

# Petroleum Reservoir Simulation

Using  
3-Phase,  
3-D Finite Elements



Husam M. Yaghi, Ph.D.

PETROLEUM RESERVOIR SIMULATION  
USING 3D, 3Phase, Parallel Finite Elements

## DEDICATION

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To my mother and father  
(Bless their Soul)

# ACKNOWLEDGEMENTS

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This book is an updated re-printed of the author's Ph.D. dissertation work.

The US Department of Energy (DoE) was kind enough to fund the research twice and also to give source code access to their reservoir simulation BOAST II, which this research had used as the foundation.

Dr. John M. Tyler of Louisiana State University (LSU) had provided unlimited support and guidance throughout the six years of research.

My mother and father (bless his sole) were encouraging till completion.

My newly-wed wife at that time had sacrificed her honeymoon to be by my side while refining my Cray's parallel code to improve accuracy of the simulation results.

To God the almighty, thanks for the blessings.

Husam Yaghi, Ph.D.

# FOREWORD

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In this book, the author presents a detailed complex mathematical and computational model for the simulation of fluid flow in reservoirs. The research was conducted in collaboration with Dr. John M. Tyler of LSU and was funded by the United States Department of Energy. The results were published in a Ph.D. dissertation and at several international conferences.

Reservoir simulation is used by oil and gas companies in the development of new fields and in developed fields where production forecasts are needed to help make investment decisions. The simulation is also used to predict the available reserves of oil or gas underground and to identify opportunities to increase oil production in heavy oil deposits.

Hydrocarbons and their associated impurities occur in rock formations that are usually buried thousands of feet or meters below the surface. Scientists and engineers often call rock formations that hold hydrocarbons "reservoirs". Oil does not flow in underground rivers or pool up in subterranean lakes, contrary to what some people think. And, as you've learned, gasoline and other refined hydrocarbons do not naturally occur in pockets under the ground, just waiting to be drilled for. Instead, crude oil and natural gas occur in buried rocks and, once produced from a well, companies have to refine the crude oil and process the natural gas into useful products. Further, not every rock can hold hydrocarbons. To serve as an oil and gas reservoir, rocks have to meet several criteria.

To understand how hydrocarbons get into buried rocks, visualize an ancient sea teeming with vast numbers of living organisms. Some are fishes and other large swimming beasts; others, however, are so small that you cannot see them without a strong magnifying glass or a microscope. Millions and millions of them live and die daily. It is these tiny and plentiful organisms that many scientists believe gave rise to oil and gas.

When these tiny organisms died millions of years ago, their remains settled to the bottom. As thousands of years went by, enormous quantities of this organic sediment accumulated in thick deposits on the seafloor. The organic material mixed with the mud and sand on the bottom. Ultimately, many layers of sediments built up until they became hundreds or thousands of feet (meters) thick.

The tremendous weight of the overlying sediments created great pressure and heat on the deep layers. The heat and pressure changed the deep layers into rock. At the same time, heat, pressure, and other forces changed the dead organic material in the layers into hydrocarbons: crude oil and natural gas.

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# TABLE OF CONTENT

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<b>DEDICATION</b> .....	ii
<b>ACKNOWLEDGEMENTS</b> .....	iii
<b>ABSTRACT</b> .....	vi
<b>CHAPTER 1. INTRODUCTION</b> .....	1
1.1 Statement of the Problem.....	4
<b>CHAPTER 2. LITERATURE REVIEW</b> .....	11
2.1 Current Reservoir Simulations.....	11
2.2 Well Models.....	12
2.3 Black-Oil Reservoir Model.....	15
2.4 Domain Decomposition.....	16
2.5 Coupling of FDM and FEM.....	17
2.6 Parallel Computing in Reservoir Simulation.....	18
<b>CHAPTER 3. FINITE ELEMENTS METHOD</b> .....	20
3.1 FEM Introduction.....	20
3.2 Finite Element Discretization.....	23
3.3 Convergence of the FEM.....	25
3.4 Possible FEM Benefits.....	26
<b>CHAPTER 4. MESH SYSTEMS</b> .....	28
4.1 Block Centered.....	28
4.2 Coarse Mesh.....	29
4.3 Cylindrical Mesh.....	31
4.4 Treatment of Irregularly Shaped Elements.....	31
4.5 Wellbore Vicinity Model.....	33
<b>CHAPTER 5. NUMERICAL MODEL</b> .....	35
5.1 Discretization of the Flow Equations.....	36
5.2 Finite Element Formulation.....	57
<b>CHAPTER 6. COMPUTATIONAL MODEL</b> .....	73
6.1 System Requirements.....	75
6.2 Simulation Process.....	75
6.3 Input Data Requirements.....	76
6.4 Coupling of Well Region and Reservoir Simulators.....	76

