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**The singularities
as ontological limits
of the general relativity**

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Abstract

The singularities from the general relativity resulting by solving Einstein's equations were and still are the subject of many scientific debates: Are there singularities in spacetime, or not? Big Bang was an initial singularity? If singularities exist, what is their ontology? Is the general theory of relativity a theory that has shown its limits in this case?

In this essay I argue that there are singularities, and the general theory of relativity, as any other scientific theory at present, is not valid for singularities. But that does not mean, as some scientists think, that it must be regarded as being obsolete.

After a brief presentation of the specific aspects of Newtonian classical theory and the special theory of relativity, and a brief presentation of the general theory of relativity, the chapter *Ontology of General Relativity* presents the ontological aspects of general relativity. The next chapter, *Singularities*, is dedicated to the presentation of the singularities resulting in general relativity, the specific aspects of the black holes and the event horizon, including the Big Bang debate as original singularity, and arguments for the existence of the singularities. In *Singularity Ontology*, I am talking about the possibilities of ontological framing of singularities in general and black holes in particular, about the hole argument highlighted by Einstein, and the arguments presented by scientists that there are no singularities and therefore that the general theory of relativity is in deadlock. In *Conclusions* I outline and summarize briefly the arguments that support my above views.

Keywords: general relativity, general theory of relativity, Albert Einstein, singularities, black hole, event horizon, Big Bang, cosmology, gravity

Introduction

The singularities from the general relativity resulting by solving Einstein's equations were and still are the subject of many scientific debates: Are there singularities in spacetime, or not? Big Bang was an initial singularity? If singularities exist, what is their ontology? Is the general theory of relativity a theory that has shown its limits in this case?

In this essay I argue that there are singularities, and the general theory of relativity, as any other scientific theory at present, is not valid for singularities. But that does not mean, as some scientists think, that it must be regarded as being obsolete. For this, I have used the studies of several physicists and philosophers: Thomas A. Ryckman, *Early Philosophical Interpretations of General Relativity* (Ryckman 2018), Don A. Howard, *Einstein's Philosophy of Science* (D. A. Howard 2017), John D. Norton, *What Can We Learn about the ontology of Space and Time from the Theory of Relativity?* (Norton 2012), Robert Weingard, *On the ontological Status of the Metric in General Relativity* (Weingard 1976), Vincent Lam and Michael Esfeld, *The Structural Metaphysics of Quantum Theory and General Relativity* (Lam and Esfeld 2012), Erik Curiel and Peter Bokulich, *Singularities and Black Holes* (Curiel and Bokulich 2018), Gustavo E. Romero, *The ontology of General Relativity* (Romero 2013c), *Philosophical Issues of Black Holes* (Romero 2014) and *Adversus singularities: The ontology of space-time singularities* (Romero 2013a), Nick Huggett and Carl Hoefer, *Absolute and Relational Theories of Space and Motion* (Huggett and Hoefer 2018), Christopher Smeenk and George Ellis, *Philosophy of Cosmology* (Smeenk and Ellis 2017), Alan D. Rendall, *The nature of spacetime singularities* (Rendall 2005), Erik Curiel, *The Analysis of Singular Spacetimes* (Curiel 1999) and C. J. S. Clarke, *Space-Time singularities* (Clarke 1976).

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Classical Theory and Special Relativity

Newtonian classic gravity admits a geometric description. Together with special relativity, it allows a heuristic description of the general relativity (GR). The inertial movement in classical mechanics is related to the geometry of space and time, practically along geodesics in which the world lines are straight lines in relativist spacetime. (Ehlers 1973) Due to the principle of equivalence between inertial and gravitational masses, when considering gravity, no distinction is made between inertial motion and gravity. This allows the definition of a new class of bodies in a free fall, defining a geometry of space and time by a geodetic motion that depends on the gradient of the gravitational potential. Hence the Newton-Cartan theory, a geometric formula of Newtonian gravity in curved spacetime using only covariant concepts. (Ehlers 1973) (Havas 1964)

Newtonian geometric gravity is a limiting case of special relativistic mechanics. Where gravity can be neglected, physics is Lorentzian invariant as in relativity, rather than Galilean invariant as in classical mechanics. (Giulini 2006)

Lorentz's symmetry involves additional structures through light cones defining a causal structure ¹. Together with the world lines for freefalling bodies, light cones can be used to reconstruct the semi-Riemannian spacetime metric, at least up to a positive scalar factor, resulting in a conforming structure (or geometry).

