

**PASSIVE MICROWAVE
COMPONENTS AND ANTENNAS**

**PASSIVE MICROWAVE
COMPONENTS AND ANTENNAS**

Edited by
VITALIY ZHURBENKO

Published by In-Teh

In-Teh

Olajnica 19/2, 32000 Vukovar, Croatia

Abstracting and non-profit use of the material is permitted with credit to the source. Statements and opinions expressed in the chapters are those of the individual contributors and not necessarily those of the editors or publisher. No responsibility is accepted for the accuracy of information contained in the published articles. Publisher assumes no responsibility liability for any damage or injury to persons or property arising out of the use of any materials, instructions, methods or ideas contained inside. After this work has been published by the In-Teh, authors have the right to republish it, in whole or part, in any publication of which they are an author or editor, and the make other personal use of the work.

© 2010 In-teh

www.intechweb.org

Additional copies can be obtained from:

publication@intechweb.org

First published April 2010

Printed in India

Technical Editor: Sonja Mujacic

Cover designed by Dino Smrekar

Passive Microwave Components and Antennas,

Edited by Vitaliy Zhurbenko

p. cm.

ISBN 978-953-307-083-4

Preface

State-of-the-art microwave systems always require higher performance and lower cost microwave components. Constantly growing demands and performance requirements of industrial and scientific applications often make employing traditionally designed components impractical. For that reason, the design and development process remains a great challenge today. This problem motivated intensive research efforts in microwave design and technology, which is responsible for a great number of recently appeared alternative approaches to analysis and design of microwave components and antennas. This book highlights these new trends focusing on passive components such as novel resonators, filters, diplexers, power dividers, directional couplers, impedance transformers, waveguides, transmission lines and transitions as well as antennas, metamaterial-based structures, and various electromagnetic analysis and design techniques.

Modelling and computations in electromagnetics is a quite fast-growing research area. The recent interest in this field is caused by the increased demand for designing complex microwave components, modeling electromagnetic materials, and rapid increase in computational power for calculation of complex electromagnetic problems. The first part of this book is devoted to the advances in the analysis techniques such as method of moments, finite-difference time-domain method, boundary perturbation theory, Fourier analysis, mode-matching method, and analysis based on circuit theory. These techniques are considered with regard to several challenging technological applications such as those related to electrically large devices, scattering in layered structures, photonic crystals, and artificial materials.

The second part of the book deals with waveguides, transmission lines and transitions. This includes microstrip lines (MSL), slot waveguides, substrate integrated waveguides (SIW), vertical transmission lines in multilayer media as well as MSL to SIW and MSL to slot line transitions.

Impedance matching is an important aspect in the design of microwave circuitry since impedance mismatches may severely deteriorate performance of the overall system. Different techniques for wideband matching are presented in the third part of this book. The design of compact microwave resonators and filters is also covered in this part. Compact, high-performance microwave filters are essential for high-efficiency miniaturized microwave systems. The filter circuit size is large in traditionally designed planar bandpass filters due to a high number of large area resonators. The rejection level in the upper stopband of the filters is usually degraded by the spurious response at twice the passband frequency. Several types of resonators have been designed to overcome these problems, such as miniaturized hairpin resonators, stepped-impedance hairpin resonators, and slow-wave open-loop resonators.

Miniaturized resonators lead to a reduced filter size, but not always improve the spurious response. Another method relies on various resonator combinations within one filter structure to reduce the circuit size, such as the loop resonator or hairpin resonator incorporated with one or several open stubs.

Recently, microwave filters based on electromagnetic bandgap structures and artificial materials have attracted a great deal of interest because of improved characteristics in comparison to traditional filter design. Such artificial materials can be realized using periodic inclusion of variously shaped metals into a host medium. The most prominent candidate for such structures has been the split-ring resonator. In addition to the split-ring resonator there are several alternative realizations based on lumped elements, quasi-lumped LC resonators and other planar microwave resonators which are in details discussed in the fourth part of this book.

Antennas are key components in most microwave devices and systems. They are used everywhere where a transformation between a guided wave and a free-space wave (or vice versa) is required. The final part of the book is dedicated mainly to the design and applications of planar antennas and arrays including metamaterial-based antennas, monopoles, slot antennas, reflector antennas and arrays.

The book concludes with a chapter considering accuracy aspects of antenna gain measurements.