6.5	Wellblock Pressure Distribution.....	80
6.6	Transmissibilities.....	84
6.7	Node Numbering.....	87
<b>CHAPTER 7. PARALLEL MODEL.....</b>		<b>90</b>
7.1	Wellbore Vicinity Parallelization.....	91
7.2	Other Parallel Implementations.....	96
<b>CHAPTER 8. MULTIMEDIA VISUALIZATION.....</b>		<b>99</b>
8.1	Wellbore Vicinity Prototype Model.....	100
8.2	Distributed Visualization.....	102
<b>CHAPTER 9. RESULTS.....</b>		<b>105</b>
9.1	Case 1.....	106
9.2	Case 2.....	117
9.3	Comparison of Results.....	135
<b>CHAPTER 10. SUMMARY.....</b>		<b>139</b>
<b>CHAPTER 11. CONCLUSION AND RECOMMENDATIONS.....</b>		<b>140</b>
<b>BIBLIOGRAPHY.....</b>		<b>146</b>
<b>NOMENCLATURE.....</b>		<b>154</b>
<b>APPENDIX A INPUT DATA: CASE 1.....</b>		<b>156</b>
<b>APPENDIX B INPUT DATA: CASE 2.....</b>		<b>161</b>
<b>ABOUT THE AUTHOR.....</b>		<b>166</b>

# CHAPTER 1

## INTRODUCTION

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Oil and natural gas are two of the world's most important natural resources. They are building materials of modern life. Together, oil and natural gas are called petroleum and remain in a reservoir until produced. In general, a reservoir is both the reservoir rock and its fluid content [1]. The life of a reservoir can be classified into primary or secondary recovery phases. In the primary recovery phase of a reservoir, oil is obtained by natural drive mechanisms. In the secondary phase a recovery process can be initiated to maintain the pressure in a reservoir by injecting water or gas. The later type is the focus of this research. Actually, there can be recovery processes after the secondary process but the mechanism for these is beyond the scope of this research.

Reservoir simulation has a key role in the development and management of petroleum resources. One objective of this research is to provide better tools for the understanding of the complex physical fluid flow processes that occur around a wellbore in a reservoir. A better understanding of this process could lead to increased recovery and reduced expenses. Classical reservoir simulation deals with the fluids on a gross average basis and does not account adequately for the flow pattern variations in the reservoir and fluid changes in the wellbore caused by pressure and time [1]. While many advances have been made in reservoir engineering and well drilling, the modeling and technology have lagged.



The accurate computer simulation of the multiphase fluid flow processes continues to be a difficult endeavor. Such problems feature near-discontinuities in the solution which are not sufficiently resolved by standard domain discretization procedures without extensive grid refinement. Meanwhile, the problem domains are often large-scale, irregularly shaped, and exhibit heterogeneous characteristics. Methods which are developed to simulate these processes are often useful in a number of other disciplines in which the differential equations are analogous (e.g. the flow of heat). Such processes as groundwater flow and hazardous waste migrations are closely related to reservoir engineering problems, and improvements in the modeling of one type of fluid flow problem may be utilized in obtaining an improved solution to another.

The direction of flow and rate of flow depend on the physical features of the flowing medium such as its viscosity, phase behavior, and the nature of the reservoir rock in which it is flowing such as permeability, pore geometry, etc. Viscosity is a fluid property responsible for the frictional drag or shear resistance which develops when one layer of a fluid slides over another. Permeability is a property that measures the ability of the reservoir rock to transport fluids through itself. Permeability is independent of the nature of the fluid and is determined solely by the structure of the porous media. Porosity measures the reservoir rock's ability to store petroleum, which may be defined as one minus the fraction of the bulk volume of the rock comprised of solid matter. Viscosity, permeability, and porosity are expressed as percentages.

New technologies have changed the way we search for the petroleum. We study the ground beneath the surface using technology that gives us a three-dimensional view of what that ground is like. All of these high-tech tools help pinpoint where the oil and natural gas are-and where they are not-so we drill fewer wells.

Engineers are constantly in search of tools that could help in enhancing the recovery of petroleum. Reservoir simulation has a key role in the development and management of this activity. Reservoir simulation is a process for predicting the behavior of a real reservoir from the analysis of a model of that reservoir. The model could be a scaled physical model examined in a laboratory, or mathematical. The mathematical model developed in this research is a set of nonlinear partial differential equations that describe the activities occurring within the reservoir. These activities are the simultaneous flow of three phases (water, oil, and gas) along with the mass transfer between these phases.