If gravity is taken into account, the temporal straight lines defining an inertial frame without gravity are curved, resulting in a change in spacetime geometry. (Schutz and Schutz 1985)

Proper time measured with clocks in a gravitational field does not follow the rules of special relativity (it is not measured by the Minkowski metric), requiring a more general, curved geometry of space, with a pseudo-Riemannian metric naturally associated with a certain type of connection, the Levi-Civita connection, which satisfies the principle of equivalence and makes the local space Minkowskian. (Ehlers 1973)

In November 1915, at the Academy of Sciences of Prussia, Einstein presented the field equations ² that include gravity, which specifies how space and time geometry is influenced by matter and radiation.

¹ For each event A , there is a set of independent events of observers which can, in principle, influence or be influenced by A through signals or interactions that do not have to travel faster than light and a set of events for which such influence is impossible.

² Einstein field equations:

$$G_{\mu\nu} \equiv R_{\mu\nu} - (1/2)Rg_{\mu\nu} = (8\pi G/c^4)T_{\mu\nu}$$

where $G_{\mu\nu}$ is the Einstein tensor, a specific combination without distinction of the Ricci tensor $R_{\mu\nu}$ and the metrics, and $T_{\mu\nu}$ is the energy-momentum tensor. The proportionality constant can be fixed as $k = 8\pi G/c^4$, where G is the gravitational constant and c the speed of light. In vacuum, $R_{\mu\nu} = 0$.

General Relativity (GR)

According to GR, the gravitational force is a manifestation of the local spacetime geometry. GR is a metric theory of gravity. It is based on Einstein's equations, which describe the relationship between the geometry of a four-dimensional, pseudo-Riemannian manifold, representing spacetime and energy-impulse contained in that spacetime. Gravity corresponds to changes in space and time properties, which in turn modify the paths of objects. Curvature is caused by the energy-impulse of matter. According to John Archibald Wheeler, spacetime tells matter how to move, and matter tells spacetime how to curve. (Wheeler 1990) For weak gravitational fields and low speeds relative to light speed, the predictions of the theory converge to those of Newton's law of universal gravity.

GR shows general covariance (the laws have the same form in all coordinate systems) and does not contain invariant geometric background structures (it is independent of the actual shape of the spacetime and the value of various fields). Basically, the principle of equivalence is valid at the local level, the space-time is Minkowskian, and the laws of physics manifest Lorentz's local invariance. (Weinberg 1972)

In GR, matter and geometry must satisfy Einstein's equations. A solution to these equations is a model of universe with possible additional laws governing matter. The most known exact solutions are those that correspond to a certain type of black hole in an empty universe (Chandrasekhar 1998) (the Schwarzschild solution, the Reissner-Nordström solution and the Kerr metric), which describe an expanding universe (the Friedmann-Lemaitre-Robertson-Walker and Sitter universes), the Gödel universe (with the possibility to travel in time), the Taub-NUT solution (a homogeneous but anisotropic universe model) and the anti-Sitter space (recently highlighted in the context of the Maldacena conjecture). (S. W. Hawking and Ellis 2008)

In Newtonian gravity, the source of gravity is the mass, and in special relativity the mass is part of a more general quantity called energy-impulse tensor that includes both energy density and impulse density, and stress (pressure and shear). In GR, the gravity field equation refers to this tensor and the Ricci tensor that describes a certain class of tidal effects.

