Kinematics fundamentals

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Chapter 1. Motion

1.1. Motion*

Motion is a state, which indicates change of position. Surprisingly, everything in this world is constantly moving and nothing is stationary. The apparent state of rest, as we shall learn, is a notional experience confined to a particular system of reference. A building, for example, is at rest in Earth's reference, but it is a moving body for other moving systems like train, motor, airplane, moon, sun etc.

Figure 1.1. Motion of an airplane

The position of plane with respect to the earth keeps changing with time.

Definition: Motion

Motion of a body refers to the change in its position with respect to time in a given frame of reference.

A frame of reference is a mechanism to describe space from the perspective of an observer. In other words, it is a system of measurement for locating positions of the bodies in space with respect to an observer (reference). Since, frame of reference is a system of measurement of positions in space as measured by the observer, frame of reference is said to be attached to the observer. For this reason, terms "frame of reference" and "observer" are interchangeably used to describe motion.

In our daily life, we recognize motion of an object with respect to ourselves and other stationary objects. If the object maintains its position with respect to the stationary objects, we say that the object is at rest; else the object is moving with respect to the stationary objects. Here, we conceive all objects moving with earth without changing their positions on earth surface as stationary objects in the earth's frame of reference. Evidently, all bodies not changing position with respect to a specific observer is stationary in the frame of reference attached with the observer.

We require an observer to identify motion

Motion has no meaning without a reference system.

An object or a body under motion, as a matter of fact, is incapable of identifying its own motion. It would be surprising for some to know that we live on this earth in a so called stationary state without ever being aware that we are moving around sun at a very high speed - at a speed faster than the fastest airplane that the man kind has developed. The earth is moving around sun at a speed of about 30 km/s (≈ 30000 m/s ≈ 100000 km/hr) – a speed about 1000 times greater than the motoring speed and 100 times greater than the aircraft's speed.

Likewise, when we travel on aircraft, we are hardly aware of the speed of the aircraft. The state of fellow passengers and parts of the aircraft are all moving at the same speed, giving the impression that passengers are simply sitting in a stationary cabin. The turbulence that the passengers experience occasionally is a consequence of external force and is not indicative of the motion of the aircraft.

It is the external objects and entities which indicate that aircraft is actually moving. It is the passing clouds and changing landscape below, which make us think that aircraft is actually moving. The very fact that we land at geographically distant location at the end of travel in a short time, confirms that aircraft was actually cruising at a very high speed.

The requirement of an observer in both identifying and quantifying motion brings about new dimensions to the understanding of motion. Notably, the motion of a body and its measurement is found to be influenced by the state of motion of the observer itself and hence by the state of motion of the attached frame of reference. As such, a given motion is evaluated differently by different observers (system of references).

Two observers in the same state of motion, such as two persons standing on the platform, perceive the motion of a passing train in exactly same manner. On the other hand, the passenger in a speeding train finds that the other train crossing it on the parallel track in opposite direction has the combined speed of the two trains $(v_1 + v_1)$. The observer on the ground, however, find them running at their individual speeds v_1 and v_2.

From the discussion above, it is clear that motion of an object is an attribute, which can **not** be stated in absolute term; but it is a kind of attribute that results from the interaction of the motions of the both object and observer (frame of reference).

Frame of reference and observer

Frame of reference is a mathematical construct to specify position or location of a point object in space. Basically, frame of reference is a coordinate system. There are plenty of coordinate systems in use, but the Cartesian coordinate system, comprising of three mutually perpendicular axes, is most common. A point in three dimensional space is defined by three values of coordinates i.e. x,y and z in the Cartesian system as shown in the figure below. We shall learn about few more useful coordinate systems in next module titled " **Coordinate systems in physics** ".

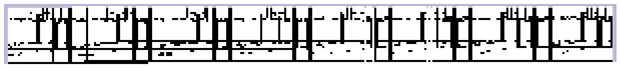

Figure 1.2. Frame of reference

A point in three dimensional space is defined by three values of coordinates

We need to be specific in our understanding of the role of the observer and its relation with frame of reference. Observation of motion is considered an human endeavor. But motion of an object is described in reference of both human and non-human bodies like clouds, rivers, poles, moon etc. From the point of view of the study of motion, we treat reference bodies capable to make observations, which is essentially a human like function. As such, it is helpful to imagine ourselves attached to the reference system, making observations. It is essentially a notional endeavor to consider that the measurements are what an observer in that frame of reference would make, had the observer with the capability to measure was actually present there.

Earth is our natural abode and we identify all non-moving ground observers equivalent and at rest with the earth. For other moving systems, we need to specify position and determine motion by virtually (in imagination) transposing ourselves to the frame of reference we are considering.