HEAT CONDUCTION – BASIC RESEARCH

Edited by **Vyacheslav S. Vikhrenko**

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Preface

Heat conduction is a fundamental phenomenon encountered in many industrial and biological processes as well as in everyday life. Economizing of energy consumption in different heating and cooling processes or ensuring temperature limitations for proper device operation requires the knowledge of heat conduction physics and mathematics. The fundamentals of heat conduction were formulated by J. Fourier in his outstanding manuscript *Théorie de la Propagation de la Chaleur dans les Solides* presented to the Institut de France in 1807 and in the monograph *ThéorieAnalytique de la Chaleur* (1822). The two century evolution of the heat conduction theory resulted in a wide range of methods and problems that have been solved or have to be solved for successful development of the world community.

The content of this book covers several up-to-date approaches in the heat conduction theory such as inverse heat conduction problems, non-linear and non-classic heat conduction equations, coupled thermal and electromagnetic or mechanical effects and numerical methods for solving heat conduction equations as well. The book is comprised of 14 chapters divided in four sections.

In the first section inverse heat conduction problems are discuss. The section is started with a review containing classification of inverse heat conduction problems alongside with the methods for their solution. The genetic algorithm, neural network and particle swarm optimization techniques, and the Marching Algorithm are considered in the next two chapters. In Chapter 4 the inverse heat conduction problem is used for evaluating from experimental data the local heat transfer coefficient for jet impingement with plane surface.

The first two chapter of the second section are devoted to construction of analytical solutions of nonlinear heat conduction problems when nonlinear terms are included in the heat conduction equation (Chapter 5) or the nonlinearity appears through boundary conditions and/or temperature dependence of the heat conduction equation coefficients (Chapter 6). In the last two chapters of this section wavelike solutions are attained due to construction of a hyperbolic heat conduction equation (Chapter 7) or because of time varying boundary conditions (Chapter 8).

The third section is devoted to combined effects of heat conduction and electromagnetic interactions in plasmas (Chapter 9) or pyroelectric material (Chapter 10), elastic deformations (Chapter 11) and hydrodynamics (Chapter 12).

Two chapters in the last section are dedicated to numerical methods for solving heat conduction problems, namely the particle transport Monte Carlo method (Chapter 13) and a meshless version of the boundary element method (Chapter 14).

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Part 1

Inverse Heat Conduction Problems

1

Inverse Heat Conduction Problems

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1. Introduction

In the heat conduction problems if the heat flux and/or temperature histories at the surface of a solid body are known as functions of time, then the temperature distribution can be found. This is termed as a direct problem. However in many heat transfer situations, the surface heat flux and temperature histories must be determined from transient temperature measurements at one or more interior locations. This is an inverse problem. Briefly speaking one might say the inverse problems are concerned with determining causes for a desired or an observed effect.

The concept of an inverse problem have gained widespread acceptance in modern applied mathematics, although it is unlikely that any rigorous formal definition of this concept exists. Most commonly, by inverse problem is meant a problem of determining various quantitative characteristics of a medium such as density, thermal conductivity, surface loading, shape of a solid body etc. , by observation over physical fields in the medium or – in other words - a general framework that is used to convert observed measurements into information about a physical object or system that we are interested in. The fields may be of natural appearance or specially induced, stationary or depending on time, (Bakushinsky & Kokurin, 2004).

Within the class of inverse problems, it is the subclass of indirect measurement problems that characterize the nature of inverse problems that arise in applications. Usually measurements only record some indirect aspect of the phenomenon of interest. Even if the direct information is measured, it is measured as a correlation against a standard and this correlation can be quite indirect. The inverse problems are difficult because they ussually are extremely sensitive to measurement errors. The difficulties are particularly pronounced as one tries to obtain the maximum of information from the input data.

A formal mathematical model of an inverse problem can be derived with relative ease. However, the process of solving the inverse problem is extremely difficult and the so-called exact solution practically does not exist. Therefore, when solving an inverse problem the approximate methods like iterative procedures, regularization techniques, stochastic and system identification methods, methods based on searching an approximate solution in a subspace of the space of solutions (if the one is known), combined techniques or straight numerical methods are used.

2. Well-posed and ill-posed problems

The concept of well-posed or correctly posed problems was introduced in (Hadamard, 1923). Assume that a problem is defined as

$$\mathbf{A}u=g \tag{1}$$

where $\boldsymbol{u} \in U$, $\boldsymbol{g} \in G$, U and G are metric spaces and $\mathbf{A}$ is an operator so that $\mathbf{A}U \subset G$. In general $\boldsymbol{u}$ can be a vector that characterize a model of phenomenon and $\boldsymbol{g}$ can be the observed attribute of the phenomenon.
A well-posed problem must meet the following requirements:

- the solution of equation (1) must exist for any $\mathbf{g} \in \mathbf{G}$,
- the solution of equation (1) must be unique,
- the solution of equation (1) must be stable with respect to perturbation on the right-hand side, i.e. the operator $\mathbf{A}^{-1}$ must be defined throughout the space G and be continuous.