There are alternative theories to GR built on the same concepts, with different rules and/or constraints resulting different field equations (Whitehead theory, Brans-Dicke theory, teleparallelism, $f(R)$ gravity, Einstein-Cartan theory, etc.). (Brans and Dicke 1961)

1 Ontology of General Relativity

In classical vision, space and time are containers; matter is the content. The distinctive property of matter is that it carries energy and impulse, preserved over time, resulting in energy and impulse being fundamental ontologically. (Norton 2012)

GR generated various early philosophical interpretations. His adherents have highlighted the "relativization of inertia" and the concept of simultaneity, Kantians and Neo-Kantians have underlined the approach of certain synthetic "intellectual forms" (especially the principle of general covariance, and logical empirics have emphasized the philosophical methodological significance of the theory.

Reichenbach approached the GR through the "relativity of geometry" thesis, trying to build a "constructive axiomatization" (Rendall 2005) of relativity based on "elementary matters of fact" (*Elementartatbestände*) for the observable behavior of light rays, rods and clocks.

The mathematician Hermann Weyl attempted a reconstruction of Einstein's theory based on the epistemology of a "pure infinitesimal geometry", an extended geometry with additional terms that formally identified with the potential of the electromagnetic field. (Weyl and Weyl 1993, 115–16)

Thomas Ryckman asserts that the unified geometric field theory program appears to be inseparably framed into a form of scientific realism, called "structural realism," with a possible tendency inspired by Platonism. (Ryckman 2018) In its contemporary form, structural realism has both an epistemic form and an "ontic" form, the latter claiming in essence that current physical theories justify the fact that the structural features of the physical world are ontologically fundamental (Ladyman and Ross 2007), subscribing to the idea that the only ontological continuity in terms of changes in fundamental physical theory is the continuity of the structure. Ontic structural realism is a metaphysical framework that provides an adequate understanding of the characteristics of fundamental physical theories. According to him, there are structures in the field of fundamental physics in the sense of networks of concrete physical relations, without these relations to depend on fundamental physical objects that possess an intrinsic identity, ie an identity consisting of intrinsic properties or primitive thisness. This position can consider significantly the fundamental characteristics of the GR of invariance of diffeomorphism and background independence (Esfeld and Lam 2008).

Some philosophers see an opposition between traditionally metaphysics committed to an ontological priority of objects over relations, and ontic structural realism that is dedicated to an ontological priority of relations over objects. Supporters of ontic structural realism think that the error leading to this conclusion lies in the supposition of existence of an ontological distinction between objects, on the one hand, and properties, including relations, on the other (Esfeld and Lam 2011). They consider that there is no ontological distinction between objects and properties, including relations, and thus no relation of ontological dependence between objects and properties, including relation, so there is no problem of ontological priority. The distinction is only conceptual, (Lam and Esfeld 2012) but it would be a mistake to deduce from this way of

representation that there are spacetime points in the world as entities distinct ontologically from the properties of the metric field. It would result that the assumption of an ontological distinction between objects and properties, including relations, must be abandoned. There is no ontological distinction between objects and their ways of being, but only a conceptual one.

Anti-metaphysical logical empiricists such as Carnap and neo-Kantians such as Cassirer (who consider the theory as a crucial test for Erkenntniskritik, the preferred term for Marburg's transcendental idealist epistemology) played an important role in the debates on GR ontology and development of modern concept of categorization in formal semantics (D. Howard 1996). Cassirer concluded that GR presents "the most determinate application and carrying through within empirical science of the standpoint of critical idealism." (Cassirer 1921)

Einstein, together with Schlick and Reichenbach, developed a new form of empiricism, appropriate to the argumentation of GR against neo-Kantian critique. (Schlick 1921) (H. Reichenbach 1928)

Mach's idea that mass and inertial motion of the body results from the influence of all other surrounding masses (eliminating the concept of absolute space) strongly influenced Einstein in the epistemological attempt to generalize the principle of relativity, combining a valid principle of invariance of the forms of natural laws (general covariance) with a false "general relativity principle" of accelerated movements. (Ryckman 2018)

Einstein was not a scientific realist, but he believed that there was a theoretical content beyond the empirical content, that the theoretical science gave us a window on nature, even if in principle there would not be a single correct explanation at the level of deep ontology. (D. A. Howard 2017)