Editor

Vitaliy Zhurbenko

Contents

Preface	V
1. Boundary Perturbation Theory for Scattering in Layered Rough Structures Pasquale Imperatore, Antonio Iodice and Daniele Riccio	001
2. Numerical Analysis of Planar Periodic Multilayer Structures by Method of Moments Stanislav Gona	027
3. The High-Order Symplectic Finite-Difference Time-Domain Scheme Wei E.I. Shaa, Xian-liang Wub, Zhi-xiang Huangb and Ming-sheng Chenc	047
4. Nonlocal Electromagnetic Media: A Paradigm for Material Engineering Said M. Mikki and Ahmed A. Kishk	073
5. Numerical Modeling of Photonic Crystal Circuits Using Fourier Series Expansion Method Based on Floquet-Modes Koki Watanabe and Kiyotoshi Yasumoto	095
6. Computer Aided Design of Waveguide Devices by Mode-Matching Methods Jorge A. Ruiz-Cruz, Jose R. Montejo-Garai and Jesus M. Rebollar	117
7. Circuitual analysis of cylindrical structures applied to the electromagnetic resolution of resonant cavities Felipe L. Penaranda-Foix and Jose M. Catala-Civera	141
8. Solving Inverse Scattering Problems Using Truncated Cosine Fourier Series Expansion Method Abbas Semnani and Manoochehr Kamyab	169
9. SLOT OPTICAL WAVEGUIDES SIMULATIONS & MODELING Muddassir Iqbal, Z. Zheng and J.S. Liu	185
10. Analysis and Design of SIW Components Based on H-Plane Planar Circuit Approach Isao Ohta and Mitsuyoshi Kishihara	211
11. Design and Modeling of Microstrip Line to Substrate Integrated Waveguide Transitions Ting-Yi Huang, Tze-Min Shen and Ruey-Beei Wu	225

12. Microstrip-Slot Transition and Its Applications in Multilayer Microwave Circuits	247
Norhudah Seman and Marek E. Bialkowski	
13. Vertical Transmission Lines in Multilayer Substrates and Highly-Integrated Filtering Components Based on These Transmission Lines	267
Taras Kushta	
14. Impedance Transformers	303
Vitaliy Zhurbenko, Viktor Krozer and Tonny Rubæk	
15. Design of Compact Planar Ultra-Wideband Bandpass Filters	323
Yasushi Horii	
16. A Dual-Mode Wide-Band Bandpass Filter Using the Microstrip Loop Resonator with Tuning Stubs	341
Jessada Konpang	
17. Tunable Dielectric Microwave Devices with Electromechanical Control	367
Yuriy Poplavlo, Yuriy Prokopenko and Vitaliy Molchanov	
18. Dual Mode Microstrip Ring Resonator with Composite-Right/Left-handed Line	383
M.K.Haldar, Hieng Tiong Su and Kian Kiong Fong	
19. Electrically small resonators for metamaterial and microwave circuit design	403
Marta Gil, Francisco Aznar, Adolfo Vélez, Miguel Durán-Sindreu, Jordi Selga, Gerard Sisó, Jordi Bonache and Ferran Martín	
20. Compact CPW Metamaterial Resonators for High Performance Filters	429
Ibraheem A. I. Al-Naib, Christian Jansen and Martin Koch	
21. Passive Diplexers and Active Filters based on Metamaterial Particles	451
Daniel Segovia-Vargas, Vicente González-Posadas, José Luis Jiménez-Martín, Luis Enrique García-Muñoz and Oscar García-Pérez	
22. Multifrequency and Multifunction Metamaterial-loaded Printed Antennas	469
Francisco Javier Herraiz-Martínez, Daniel Segovia-Vargas, Eduardo Ugarte-Muñoz, Luis Enrique García-Muñoz and Vicente González-Posadas	
23. Wideband planar plate monopole antenna	487
H. R. Hassani and S. M. Mazinani	
24. Collinear Microstrip Patch Antennas	513
Alois Holub and Milan Polívka	
25. Design of Non-Uniformly Excited Linear Slot Arrays Fed by Coplanar Waveguide	533
JP Jacobs, J Joubert and JW Odendaal	

Boundary Perturbation Theory for Scattering in Layered Rough Structures

Pasquale Imperatore, Antonio Iodice and Daniele Riccio
*University of Naples "Federico II",
Department of Biomedical, Electronic and Telecommunication Engineering,
Napoli, Italy*

1. Introduction

The electromagnetic wave interaction with layered structures constitutes a crucial topic of current interest in theoretical and experimental research. Generally speaking, several modelling and design problems, encountered, for instance, in SAR (*Synthetic Aperture Radar*) application, GPR (*Ground Penetrating Radar*) sensing, radar altimeter for planetary exploration, microstrip antennas and MMICs (*Monolithic Microwave Integrated Circuits*), radio-propagation in urban environment for wireless communications, through-the-wall detection technologies, optics, biomedical diagnostic of layered biological tissues, geophysical and seismic exploration, lead to the analysis of the electromagnetic wave interaction with multilayered structure, whose boundaries can exhibit some amount of roughness.

This chapter is aimed primarily at providing a comprehensive analytical treatment of electromagnetic wave propagation and scattering in three-dimensional multilayered structures with rough interfaces. The emphasis is placed on the general formulation of the scattering problem in the analytic framework of the *Boundary Perturbation Theory (BPT)* developed by Imperatore et al. A systematic perturbative expansion of the fields in the layered structure, based on the gently rough interfaces assumption, enables the transferring of the geometry randomness into a non-uniform boundary conditions formulation. Subsequently, the fields' expansion can be analytically evaluated by using a recursive matrix formalism approach encompassing a proper scattered field representation in each layer and a matrix reformulation of non-uniform boundary conditions. A key-point in the development resides in the appropriate exploitation of the *generalized reflection/transmission* notion, which has strong implications in order to make the mathematical treatment manageable and to effectively capture the physics of the problem. Two relevant compact closed-form solutions, derived in the first-order limit of the perturbative development, are presented. They refer to two complementary *bi-static* configurations for the scattering, respectively, from and through layered structures with arbitrary number of rough interfaces. The employed formalism is *fully-polarimetric* and suitable for applications. In addition, it is demonstrated how the symmetrical character of the *BPT* formalism reflects the inherent conformity with the *reciprocity* theorem of the electromagnetic theory.