The mathematical model accounts for various factors affecting the behavior of the fluids. It takes into account gravity, pressure, heterogeneity, and geometry. The mathematical model begins by combining Darcy's flow for each phase with a simple differential material balance for each phase. Henry Darcy originally designed a flow tube to determine the most efficient means of filtrating the municipal sewage water in Dijon, France, in 1865. He found that the rate of water flow through a porous bed of a "given nature" is proportional to the pressure and to the cross-sectional area normal to the direction of flow and inversely proportional to the length of the flow path. He also determined that the quantity of flow is related to the

nature of the porous medium. Darcy's Law was initially developed for one-dimensional flow through a porous media. However, combining this relationship with calculus, this law has been extended to flow in two or three dimensions .

### **1.1 Statement of the Problem**

The purpose of this research is to develop a more accurate numerical model of three dimensional three-phase fluid flow in a porous media; specialized for the treatment of the wellbore vicinity where majority of fluid activity occurs. Most reservoir simulations models available today, obtain solutions to fluid flow equations that are usually nonlinear partial differential equations by replacing derivatives with finite-difference approximations [4,6]. The use of these approximations introduces an error known as truncation error. For many problems the error is small and the approximate solutions of the subsequent finite difference equations are sufficiently accurate. However, truncation errors can cause significant solution inaccuracies for certain types of problems in which viscous forces are much larger than capillary forces.

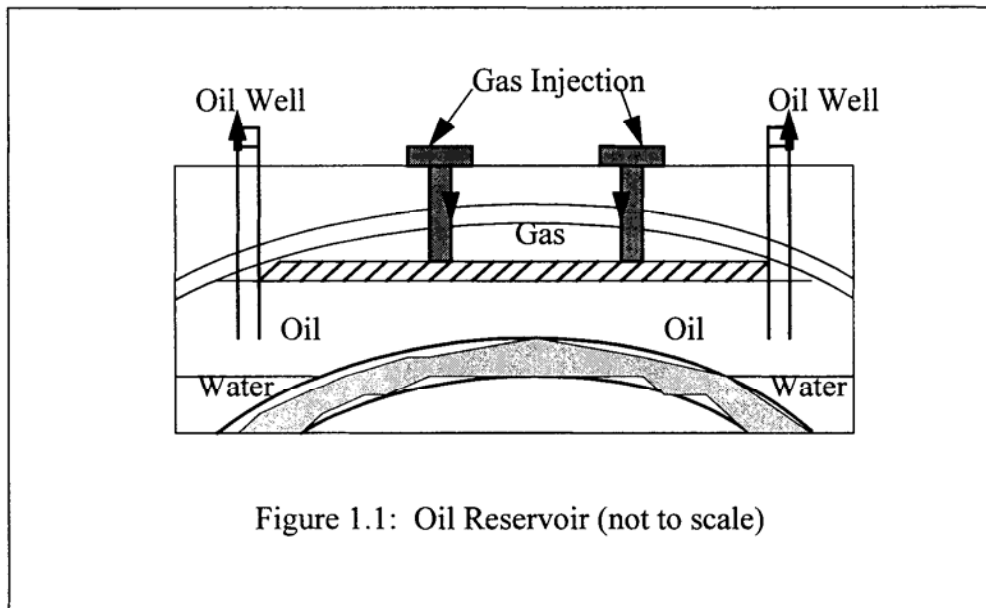
Available public domain reservoir simulators represent the behavior of fluid flow in the wellbore vicinity with Cartesian coordinates and fail to provide robust and correct answers for the large pressure and saturation changes in this vicinity. One reason is that these reservoir simulators often make simplifications of the equations that are not physically realistic. Another reason is that they use a finite difference method to represent the wellbore vicinity where high mobility and large changes in the saturation of the fluids and pressure occur.

An important step for this simulation of physical phenomena is the transformation of the underlying differential equations into a finite discretized space. In the considered domain, the resulting partial differential equations are approximated using numerical methods on finite discrete intervals.

In spite of the enormous commercial potential and interest in this field, an extensive literature survey has shown that commercial modeling of the “wellbore” region has been extremely limited. Commercial simulators are proprietary in order for the companies to protect their investment from competitors. Therefore, commercial tools are not available to us in a form that can be modified for use with this research.

In order to develop a reservoir simulation tool, understanding of the physical problem is required. Mistakenly, it is thought that petroleum is found underground in a pool from which production occurs. On the contrary, oil and natural gas are trapped inside tiny rock holes or pores of rock which complicates the task of simulating the process. In the secondary recovery phase, petroleum is forced out by injecting another fluid through another well (injection well), Figure 1.1.

To produce petroleum from a production well, engineers generally drill in an approximate range from 1,000 to 20,000 feet deep for vertical wells and the same depth plus a horizontal length for horizontal wells. The well radius usually ranges between 4 to 6 inches at the reservoir depth. There are other issues associated with drilling wells that affect reservoir simulation. However drilling engineering is beyond the scope of this research.



