 3^6$$

Example 1.2

$8 \cdot 8$. Since 8 occurs as a factor 2 times,

$$8 \cdot 8 = 8^2$$

Example 1.3

$5 \cdot 5 \cdot 5 \cdot 9 \cdot 9$. Since 5 occurs as a factor 3 times, we have 5^3. Since 9 occurs as a factor 2 times, we have 9^2. We should see the following replacements.

$$\underbrace{5 \cdot 5 \cdot 5}_{5^3} \cdot \underbrace{9 \cdot 9}_{9^2}$$

Then we have

$5 \cdot 5 \cdot 5 \cdot 9 \cdot 9 = 5^3 \cdot 9^2$

Example 1.4
Expand 3^5. The base is 3 so it is the repeated factor. The exponent is 5 and it records the number of times the base 3 is repeated. Thus, 3 is to be repeated as a factor 5 times.

$3^5 = 3 \cdot 3 \cdot 3 \cdot 3 \cdot 3$

Example 1.5
Expand $6^2 \cdot 10^4$. The notation $6^2 \cdot 10^4$ records the following two facts: 6 is to be repeated as a factor 2 times and 10 is to be repeated as a factor 4 times. Thus,

$6^2 \cdot 10^4 = 6 \cdot 6 \cdot 10 \cdot 10 \cdot 10 \cdot 10$

1.5 Exercises

For the following problems, express each product using exponents.

Exercise 1.1 *(Solution on p. 6.)*
$8 \cdot 8 \cdot 8$

Exercise 1.2
$12 \cdot 12 \cdot 12 \cdot 12 \cdot 12$

Exercise 1.3 *(Solution on p. 6.)*
$5 \cdot 5 \cdot 5 \cdot 5 \cdot 5 \cdot 5 \cdot 5$

Exercise 1.4
$1 \cdot 1$

Exercise 1.5 *(Solution on p. 6.)*
$3 \cdot 3 \cdot 3 \cdot 3 \cdot 3 \cdot 4 \cdot 4$

Exercise 1.6
$8 \cdot 8 \cdot 8 \cdot 15 \cdot 15 \cdot 15 \cdot 15$

Exercise 1.7 *(Solution on p. 6.)*
$2 \cdot 2 \cdot 2 \cdot 9 \cdot 9 \cdot 9 \cdot 9 \cdot 9 \cdot 9 \cdot 9 \cdot 9$

Exercise 1.8
$3 \cdot 3 \cdot 10 \cdot 10 \cdot 10$

Exercise 1.9 *(Solution on p. 6.)*
Suppose that the letters x and y are each used to represent numbers. Use exponents to express the following product.

$x \cdot x \cdot x \cdot y \cdot y$

Exercise 1.10
Suppose that the letters x and y are each used to represent numbers. Use exponents to express the following product.

$x \cdot x \cdot x \cdot x \cdot x \cdot y \cdot y \cdot y$

For the following problems, expand each product (do not compute the actual value).

Exercise 1.11 *(Solution on p. 6.)*
3^4

Exercise 1.12
4^3

Exercise 1.13 *(Solution on p. 6.)*
2^5

Exercise 1.14
9^6

Exercise 1.15 *(Solution on p. 6.)*
$5^3 \cdot 6^2$

Exercise 1.16
$2^7 \cdot 3^4$

Exercise 1.17 *(Solution on p. 6.)*
$x^4 \cdot y^4$

Exercise 1.18
$x^6 \cdot y^2$

For the following problems, specify all the whole number factors of each number. For example, the complete set of whole number factors of 6 is 1, 2, 3, 6.

Exercise 1.19 *(Solution on p. 6.)*
20

Exercise 1.20
14

Exercise 1.21 *(Solution on p. 6.)*
12

Exercise 1.22
30

Exercise 1.23 *(Solution on p. 6.)*
21

Exercise 1.24
45

Exercise 1.25 *(Solution on p. 6.)*
11

Exercise 1.26
17

Exercise 1.27 *(Solution on p. 6.)*
19

Exercise 1.28
2

Solutions to Exercises in Chapter 1

Solutions to Arithmetic Review: Factors, Products, and Exponents

Solution to Exercise 1.1 (p. 4)
8^3
Solution to Exercise 1.3 (p. 4)
5^7
Solution to Exercise 1.5 (p. 4)
$3^5 \cdot 4^2$
Solution to Exercise 1.7 (p. 4)
$2^3 \cdot 9^8$
Solution to Exercise 1.9 (p. 4)
$x^3 \cdot y^2$
Solution to Exercise 1.11 (p. 4)
$3 \cdot 3 \cdot 3 \cdot 3$
Solution to Exercise 1.13 (p. 4)
$2 \cdot 2 \cdot 2 \cdot 2 \cdot 2$
Solution to Exercise 1.15 (p. 4)
$5 \cdot 5 \cdot 5 \cdot 6 \cdot 6$
Solution to Exercise 1.17 (p. 4)
$x \cdot x \cdot x \cdot x \cdot y \cdot y \cdot y \cdot y$
Solution to Exercise 1.19 (p. 5)
1, 2, 4, 5, 10, 20
Solution to Exercise 1.21 (p. 5)
1, 2, 3, 4, 6, 12
Solution to Exercise 1.23 (p. 5)
1, 3, 7, 21
Solution to Exercise 1.25 (p. 5)
1, 11
Solution to Exercise 1.27 (p. 5)
1, 19

Chapter 2

Arithmetic Review: Prime Factorization[1]

2.1 Overview

- Prime And Composite Numbers
- The Fundamental Principle Of Arithmetic
- The Prime Factorization Of A Whole Number

2.2 Prime And Composite Numbers

Notice that the only factors of 7 are 1 and 7 itself, and that the only factors of 23 are 1 and 23 itself.