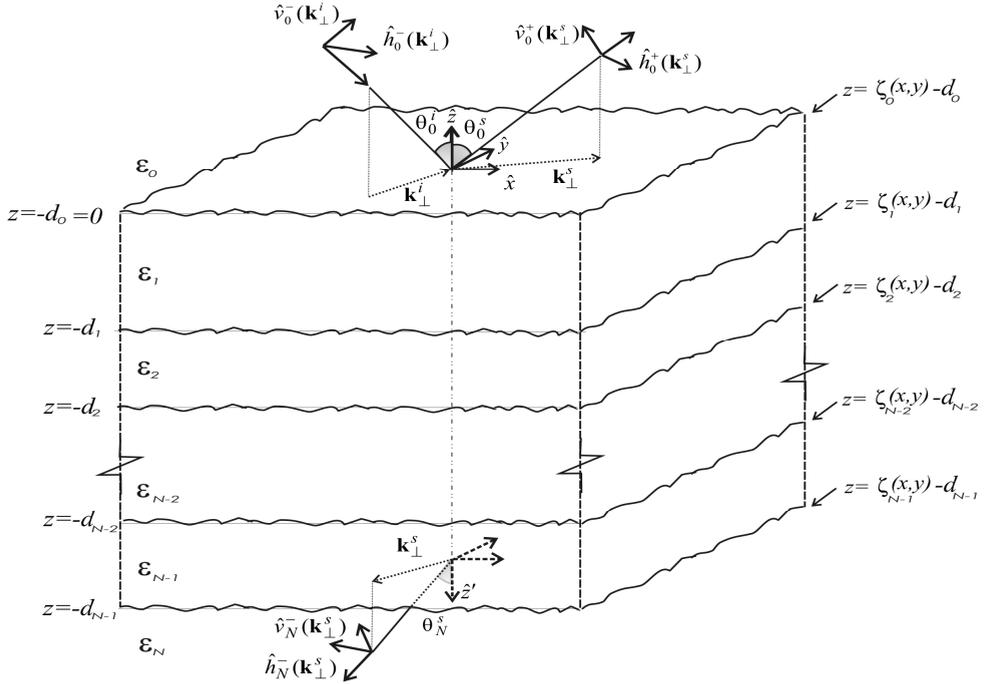


Fig. 1. Geometry for an N-rough boundaries layered medium

2. Statement of the problem

When stratified media with rough interfaces are concerned, the possible approaches to cope with the EM scattering problem fall within three main categories. First, the numerical approaches do not permit to attain a comprehensive understanding of the general functional dependence of the scattering response on the structure parameters, as well as do not allow capturing the physics of the involved scattering mechanisms. In addition, the numerical approach turns out to be feasible for non-fully 3D geometry or configurations in which a very limited number of rough interfaces is accounted for. Layered structures with rough interfaces have been also treated resorting to *radiative transfer theory* (RT). However, coherent effects are not accounted for in RT theory and could not be contemplated without employing full wave analysis, which preserves phase information. Another approach relies on the full-wave methods. Although, to deal with the electromagnetic propagation and scattering in complex random layered media, several analytical formulation involving some idealized cases and suitable approximations have been conducted in last decades, the relevant solutions usually turn out to be too complicated to be generally useful in applications, even if simplified geometries are accounted for. The proliferation of the proposed methods for the simulation of wave propagation and scattering in stratified media and the continuous interest in this topic are indicative of the need of appropriate modelling and interpretation of the complex physical phenomena that take place in layered structures. Indeed, the availability of accurate, sound physical and manageable models turns out still to

be a strong necessity, in perspective to apply them, for instance, in retrieving of add-valued information from the data acquired by microwave sensors.

Generally speaking, an exact analytical solution of *Maxwell* equations can be found only for a few idealized problems. Subsequently, appropriate approximation methods are needed. Regarding the perturbative approaches, noticeable progress has been attained in the analytic investigation on the extension of the classical SPM (small perturbation method) solution for the scattering from rough surface to specific layered configurations. Most of previous existing works analyze different layered configurations in the first-order limit, using procedures, formalisms and final solutions that can appear of difficult comparison (Yarovoy et al., 2000), (Azadegan and Sarabandi, 2003), (Fuks, 2001). All these formulations, which refer to the case of a single rough interface, have been recently unified in (Franceschetti et al, 2008). On the other hand, solution for the case of two rough boundaries has also been proposed in (Tabatabaenejad and Moghaddam, 2006).