A readily available public reservoir simulator BOAST II (Black Oil Applied Simulation Tool) from the United States Department of Energy could be modified to simulate the conditions encountered in the wellblock region of interest. Hence, our reservoir wellblock region model uses BOAST II as the basic building block because it was the best working simulator that supplied the source code to which our model could be interfaced.

BOAST II has several drawbacks, some of which are:

1. It is based on the finite difference model, which divides the entire region of interest into equally spaced blocks. This arrangement gives no special consideration to areas of high activity. For the production wellblock, we use finite elements to create more points (sub-blocks) closer to the wellbore. Our results show that most activity takes place within the

first 20 feet or so from the well in a wellblock. Finite elements also better model radial flow in this region.

2. It uses one average pressure value for each reservoir block. To make modeling of this highly active block more accurate, we developed a finite element model to further examine this wellblock. Once we are finished producing a detailed map of pressure and saturation activities at various points in that block, all these values are averaged and inserted back into BOAST as an improved value for the block.

In building our simulator, we follow four basic major steps. First, a physical model of the flow process is developed incorporating as much physics as is deemed necessary to describe the essential phenomena. Second, a mathematical formulation of the physical model is developed, usually involving coupled systems of nonlinear partial differential equations. Third, once the properties of the mathematical model, such as existence, uniqueness, and regularity of the solution, are sufficiently well understood and the properties seem compatible with the physical model, discretized numerical approximations of the mathematical equations are produced [9]. Finally, a computer wellbore model is developed, executed, and results obtained which are compared with actual observations of this physical process to demonstrate its validity. The actual observations for testing were furnished by the United States Department of Energy - National Institute of Petroleum and Energy Research.

An important aspect of the wellbore research problem is that the geometry of flow is radial [10], which must be taken into account to accurately simulate

multiphase flow in the neighborhood of the wellbore. The simplest case is that of perfect radial flow of homogeneous fluid into a well. Such flow is obtained if the well completely penetrates the rock stratum and the distant fluid acts uniformly in all directions radiating from the axis of the well bore. From a practical point of view, this perfectly radial flow is too idealized because it implies an exactly uniform pressure imposed on a circular boundary.

It is anticipated that even cases with only a single well will, in general, have nonuniform pressure distribution over their external boundaries and the boundaries themselves over which the pressure distributions are pre-assigned and known will be other than circular in shape. In all cases, the flow into the wells will be unsymmetrical and the pressure distributions on the external boundary will be nonuniform.

If a localized region around the wellbore can be considered homogeneous, the flow is radial, and the fluid flow equations for a single phase can be formulated and solved analytically in cylindrical coordinates. In this case, it is known that a "radial" grid system provides much better results than the use of a rectangular grid system. As the distance from the wellbore increases, the flow becomes more linear. This means that a rectangular grid system can be applied at some distance from the wellbore with confidence to discretize the fluid flow equations and a Cartesian coordinate system would be accurate. Therefore it is necessary for reservoir simulators to properly represent the fluid flow in the regions of interest and then couple the various systems together.

Since the regions in the vicinity of the wellbore often have significant pressure and saturation changes, an implicit treatment of the transmissibility in this region is needed. On the other hand, in reservoir zones distant from the wellbore, the transmissibility may be treated explicitly. We employ a finite element method (FEM) for the wellbore region [93] and the finite difference method (FDM) from BOAST II for the other regions of the reservoir. The FEM is necessary for the wellbore region where large condition changes are exhibited. The FDM is adequate for other regions of the reservoir where more uniform conditions are likely to be found. The simulator then has to couple both regions together for the exchange of data. As part of this research a FEM/FDM grid interface is developed and employed as illustrated in Figure 4.3. Boundary conditions between these two methods are also developed. All fluids are treated as compressible and transient, unlike a common practice of assuming the opposite to reduce complexity.

To accomplish the objectives of this research, the following aspects of reservoir simulation were addressed:

- an improved representation for the wellbore vicinity;
- more accurate treatment of the wellbore/reservoir interaction;
- a coupling of the FEM wellbore region model with the FDM reservoir model;
- utilization of parallel computing.

Our wellbore model can be used with other reservoir simulators provided the interface is done at the source code level. The model works independently except



for the initial input of data. However, if the input data is faulty, the wellbore model will produce faulty results. The model can predict the wellbore behavior and can produce detailed history information about the fluid pressures and saturations at various locations within a wellblock; these were not available from BOAST II.

In addition to the output files that our wellbore model produces, we also developed and added a web-based multimedia visualization tool [90,91]. This tool reads the simulation output files, generates a web-based table showing the results, and then passes that data to a Java applet for colored and animated visualization. This prototype model also suggested a bilingual graphical user interface and was presented at a visualization conference in Toronto, Canada [90].

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