In this context, there has been a permanent discussion of the nature and role of the conventions in science continued until the end and after Einstein's life, (Schilpp and Schilpp 1959) whether the choice of geometry is empirical, conventional, or *a priori*. Duhem (Duhem, Vuillemin, and Broglie 1991) believes that in physics, assumptions are not tested in isolation, but only as part of theory as a whole (theoretical holism and the underestimation of choice of theory through empirical evidence). In a 1918 letter to Max Planck, Einstein approached the question of underdetermination (translation by Don A Howard):

"The supreme task of the physicist is ... the search for those most general, elementary laws from which the world picture is to be obtained through pure deduction. No logical path leads to these elementary laws; it is instead just the intuition that rests on an empathic understanding of experience. In this state of methodological uncertainty, one can think that arbitrarily many, in themselves equally justified systems of theoretical principles were possible; and this opinion is, *in principle*, certainly correct. But the development of physics has shown that of all the conceivable theoretical constructions a single one has, at any given time, proved itself unconditionally superior to all others. No one who has really gone deeply into the subject will deny that, in practice, the world of perceptions determines the theoretical system unambiguously, even though no logical path leads from the perceptions to the basic principles of the theory." (A. (Author) Einstein 1918, 31)

Einstein argued why the theoretical choice is empirically determined in a letter addressed to Schlick, where he used Schlick's argument on the elements of a theoretical ontology:

"It appears to me that the word "real" is taken in different senses, according to whether impressions or events, that is to say, states of affairs in the physical sense, are spoken of.

If two different peoples pursue physics independently of one another, they will create systems that certainly agree as regards the impressions ("elements" in Mach's sense). The mental constructions that the two devises for connecting these "elements" can be vastly different. And the two constructions need not agree as regards the "events"; for these surely belong to the conceptual constructions. Certainly, on the "elements," but not the "events," are real in the sense of being "given unavoidably in experience.

"But if we designate as "real" that which we arrange in the space-time-schema, as you have done in the theory of knowledge, then without doubt the "events," above all, are real.... *I would like to recommend a clean conceptual distinction here.*" (D. A. Howard 2017)

Einstein's point of view, according to which physical reality consists exclusively of what can be built based on spacetime coincidences, spacetime points, for example, being considered as intersections of the world lines, is now known as the "point-coincidence argument." (D. A. Howard 2017) Coincidences thus have a privileged ontic role because they are invariant and thus univocally determined.

Einstein's new perspective on spacetime ontology has led Schlick to assert that Mach has only erroneously considered elements of sensation to be real, spacetime events individualized invariantly as spacetime coincidences also having the right to be considered real due to the univocal way of their determination. (D. A. Howard 2017) Einstein agreed, provided that it is possible to distinguish between the two types of reality, the elements and the spacetime events, that "two different peoples" pursued physics independently will agree on the elements but would disagree at the level of the spacetime event ontology.

Right after the apparition of GR, a reduction of physics to geometry was discussed: "physics is a four-dimensional pseudo-geometry [i.e., a geometry distinguishing spatial and temporal dimensions] whose metric determination $g_{\mu\nu}$ is bound, according to the fundamental equations ... of my first [1915] contribution, to the electromagnetic quantities, that is, to matter. ((Hilbert 1917, 63), translation by Thomas A. Ryckman)

In GR, the density of non-gravitational energy and impulse for an event is represented by the stress-energy tensor of matter (T), being the structure that encodes total energy and momentum densities due to all non-gravitational forms. Einstein defined an analogous quantity, the stress-energy tensor for the gravitational field (t). T is a true tensor, but t is a pseudotensor, which means that T can be represented independently of a particular coordinate system, unlike t . Thus, no change in the coordinate system cannot cause T to disappear, unlike t that can be made

null for a particular event. (Norton 2012) The total energy and impulse of the system are no longer well defined.