Methodologically, we underline that all the previously mentioned existing perturbative approaches, followed by different authors in analyzing scattering from simplified geometry, imply an inherent analytical complexity, which precludes the treatment to structures with more than one (Fuks, 2001) (Azadegan et al., 2003) (Yarovoy et al., 2000) or two (Tabatabaenejad et al., 2006) rough interfaces.

The general problem we intend to deal with here refers to the analytical evaluation of the electromagnetic scattering from and through layered structure with an arbitrary number of rough interfaces (see Fig.1). As schematically shown in fig.1, an arbitrary polarized monochromatic plane wave

$$\mathbf{E}_0^i(\mathbf{r}) = [E_0^{ih} \hat{h}_0^-(\mathbf{k}_\perp^i) + E_0^{iv} \hat{v}_0^-(\mathbf{k}_\perp^i)] e^{j(\mathbf{k}_\perp^i \cdot \mathbf{r}_\perp - k_{z0}^i z)} \quad (1)$$

is considered to be incident on the layered medium at an angle θ_0^i relative to the \hat{z} direction from the upper half-space, where in the field expression a time factor $\exp(-j\omega t)$ is understood, and where, using a spherical frame representation, the incident vector wave direction is individuated by θ_0^i, φ_0^i :

$$k_0 \hat{k}_0^i = \mathbf{k}_0^i = \mathbf{k}_\perp^i - \hat{z} k_{z0}^i = k_0 (\hat{x} \sin \theta_0^i \cos \varphi_0^i + \hat{y} \sin \theta_0^i \sin \varphi_0^i - \hat{z} \cos \theta_0^i), \quad (2)$$

with

$$\hat{h}_0^-(\mathbf{k}_\perp^i) = \frac{\hat{k}_0^i \times \hat{z}}{|\hat{k}_0^i \times \hat{z}|} = \sin \varphi_0^i \hat{x} - \cos \varphi_0^i \hat{y}, \quad (3)$$

$$\hat{v}_0^-(\mathbf{k}_\perp^i) = \hat{h}_0^-(\mathbf{k}_\perp^i) \times \hat{k}_0^i = (\hat{x} \cos \varphi_0^i + \hat{y} \sin \varphi_0^i) \cos \theta_0^i + \hat{z} \sin \theta_0^i, \quad (4)$$

where $\mathbf{k}_\perp^i = k_x^i \hat{x} + k_y^i \hat{y}$ is the two-dimensional projection of incident wave-number vector on the plane $z=0$. The parameters pertaining to layer m with boundaries $-d_{m-1}$ and $-d_m$ are distinguished by a subscript m . Each layer is assumed to be homogeneous and characterized by arbitrary and deterministic parameters: the *dielectric relative permittivity* ε_m , the *magnetic relative permeability* μ_m and the *thickness* $\Delta_m = d_m - d_{m-1}$. With reference to Fig.1, it has been assumed that in particular, $d_0=0$. In the following, the symbol \perp denotes the projection of the corresponding vector on the plan $z=0$. Here $\mathbf{r} = (\mathbf{r}_\perp, z)$, so we distinguish the transverse

spatial coordinates $\mathbf{r}_\perp = (x, y)$ and the longitudinal coordinate z . In addition, each m th rough interface is assumed to be characterized by a zero-mean two-dimensional *stochastic process* $\zeta_m = \zeta_m(\mathbf{r}_\perp)$ with normal vector $\hat{\mathbf{n}}_m$. No constraints are imposed on the degree to which the rough interfaces are correlated.

A general methodology has been developed by *Imperatore et al.* to analytically treat EM bistatic scattering from this class of layered structures that can be described by small changes with respect to an idealized (unperturbed) structure, whose associated problem is exactly solvable. A thorough analysis of the results of this theoretical investigation (BPT), which is based on perturbation of the boundary condition, will be presented in the following, methodologically emphasizing the development of the several inherent aspects.