If one of the requirements is not fulfilled the problem is termed as an ill-posed. For ill-posed problems the inverse operator $\mathbf{A}^{-1}$ is not continuous in its domain $\mathbf{A}U \subset G$ which means that the solution of the equation (1) does not depend continuously on the input data $\boldsymbol{g} \in G$, (Kurpisz & Nowak, 1995; Hohage, 2002; Grysa, 2010). In general we can say that the (usually approximate) solution of an ill-posed problem does not necessarily depend continuously on the measured data and the structure of the solution can have a tenuous link to the measured data. Moreover, small measurement errors can be the source for unacceptable perturbations in the solution. The best example of the last statement is numerical differentiation of a solution of an inverse problem with noisy input data. Some interesting remarks on the inverse and ill-posed problems can be found in (Anderssen, 2005).
Some typical inverse and ill-posed problems are mentioned in (Tan & Fox, 2009).

3. Classification of the inverse problems

Engineering field problems are defined by governing partial differential or integral equation(s), shape and size of the domain, boundary and initial conditions, material properties of the media contained in the field and by internal sources and external forces or inputs. As it has been mentioned above, if all of this information is known, the field problem is of a direct type and generally considered as well posed and solvable. In the case of heat conduction problems the governing equations and possible boundary and initial conditions have the following form:

$$\rho c \frac{\partial T}{\partial t} = \nabla \cdot (k \nabla T) + \dot{Q}_v, \quad (x,y,z) \in \Omega \subset R^3, \; t \in (0, t_f], \tag{2}$$

$$T(x,y,z,t) = T_b(x,y,z,t) \quad \text{for} \quad (x,y,z,t) \in S_D, \quad t \in (0, t_f], \tag{3}$$

$$-k \frac{\partial T(x,y,z,t)}{\partial n} = q_b(x,y,z,t) \quad \text{for} \quad (x,y,z,t) \in S_N, \quad t \in (0, t_f], \tag{4}$$

$$-k \frac{\partial T(x,y,z,t)}{\partial n} = h_c\big(T(x,y,z,t) - T_e(x,y,z,t)\big) \quad \text{for} \quad (x,y,z,t) \in S_R, \quad t \in (0, t_f], \tag{5}$$

$$T(x,y,z,0) = T_0(x,y,z) \quad \text{for} \quad (x,y,z) \in \Omega, \tag{6}$$

where $\nabla = (\partial/\partial x, \partial/\partial y, \partial/\partial z)$ stands for gradient differential operator in 3D; ρ denotes density of mass, [kg/m³]; c is the constant-volume specific heat, [J/kg K]; T is temperature, [K]; k denotes thermal conductivity, [W/m K]; $\dot{Q}_v$ is the rate of heat generation per unit volume, [W/m³], frequently termed as source function; $\partial/\partial n$ means differentiation along the outward normal; h_c denotes the heat transfer coefficient, [W/m² K]; T_b , q_b and T_0 are given functions and T_e stands for environmental temperature, t_f – final time. The boundary $\partial\Omega$ of the domain Ω is divided into three disjoint parts denoted with subscripts D for Dirichlet, N for Neumann and R for Robin boundary condition; $S_D \cup S_N \cup S_R = \partial\Omega$. Moreover, it is also possible to introduce the fourth-type or radiation boundary condition, but here this condition will not be dealt with.

The equation (2) with conditions (3) to (6) describes an initial-boundary value problem for transient heat conduction. In the case of stationary problem the equation (2) becomes a Poisson equation or – when the source function $\dot{Q}_v$ is equal to zero – a Laplace equation.

Broadly speaking, inverse problems may be subdivided into the following categories: inverse conduction, inverse convection, inverse radiation and inverse phase change (melting or solidification) problems as well as all combination of them (Özisik & Orlande, 2000). Here we have adopted classification based on the type of causal characteristics to be estimated:

1. Boundary value determination inverse problems,
2. Initial value determination inverse problems,
3. Material properties determination inverse problems,
4. Source determination inverse problems
5. Shape determination inverse problems.

3.1 Boundary value determination inverse problems

In this kind of inverse problem on a part of a boundary the condition is not known. Instead, in some internal points of the considered body some results of temperature measurements or anticipated values of temperature or heat flux are prescribed. The measured or anticipated values are called internal responses. They can be known on a line or surface inside the considered body or in a discrete set of points. If the internal responses are known as values of heat flux, on a part of the boundary a temperature has to be known, i.e. Dirichlet or Robin condition has to be prescribed. In the case of stationary problems an inverse problem for Laplace or Poisson equation has to be solved. If the temperature field depends on time, then the equation (2) becomes a starting point. The additional condition can be formulated as

$$T(x,y,z,t) = T_a(x,y,z,t) \quad \text{for} \quad (x,y,z) \in L \subset \Omega, \; t \in (0, t_f] \tag{7}$$

or

$$T(x_i,y_i,z_i,t_i) = T_{ik} \quad \text{for} \quad (x_i,y_i,z_i) \in \Omega, \; t_k \in (0, t_f], \; i=1,2,\ldots, I; \; k=1,2,..,K \tag{8}$$

with T_a being a given function and T_{ik} known from e.g. measurements. As examples of such problems can be presented papers (Reinhardt et al., 2007; Soti et al., 2007; Ciałkowski & Grysa, 2010) and many others.