In GR, "the gravitational field energy cannot be located". We can speak only about the gravitational energy and the momentum of an extended system, not about the density of the energy and the gravitational momentum at a certain event. (Misner et al. 2017, §20.3-20.4)

Also, GR no longer offers a precise notion of gravitational force, this being "geometrized". The restoration of the Minkowski spacetime in the flat asymptomatic regions of space allows us to use the resources of special relativity to reintroduce the notion of gravitational force, identified with the geometric disturbances of the metric structure of the exact planeness required by a Minkowski spacetime. (Norton 2012)

The material metric (metric structure) of spacetime in GR is reducible to the behavior of material entities (clocks, ray, light, geodesic, etc.) from spacetime. (Grünbaum 2012)

Respectively, spacetime measurement always depends on measuring instruments chosen as measurement standards, and metric relations involve the chosen standards. It follows that the metric relations between the material content of spacetime are not explained by the spacetime metric, but they are constitutive of it. At the same time, in the metric of the physical field, the metric relations of a spacetime are determined by an irreducible physical field, the second order metric tensor field, which, although separated from the material entities of spacetime, explains the metric relations between those entities. (Weingard 1976)

From this point of view, the epistemological status of our belief that there is a tensor metric field is the same as our beliefs about other theoretical entities, such as neutrinos. As we postulate the existence of neutrino to explain the energy deficit observed in beta decay, we will postulate the metric field, according to the physical metrical field, to explain the different

phenomena observed, such as why the free particles in a gravitational field have the trajectories they have. And in this process, the metric tensors field helps explain the metric relations observed between material entities. Robert Weingard asserts that there is an ontological disagreement between the two metrics, the first being the relations between material entities in spacetime, while the latter is a self-contained physical field, distinct and indivisible to the material content of spacetime. Robert Weingard argues that the physical field metric provides a more appropriate ratio of the ontological state of metrics in GR spacetime. According to this thesis, an empty spacetime with a well-defined metric is perfectly understandable. This idea was contradicted by Grünbaum:

”If there are no extra geochronometric physical entities to specify (individuate) the *homogeneous* elements of space-time . . . then whence do these elements of otherwise equivalent punctual constitution derive their individual identities? Must the world points not be individuated before the space-time manifold can even be meaningfully said to have a metric? I see no answer to this question as to the principle of individuation here within the framework of the ontology of the Leibnizian identity of indiscernibles. Nor do I know of any other ontology which provides an intelligible answer to this *particular* problem of individuating avowedly *homogeneous* individuals.” (Grünbaum 1970)

Since 2000, a new approach to the nature of space-time structures has emerged, particularly in Oliver Pooley's (Pooley 2012) and Harvey Brown works. (Brown 2015) The dynamic approach asserts that the spacetime structure of our world is due to the dynamic (fundamental) laws of their nature and symmetry, the spacetime structure being derived. A given geometry for spacetime constrains formally the accepted theories to those with a straight symmetry. An assumption of many substantialists was that this constraint was not only formal but ontological: geometry (hence the manifestation itself) is more fundamental than laws, or that geometry provides a "real" explanation of the form of laws. (John Earman 1992, 125). But symmetry could be reversed so that symmetry is determined ontologically by the laws of theory, resulting that geometry itself is an expression of matter dynamics. (Huggett and Hoefer 2018)

Gustavo E. Romero states that GR is a "space and time theory". (Misner et al. 2017) Spacetime is the emergence of the ontological composition of all events, (Romero 2013c) being able to be represented by a concept with a four-dimensional representation of a metric field.

2 Singularities

Within the classical theory of Newton's gravity there is the fundamental possibility of singularity. No signal can propagate from within a singularity, but its gravitational influence is permanently present externally and depends only on the total mass, the angular momentum, and the electric charge of the singularity. Singularities can be detected by the influence of their strong gravity in the immediate vicinity.

In the classical theory of Newton's gravity, an energy argument tells us that there is a speed of escape from the surface of any object.