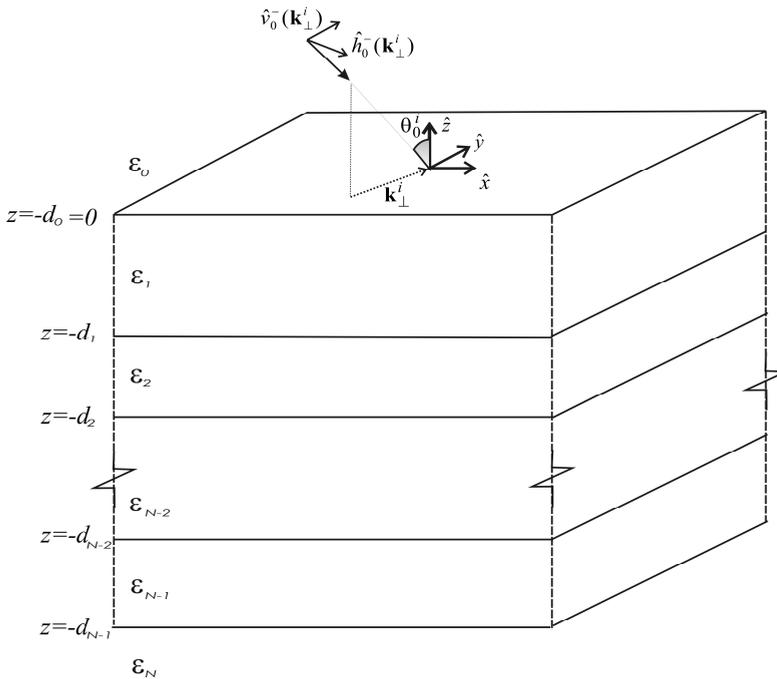


Fig. 2. Geometry for a flat boundaries layered medium

3. Basic definition and notations

This section is devoted preliminary to introduce the formalism used in the following of this chapter. The Flat Boundaries layered medium (*unperturbed structure*) is defined as a stack of parallel slabs (Fig.2), sandwiched in between two half-spaces, whose structure is shift invariant in the direction of x and y (infinite lateral extent in x - y directions). With the notations $T_{m|m+1}^p$ and $R_{m|m+1}^p$, respectively, we indicate the *ordinary transmission* and *reflection coefficients* at the interface between the regions $m-1$ and $m+1$,

$$R_{m|m+1}^h = \frac{\mu_{m+1}k_{zm} - \mu_m k_{z(m+1)}}{\mu_{m+1}k_{zm} + \mu_m k_{z(m+1)}}, \quad (5)$$

$$R_{m|m+1}^v = \frac{\varepsilon_{m+1}k_{zm} - \varepsilon_m k_{z(m+1)}}{\varepsilon_{m+1}k_{zm} + \varepsilon_m k_{z(m+1)}}, \quad (6)$$

$$T_{m|m+1}^h = \frac{2\mu_{m+1}k_{zm}}{\mu_{m+1}k_{zm} + \mu_m k_{z(m+1)}}, \quad (7)$$

$$T_{m|m+1}^v = \frac{2\varepsilon_{m+1}k_{zm}}{\varepsilon_{m+1}k_{zm} + \varepsilon_m k_{z(m+1)}}, \quad (8)$$

with the superscript $p \in \{v, h\}$ indicating the polarization state for the incident wave and may stand for *horizontal* (h) or *vertical* (v) polarization (Tsang et al., 1985) (Imperatore et al. 2009a), and where

$$k_{zm} = \sqrt{k_m^2 - |\mathbf{k}_\perp|^2} = k_m \cos \theta_m, \quad (9)$$

where $k_m = k_0 \sqrt{\mu_m \varepsilon_m}$ is the *wave number* for the electromagnetic medium in the m th layer, with $k_0 = \omega / c = 2\pi / \lambda$, and where $\mathbf{k}_\perp = k_x \hat{x} + k_y \hat{y}$ is the two-dimensional projection of vector wave-number on the plane $z=0$. In addition, we stress that:

$$T_{i|j}^p = 1 + R_{i|j}^p, \quad R_{i|j}^p = -R_{j|i}^p, \quad i=j\pm 1. \quad (10)$$

3.1 Generalized reflection formalism

The *generalized reflection coefficients* $\mathfrak{R}_{m-1|m}^p$ at the interface between the regions ($m-1$) and m , for the p -polarization, are defined as the ratio of the amplitudes of *upward*- and *downward*-propagating waves immediately above the interface, respectively. They can be expressed by recursive relations as in (Chew W. C., 1997) (Imperatore et al. 2009a):

$$\mathfrak{R}_{m-1|m}^p = \frac{R_{m-1|m}^p + \mathfrak{R}_{m|m+1}^p e^{j2k_{zm}\Delta_m}}{1 + R_{m-1|m}^p \mathfrak{R}_{m|m+1}^p e^{j2k_{zm}\Delta_m}}. \quad (11)$$

Likewise, at the interface between the regions ($m+1$) and m , $\mathfrak{R}_{m+1|m}^p$ is given by:

$$\mathfrak{R}_{m+1|m}^p = \frac{R_{m+1|m}^p + \mathfrak{R}_{m|m-1}^p e^{j2k_{zm}\Delta_m}}{1 + R_{m+1|m}^p \mathfrak{R}_{m|m-1}^p e^{j2k_{zm}\Delta_m}}, \quad (12)$$

Furthermore, it should be noted that the factors

$$\bar{M}_m^p(k_\perp) = 1 - R_{m|m-1}^p \mathfrak{R}_{m|m+1}^p e^{j2k_{zm}\Delta_m}, \quad (13)$$

$$\bar{M}_m^p(k_\perp) = 1 - \mathfrak{R}_{m|m-1}^p R_{m|m+1}^p e^{j2k_{zm}\Delta_m}, \quad (14)$$

$$\bar{M}_m^p(k_\perp) = 1 - \mathfrak{R}_{m|m-1}^p \mathfrak{R}_{m|m+1}^p e^{j2k_{zm}\Delta_m}, \quad (15)$$

take into account the multiple reflections in the m th layer.

