Mathematics Framework for the 2013 National Assessment of Educational Progress



WHAT IS NAEP?

The National Assessment of Educational Progress (NAEP) is a continuing and nationally representative measure of trends in academic achievement of U.S. elementary and secondary students in various subjects. For nearly four decades, NAEP assessments have been conducted periodically in reading, mathematics, science, writing, U.S. history, civics, geography, and other subjects. By collecting and reporting information on student performance at the national, state, and local levels, NAEP is an integral part of our nation's evaluation of the condition and progress of education.

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CHAPTER ONE

OVERVIEW

Since 1973, the National Assessment of Educational Progress (NAEP) has gathered information about student achievement in mathematics. Results of these periodic assessments, produced in print and web-based formats, provide valuable information to a wide variety of audiences. They inform citizens about the nature of students' comprehension of the subject, curriculum specialists about the level and nature of student achievement, and policymakers about factors related to schooling and its relationship to student proficiency in mathematics.

The NAEP Assessment in mathematics has two components that differ in purpose. One assessment measures long-term trends in achievement among 9-, 13-, and 17-year-old students by using the same basic design each time. This unique measure allows for comparisons of students' knowledge of mathematics since it was first administered in 1973. The main NAEP Assessment is administered at the national, state, and selected urban district levels. Results are reported on student achievement in grades 4, 8, and 12 at the national level, and for grades 4 and 8 at the state level and for large urban districts that volunteered to participate. The main NAEP Assessment is based on a framework (such as this one) that can be updated periodically. The *2013 Mathematics Framework* reflects changes from 2005 in grade 12 only; mathematics content objectives for grades 4 and 8 have not changed. Therefore, main NAEP trend lines from the early 1990s can continue at fourth and eighth grades for the 2013 assessment. Special analyses have also determined that main NAEP trend lines from 2005 can continue at 12th grade for the 2013 assessment.

Taken together, the NAEP Assessments provide a rich, broad, and deep picture of student mathematics achievement in the United States. Results are reported in terms of scale scores and percentiles. These reports provide comprehensive information about what students in the United States know and can do in the area of mathematics. These reports present information on strengths and weaknesses in students' knowledge of mathematics and their ability to apply that knowledge in problem-solving situations. In addition, these reports provide comparative student data according to gender, race/ethnicity, socioeconomic status, and geographic region; describe trends in student performance over time; and report on relationships between student proficiency and certain background variables.

Student results on the main NAEP Assessment are reported for three achievement levels (*Basic*, *Proficient*, and *Advanced*) as described below:

- *Basic* denotes partial mastery of prerequisite knowledge and skills that are fundamental for proficient work at each grade.
- *Proficient* represents solid academic performance for each grade assessed. Students reaching this level have demonstrated competency over challenging subject

matter, including subject-matter knowledge, application of such knowledge to real-world situations, and appropriate analytical skills.

• *Advanced* represents superior performance.

These levels are intended to provide descriptions of what students **should** know and be able to do in mathematics. Established for the 1992 mathematics scale at grades 4 and 8 and for the 2005 and 2009 mathematics scale at grade 12 through a broadly inclusive process and adopted by the National Assessment Governing Board, the three levels per grade are the primary means of reporting NAEP data. Compared with 2005, the 2009 achievement level descriptions for grade 12 reflect updated content. See appendix A for the NAEP Mathematics Achievement Level Descriptions.

WHAT IS AN ASSESSMENT FRAMEWORK?

An assessment framework is like a blueprint. It lays out the basic design of the assessment by describing the mathematics content that should be tested and the types of assessment questions that should be included. It also describes how the various design factors should be balanced across the assessment. A companion document to this framework, *Assessment and Item Specifications for the NAEP Mathematics Assessment*, gives more detail about development of the items and conditions for the 2013 NAEP Mathematics Assessment.

This is an assessment framework, not a curriculum framework. In broad terms, this framework attempts to answer the question: What mathematics skills should be assessed on NAEP at grades 4, 8, and 12? The answer to this question must necessarily take into account the constraints of a large-scale assessment such as NAEP with its limitations on time and resources. Of critical importance is the fact that this document does **not** attempt to answer the question: What (or how) mathematics should be **taught**? The framework was developed with the understanding that some concepts, skills, and activities in school mathematics are not suitable to be assessed on NAEP, although they may well be important components of a school curriculum. Examples would be an extended project that involves gathering data or a group project.

This framework describes a design for the main NAEP assessments at the national, state, and district levels, but it is not the framework for the long-term trend NAEP Assessment described earlier.

NEED FOR A NEW FRAMEWORK AT GRADE 12

For several years, the Governing Board has focused special attention on ways to improve the assessment of 12th graders by NAEP. The goal for this 12th-grade initiative is to enable NAEP to report on how well prepared 12th-grade students are for postsecondary education and training. To accomplish this goal, the content of the assessments as described in the 2005 Mathematics Framework was analyzed and revisions considered. The challenge was to find the essential mathematics that can form the foundation for these postsecondary paths. These should include use of quantitative tools, broad competence in mathematical reasoning, mathematics required for postsecondary courses, and the ability to integrate and apply mathematics in diverse problem-solving contexts. Analysis of the 2005 Mathematics Framework revealed that some revisions would be necessary to meet this challenge.

FRAMEWORK DEVELOPMENT PROCESS

To implement this change at the 12th grade, the Governing Board contracted with Achieve, Inc., to examine NAEP's Mathematics Framework in relation to benchmarks set by the American Diploma Project. An Achieve panel of mathematicians, mathematics educators, and policymakers proposed increasing the scope and rigor of 12th-grade NAEP. Achieve developed new assessment objectives, and a panel of mathematicians and mathematics educators (including classroom teachers) reviewed and revised the objectives and matched them against the current set of objectives for grades 4 and 8. The panel conducted focus groups with the Association of State Supervisors of Mathematics and survey reviews with various NAEP constituents, using repeated rounds of reviews. The Governing Board approved the final set of grade 12 objectives in August 2006.

CHANGES FROM 2005 FRAMEWORK

The exhibit below compares the 2009-2013 and 2005 mathematics frameworks.

_	
Mathematics content	 Objectives for grades 4 and 8 remain the same New subtopic of "mathematical reasoning" at grades 4, 8, and 12 Distribution of items for each content area at all grades remains the same New objectives for grade 12
Mathematical complexity	New clarifications and new examples to describe levels of mathematical complexity
Calculator policy	Remains the same
Item formats	Remains the same
Tools and manipulatives	Remains the same

Exhibit 1. Comparison of 2005 and 2009–2013 mathematics frameworks

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