In Newtonian theory, gravity is described by potential. Similarly, in GR, the symmetrical external (time independent) solution, called Schwarzschild spacetime, depends only on the mass of the inner object. The Schwarzschild radius in GR is the maximum radius of a surface under which light can no longer escape. This "horizon radius" is, coincidentally, the same as the critical ray for objects in Newtonian "singularities".

Gravitational singularities in GR are spacetime locations where the gravitational field becomes infinite. Scalar invariant curves of spacetime include a measure of matter density. Some physicists and philosophers believe that because the density of matter tends to become infinite in singularity, spacetime laws are no longer valid there.

A gravitational singularity almost universally accepted in astrophysics and cosmology as the earliest state of the universe, is the Big Bang. (Wald 1984) In this case also, the known laws of physics are no longer valid. (S. Hawking 2012)

GR predicts that any object that goes beyond a certain point (for stars, the Schwarzschild radius) forms a black hole with a singularity, with an action limit defined by an event horizon. (Curiel and Bokulich 2018) Penrose-Hawking's theorems of singularity state that in this case geodesics end in singularity. (Moulay 2012)

The theory of loop quantum gravity suggests that singularities cannot exist (Gambini, Olmedo, and Pullin 2013) because, due to the effects of quantum gravity, there is a minimum distance beyond which the force of gravity is no longer increasing.

The Schwarzschild solution to the GR equations describes a non-rotating, uncharged black hole. In convenient coordinate systems, part of the metric becomes infinite at event horizon. In a rotating black hole (Kerr black hole) the singularity appears on a ring and can become theoretically a "wormhole". (Wald 1984)

A special type of singularity is the "naked singularity" which, although it is forbidden by the cosmic censorship hypothesis, in 1991 physicists Stuart Shapiro and Saul Teukolsky performed computer simulations of a planetary rotation of cosmic dust resulting in GR allowing for "naked singularities". (Goswami, Joshi, and Singh 2005) Moreover, the cosmic censorship assertion states that there can be realistic singularities (without perfect symmetries, matter with realistic properties), but they are hidden behind the horizon and thus invisible.³ (Wald 1984).

Stephen Hawking suggested that black holes can radiate energy, preserving entropy and solving problems of incompatibility with the second law of thermodynamics. This means that the black hole has limited cosmic life.

³ Restrictions of singularities in the future exclude original singularities, such as Big Bang, which are, in principle, visible to observers at a later cosmic moment. The cosmic censorship conjecture was first presented by Penrose in a work in 1969. (Penrose 1969)

Paul Townsend states that singularities are a generic feature of GR and are inevitable if a body has gone through a certain stage (Townsend 1997) and also at the beginning of a broad class of expanding universes. (S. W. Hawking 1966) The generic structure of these entities is currently being investigated (for example, BKL conjecture). (Berger 2002)

Regarding the definition of the singularity, there is a clear disagreement: although it changes the local geometry, difficulties arise in speaking of it as something that is found in a certain space-time location, which is why some physicists and philosophers propose to speak of "singular spacetimes" instead of "singularities." The most important definitions refer either to incomplete paths or to the idea of space-time "missing points" or an idea combining the two above concepts, respectively a single structure with "pathological" behavior (spacetime deformation which manifests himself as a gravitational field). (Curiel and Bokulich 2018)

Black Holes

Black holes raise some conceptual aspects. Although they are regions of spacetime, black holes are also thermodynamic entities with a temperature and entropy; and the evolution of the black holes is apparently in conflict with standard quantum physics because it excludes entropy growth. (Curiel and Bokulich 2018)

In the center of a black holes of RG there is a gravitational singularity, a region in which the spacetime curve becomes infinite. Singularity contains the entire black hole mass, resulting in infinite density. (Carroll and Carroll 2004) In the case of a charged (Reissner-Nordström) black hole, or rotating (Kerr) black hole, it is possible to avoid singularity, but it is hypothetical to exit black hole in a different space-time, the black hole acting as a wormhole, and thus the possibility of traveling in another universe or in time. Droz considers this possibility only theoretical, because any disturbance would destroy this possibility. (Droz, Israel, and Morsink

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