3.2 Generalized transmission formalism

The *generalized transmission coefficients* in *downward* direction $\mathfrak{Z}_{0|m}^p$ can be defined as:

$$\mathfrak{Z}_{0|m}^p(k_\perp) = \exp \left[j \sum_{n=1}^{m-1} k_{zn} \Delta_n \right] \prod_{n=0}^{m-1} T_{n|n+1}^p \left[\prod_{n=1}^m \bar{M}_n^p \right]^{-1}, \quad (16)$$

where $p \in \{v, h\}$. The *generalized transmission coefficients* in *upward* direction $\mathfrak{Z}_{m|0}^p$ are then given by

$$\mathfrak{Z}_{m|0}^p = \begin{cases} \mathfrak{Z}_{0|m}^p \frac{\mu_0 k_{zm}}{\mu_m k_{z0}} & \text{for } p = h \\ \mathfrak{Z}_{0|m}^p \frac{\varepsilon_0 k_{zm}}{\varepsilon_m k_{z0}} & \text{for } p = v \end{cases}. \quad (17)$$

As a counterpart of (17), we have

$$\mathfrak{Z}_{m+1|N}^p = \begin{cases} \mathfrak{Z}_{N|m+1}^p \frac{\mu_N k_{z(m+1)}}{\mu_{m+1} k_{zN}} & \text{for } p = h \\ \mathfrak{Z}_{N|m+1}^p \frac{\varepsilon_N k_{z(m+1)}}{\varepsilon_{m+1} k_{zN}} & \text{for } p = v \end{cases}. \quad (18)$$

Equations (17) and (18) formally express the *reciprocity* of the generalized transmission coefficients for an arbitrary flat-boundaries layered structure (Imperatore et al. 2009b).

Here we introduce notion of *layered slab*, which refers to a layered structure sandwiched between two half-spaces. Accordingly, the *generalized transmission coefficients* in *downward* direction for a layered slab between two half-spaces (0, N), $\mathfrak{Z}_{0|N}^{p(slab)}$, can be defined as:

$$\mathfrak{Z}_{0|N}^{p(slab)}(k_\perp) = \exp \left[j \sum_{n=1}^{N-1} k_{zn} \Delta_n \right] \prod_{n=0}^{N-1} T_{n|n+1}^p \left[\prod_{n=1}^{N-1} \bar{M}_n^p \right]^{-1}. \quad (19)$$

It should be noted that the parenthesized superscript *slab* indicates that both the media 0 and N are half-space. Similarly, the *generalized transmission coefficients* in *downward* direction for the layered slab between two half-spaces (m+1, N), $\mathfrak{Z}_{m+1|N}^{p(slab)}$, are defined as:

$$\mathfrak{T}_{m+1|N}^{p(slab)}(k_{\perp}) = \exp \left[j \sum_{n=m+2}^{N-1} k_{zn} \Delta_n \right] \prod_{n=m+1}^{N-1} T_{n|n+1}^p \left[\prod_{n=m+2}^{N-1} \bar{M}_n^p \right]^{-1}. \quad (20)$$

Note also that

$$\mathfrak{T}_{m+1|N}^p = [\bar{M}_{m+1}^p]^{-1} \mathfrak{T}_{m+1|N}^{p(slab)}. \quad (21)$$

Moreover, we consider the *generalized transmission coefficients* in *upward* direction for the layered *slab* between two half-spaces $(m, 0)$, $\mathfrak{T}_{m|0}^{p(slab)}$, which are defined as

$$\mathfrak{T}_{m|0}^{p(slab)}(k_{\perp}) = \exp \left[j \sum_{n=1}^{m-1} k_{zn} \Delta_n \right] \prod_{n=0}^{m-1} T_{n+1|n}^p \left[\prod_{n=1}^{m-1} \bar{M}_n^p \right]^{-1}. \quad (22)$$

Note also that

$$\mathfrak{T}_{m|0}^p(k_{\perp}) = [\bar{M}_m^p(k_{\perp})]^{-1} \mathfrak{T}_{m|0}^{p(slab)}(k_{\perp}). \quad (23)$$

The *generalized transmission coefficients* in *downward* direction for the layered *slab* between two half-spaces $(0, m)$, $\mathfrak{T}_{0|m}^{p(slab)}$, can be defined as

$$\mathfrak{T}_{0|m}^{p(slab)}(k_{\perp}) = \exp \left[j \sum_{n=1}^{m-1} k_{zn} \Delta_n \right] \prod_{n=0}^{m-1} T_{n|n+1}^p \left[\prod_{n=1}^{m-1} \bar{M}_n^p \right]^{-1}. \quad (24)$$

It should be noted that the $\mathfrak{T}_{0|m}^p$ are distinct from the coefficients $\mathfrak{T}_{0|m}^{p(slab)}$, because in the evaluation of $\mathfrak{T}_{0|m}^p$ the effect of all the layers under the layer m is taken into account, whereas $\mathfrak{T}_{0|m}^{p(slab)}$ are evaluated referring to a different configuration in which the intermediate layers $1 \dots m$ are bounded by the half-spaces 0 and m .