Prime Number

A whole number greater than 1 whose only whole number factors are itself and 1 is called a **prime number.**

The first seven prime numbers are

2, 3, 5, 7, 11, 13, and 17

The number 1 is not considered to be a prime number, and the number 2 is the first and only even prime number.

Many numbers have factors other than themselves and 1. For example, the factors of 28 are 1, 2, 4, 7, 14, and 28 (since each of these whole numbers and only these whole numbers divide into 28 without a remainder).

Composite Numbers

A whole number that is composed of factors other than itself and 1 is called a **composite number.** Composite numbers are not prime numbers.

Some composite numbers are 4, 6, 8, 10, 12, and 15.

2.3 The Fundamental Principle Of Arithmetic

Prime numbers are very important in the study of mathematics. We will use them soon in our study of fractions. We will now, however, be introduced to an important mathematical principle.

The Fundamental Principle of Arithmetic

Except for the order of the factors, every whole number, other than 1, can be factored in one and only one way as a product of prime numbers.

[1] This content is available online at <http://cnx.org/content/m21868/1.5/>.

Prime Factorization
When a number is factored so that all its factors are prime numbers, the factorization is called the **prime factorization** of the number.

2.4 Sample Set A

Example 2.1
Find the prime factorization of 10.

$10 = 2 \cdot 5$

Both 2 and 5 are prime numbers. Thus, $2 \cdot 5$ is the prime factorization of 10.

Example 2.2
Find the prime factorization of 60.

$$\begin{array}{rcll} 60 & = & 2 \cdot 30 & 30 \text{ is not prime. } 30 = 2 \cdot 15 \\ & = & 2 \cdot 2 \cdot 15 & 15 \text{ is not prime. } 15 = 3 \cdot 5 \\ & = & 2 \cdot 2 \cdot 3 \cdot 5 & \text{We'll use exponents. } 2 \cdot 2 = 2^2 \\ & = & 2^2 \cdot 3 \cdot 5 & \end{array}$$

The numbers 2, 3, and 5 are all primes. Thus, $2^2 \cdot 3 \cdot 5$ is the prime factorization of 60.

Example 2.3
Find the prime factorization of 11.

11 is a prime number. Prime factorization applies only to composite numbers.

2.5 The Prime Factorization Of A Whole Number

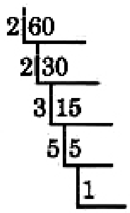
The following method provides a way of finding the prime factorization of a whole number. The examples that follow will use the method and make it more clear.

1. Divide the number repeatedly by the smallest prime number that will divide into the number without a remainder.
2. When the prime number used in step 1 no longer divides into the given number without a remainder, repeat the process with the next largest prime number.
3. Continue this process until the quotient is 1.
4. The prime factorization of the given number is the product of all these prime divisors.

2.6 Sample Set B

Example 2.4
Find the prime factorization of 60.

Since 60 is an even number, it is divisible by 2. We will repeatedly divide by 2 until we no longer can (when we start getting a remainder). We shall divide in the following way.



30 is divisible by 2 again.

15 is not divisible by 2, but is divisible by 3, the next largest prime.

5 is not divisible by 3, but is divisible by 5, the next largest prime.

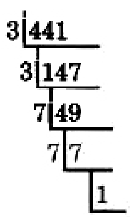
The quotient is 1 so we stop the division process.

The prime factorization of 60 is the product of all these divisors.

$$60 = 2 \cdot 2 \cdot 3 \cdot 5 \quad \text{We will use exponents when possible.}$$
$$60 = 2^2 \cdot 3 \cdot 5$$

Example 2.5
Find the prime factorization of 441.

Since 441 is an odd number, it is not divisible by 2. We'll try 3, the next largest prime.



147 is divisible by 3.

49 is not divisible by 3 nor by 5, but by 7.

7 is divisible by 7.

The quotient is 1 so we stop the division process.

The prime factorization of 441 is the product of all the divisors.

$$441 = 3 \cdot 3 \cdot 7 \cdot 7 \quad \text{We will use exponents when possible.}$$
$$441 = 3^2 \cdot 7^2$$

2.7 Exercises

For the following problems, determine which whole numbers are prime and which are composite.

Exercise 2.1 *(Solution on p. 12.)*
23

Exercise 2.2
25

Exercise 2.3 *(Solution on p. 12.)*
27

Exercise 2.4
2

Exercise 2.5 *(Solution on p. 12.)*
3

Exercise 2.6
5

Exercise 2.7 *(Solution on p. 12.)*
7

Exercise 2.8
9

Exercise 2.9 *(Solution on p. 12.)*
11

Exercise 2.10
34

Exercise 2.11 *(Solution on p. 12.)*
55

Exercise 2.12
63

Exercise 2.13 *(Solution on p. 12.)*
1044

Exercise 2.14
339

Exercise 2.15 *(Solution on p. 12.)*
209

For the following problems, find the prime factorization of each whole number. Use exponents on repeated factors.

Exercise 2.16
26

Exercise 2.17 *(Solution on p. 12.)*
38

Exercise 2.18
54

Exercise 2.19 *(Solution on p. 12.)*
62

Exercise 2.20
56

Exercise 2.21 *(Solution on p. 12.)*
176

Exercise 2.22
480

Exercise 2.23 *(Solution on p. 12.)*
819

Exercise 2.24
2025

Exercise 2.25 *(Solution on p. 12.)*
148,225

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