We stress that generalized reflection and transmission coefficients do not depend on the direction of \mathbf{k}_{\perp} . In the following, we shown how the employing the generalized reflection/transmission coefficient notions not only is crucial in obtaining a compact closed-form perturbation solution, but it also permit us to completely elucidate the obtained analytical expressions from a physical point of view, highlighting the role played by the *equivalent reflecting interfaces* and by the *equivalent slabs*, so providing the inherent connection between the global scattering response.

4. Stochastic characterization for the 3-D geometry description

In this section, the focus is on stochastic description for the geometry of the investigated structure, and the notion of wide-sense stationary process is detailed. First of all, when the description of a rough interface by means of deterministic function $\zeta_m(\mathbf{r}_{\perp})$ is concerned, the corresponding *ordinary* 2-D *Fourier Transform* pair can be defined as

$$\tilde{\zeta}_m(\mathbf{k}_\perp) = (2\pi)^{-2} \iint d\mathbf{r}_\perp e^{-j\mathbf{k}_\perp \cdot \mathbf{r}_\perp} \zeta_m(\mathbf{r}_\perp), \quad (25)$$

$$\zeta_m(\mathbf{r}_\perp) = \iint d\mathbf{k}_\perp e^{j\mathbf{k}_\perp \cdot \mathbf{r}_\perp} \tilde{\zeta}_m(\mathbf{k}_\perp). \quad (26)$$

Let us assume now that $\zeta_m(\mathbf{r}_\perp)$, which describes the generic (m)th rough interface, is a 2-D stochastic process satisfying the conditions

$$\langle \zeta_m(\mathbf{r}_\perp) \rangle = 0, \quad (27)$$

$$\langle \zeta_m(\mathbf{r}_\perp + \boldsymbol{\rho}) \zeta_m(\mathbf{r}_\perp) \rangle = B_{\zeta_m}(\boldsymbol{\rho}), \quad (28)$$

where the *angular bracket* denotes statistical ensemble averaging, and where $B_{\zeta_m}(\boldsymbol{\rho})$ is the interface *autocorrelation* function, which quantifies the similarity of the spatial fluctuations with a displacement $\boldsymbol{\rho}$. Equations (27)-(28) constitute the basic assumptions defining a *wide sense stationary* (WSS) stochastic process: the statistical properties of the process under consideration are invariant to a spatial shift. Similarly, concerning two mutually correlated random rough interfaces ζ_m and ζ_n , we also assume that they are *jointly WSS*, i.e.

$$\langle \zeta_m(\mathbf{r}_\perp + \boldsymbol{\rho}) \zeta_n(\mathbf{r}_\perp) \rangle = B_{\zeta_m \zeta_n}(\boldsymbol{\rho}), \quad (29)$$

where $B_{\zeta_m \zeta_n}(\boldsymbol{\rho})$ is the corresponding *cross-correlation function* of the two random processes.

It can be readily derived that

$$B_{\zeta_m \zeta_n}(\boldsymbol{\rho}) = B_{\zeta_n \zeta_m}(-\boldsymbol{\rho}). \quad (30)$$

The integral in (25) is a *Riemann* integral representation for $\zeta_m(\mathbf{r}_\perp)$, and it exists if $\zeta_m(\mathbf{r}_\perp)$ is piecewise continuous and *absolutely integrable*. On the other hand, when the spectral analysis of a stationary random process is concerned, the integral (25) does not in general exist in the framework of theory of the ordinary functions. Indeed, a WSS process describing an interface $\zeta_m(\mathbf{r}_\perp)$ of infinite lateral extension, for its proper nature, is not *absolutely integrable*, so the conditions for the existence of the Fourier Transform are not satisfied. In order to obtain a spectral representation for a WSS random process, this difficulty can be circumvented by resorting to the more general *Fourier-Stieltjes* integral (Ishimaru, 1978); otherwise one can define space-truncated functions. When a finite patch of the rough interface with area A is concerned, the space-truncated version of (25) can be introduced as

$$\tilde{\zeta}_m(\mathbf{k}_\perp; A) = (2\pi)^{-2} \iint_A d\mathbf{r}_\perp e^{-j\mathbf{k}_\perp \cdot \mathbf{r}_\perp} \zeta_m(\mathbf{r}_\perp), \quad (31)$$

subsequently, $\tilde{\zeta}_m(\mathbf{k}_\perp) = \lim_{A \rightarrow \infty} \tilde{\zeta}_m(\mathbf{k}_\perp; A)$ is not an ordinary function. Nevertheless, we will use again the (25)-(26), regarding them as symbolic formulas, which hold a rigorous mathematical meaning beyond the ordinary function theory (generalized Fourier Transform). We underline that by virtue of the condition (27) directly follows also that $\langle \tilde{\zeta}_m(\mathbf{k}_\perp) \rangle = 0$. Let us consider

$$\langle \zeta_m(\mathbf{r}'_{\perp}) \zeta_n^*(\mathbf{r}''_{\perp}) \rangle = \iint d\mathbf{k}'_{\perp} \iint d\mathbf{k}''_{\perp} e^{j(\mathbf{k}'_{\perp} \cdot \mathbf{r}'_{\perp} - \mathbf{k}''_{\perp} \cdot \mathbf{r}''_{\perp})} \langle \tilde{\zeta}_m(\mathbf{k}'_{\perp}) \tilde{\zeta}_n^*(\mathbf{k}''_{\perp}) \rangle, \quad (32)$$

where the asterisk denotes the complex conjugated, and where the operations of average and integration have been interchanged. When *jointly* WSS processes ζ_m and ζ_n are concerned, accordingly to (29), the LHS of (32) must be a function of $\mathbf{r}'_{\perp} - \mathbf{r}''_{\perp}$ only; therefore, it is required that

$$\langle \tilde{\zeta}_m(\mathbf{k}'_{\perp}) \tilde{\zeta}_n^*(\mathbf{k}''_{\perp}) \rangle = W_{mn}(\mathbf{k}'_{\perp}) \delta(\mathbf{k}'_{\perp} - \mathbf{k}''_{\perp}), \quad (33)$$

where $\delta(\cdot)$ is the *Dirac* delta function, and where $W_{mn}(\mathbf{k})$ is called the (spatial) *cross power spectral density* of two interfaces ζ_m and ζ_n , for the spatial frequencies of the roughness. Equation (33) states that the different spectral components of the two considered interfaces must be uncorrelated. This is to say that the (generalized) Fourier transform of jointly WSS processes are *jointly non stationary* white noise with average power $W_{mn}(\mathbf{k}'_{\perp})$. Indeed, by using (33) into (32), we obtain

$$\langle \zeta_m(\mathbf{r}'_{\perp}) \zeta_n(\mathbf{r}''_{\perp}) \rangle = \iint d\mathbf{k}''_{\perp} e^{j \cdot \mathbf{k}''_{\perp} \cdot (\mathbf{r}'_{\perp} - \mathbf{r}''_{\perp})} W_{mn}(\mathbf{k}''_{\perp}), \quad (34)$$

where the RHS of (34) involves an (ordinary) 2D Fourier Transform. Note also that as a direct consequence of the fact that $\zeta_n(\mathbf{r}_{\perp})$ is real we have the relation $\tilde{\zeta}_n(\mathbf{k}_{\perp}) = \tilde{\zeta}_n^*(-\mathbf{k}_{\perp})$. Therefore, setting $\boldsymbol{\rho} = \mathbf{r}'_{\perp} - \mathbf{r}''_{\perp}$ in (34), we have

$$B_{\zeta_m \zeta_n}(\boldsymbol{\rho}) = \iint d\boldsymbol{\kappa} e^{j \boldsymbol{\kappa} \cdot \boldsymbol{\rho}} W_{mn}(\boldsymbol{\kappa}). \quad (35)$$

The *cross-correlation function* $B_{\zeta_m \zeta_n}(\boldsymbol{\rho})$ of two interfaces ζ_m and ζ_n is then given by the (inverse) 2D Fourier Transform of their (spatial) *cross power spectral density*, and Equation (35) together with its Fourier inverse

$$W_{mn}(\boldsymbol{\kappa}) = (2\pi)^{-2} \iint d\boldsymbol{\rho} e^{-j \boldsymbol{\kappa} \cdot \boldsymbol{\rho}} B_{\zeta_m \zeta_n}(\boldsymbol{\rho}), \quad (36)$$

may be regarded as the (generalized) *Wiener-Khinchin* theorem. In particular, when $n=m$, (33) reduces to

$$\langle \tilde{\zeta}_m(\mathbf{k}'_{\perp}) \tilde{\zeta}_m^*(\mathbf{k}''_{\perp}) \rangle = W_m(\mathbf{k}'_{\perp}) \delta(\mathbf{k}'_{\perp} - \mathbf{k}''_{\perp}), \quad (37)$$

where $W_m(\boldsymbol{\kappa})$ is called the (spatial) *power spectral density* of n th corrugated interface ζ_m and can be expressed as the (ordinary) 2D Fourier transform of n -corrugated interface autocorrelation function, i.e., satisfying the transform pair:

$$W_m(\boldsymbol{\kappa}) = (2\pi)^{-2} \iint d\boldsymbol{\rho} e^{-j \boldsymbol{\kappa} \cdot \boldsymbol{\rho}} B_{\zeta_m}(\boldsymbol{\rho}), \quad (38)$$

$$B_{\zeta_m}(\boldsymbol{\rho}) = \iint d\boldsymbol{\kappa} e^{j \boldsymbol{\kappa} \cdot \boldsymbol{\rho}} W_m(\boldsymbol{\kappa}), \quad (39)$$

Thank You for previewing this eBook

You can read the full version of this eBook in different formats:

- HTML (Free /Available to everyone)
- PDF / TXT (Available to V.I.P. members. Free Standard members can access up to 5 PDF/TXT eBooks per month each month)
- Epub & Mobipocket (Exclusive to V.I.P. members)

To download this full book, simply select the format you desire below

