Konstantin K. Likharev

Essential Graduate Physics

Lecture Notes and Problems

Beta version

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Beta version, December 2013
with later problem additions and error corrections

[1] To the best of the author's knowledge, the fair use clause of the U.S. copyright law (see, e.g., http://www.copyright.gov/fls/fl102.html) allows viewing, browsing, and/or downloading copyrighted materials for temporary, non-commercial, personal purposes, without requesting the copyright holder's permission.

Preface

This is a series of lecture notes and problems on "Essential Graduate Physics", consisting of the following four parts:

CM: *Classical Mechanics* (for a 1-semester course),
EM: *Classical Electrodynamics* (2 semesters),
QM: *Quantum Mechanics* (2 semesters), and
SM: *Statistical Mechanics* (1 semester).

The parts share a teaching style, structure, and (with a few exceptions) notation, and are interlinked by extensive cross-referencing. I believe that due to this unity, the notes may be used for teaching these courses not only in the (preferred) sequence shown above but in almost any order – or in parallel.

Each part is a two-component package consisting of:

(i) *Lecture Notes* chapter texts,[2] with a list of exercise problems at the end of each chapter, and
(ii) *Exercise and Test Problems with Model Solutions* files.

The series also includes this front matter, two brief reference appendices, *MA: Selected Mathematical Formulas* (16 pp.) and *CA: Selected Physics Constants* (2 pp), and a list of references.

The series is a by-product of the so-called *core physics* courses I taught at Stony Brook University from 1991 to 2013. Reportedly, most physics departments require their graduate students to either take a set of similar courses or pass comprehensive exams based on an approximately similar body of knowledge. This is why I hope that my notes may be useful for both instructors and students of such courses, as well as for individual learners.

The motivation for composing the lecture notes (which had to be typeset because of my horrible handwriting) and their distribution to Stony Brook students was my desperation to find textbooks I could actually use for teaching. First of all, the textbooks I could find, including the most influential *Theoretical Physics* series by Landau and Lifshitz, did not match my class audiences, which included experiment-oriented students, some PhD candidates from other departments, some college graduates with substandard undergraduate background, and a few advanced undergraduates. Second, for the rigid time restrictions imposed on the core physics courses, most available textbooks are way too long, and using them would mean hopping from one topic to another, picking up a chapter here and a section there, at a high risk of losing the necessary background material and logical connections between course components – and students' interest with them. On the other hand, many textbooks lack even brief discussions of several traditional and modern topics that I believe are necessary parts of *every* professional physicist's education.[3,4]

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[2] The texts are saved as separate .pdf files of each chapter, optimized for two-page viewing and double-side printing; merged files for each part and the series as a whole, convenient for search purposes, are also provided.

[3] To list just a few: statics and dynamics of elastic and fluid continua, basic notions of physical kinetics, turbulence and deterministic chaos, physics of reversible and quantum computation, energy relaxation and dephasing of open quantum systems, the van der Pol method (a.k.a. the Rotating-Wave Approximation, RWA) in classical and quantum mechanics, physics of electrons and holes in semiconductors, the weak-potential and tight-binding approximations in the energy band theory, optical fiber electrodynamics, macroscopic quantum effects in

The main goal of my courses was to make students familiar with the basic notions and ideas of physics (hence the series' title), and my main effort was to organize the material in a logical sequence the students could readily follow and enjoy, at each new step understanding why exactly they need to swallow the next knowledge pill. As a backside of such a minimalistic goal, I believe that my texts may be used by advanced undergraduate physics students as well. Moreover, I hope that selected parts of the series may be useful for graduate students of other disciplines, including astronomy, chemistry, mechanical engineering, electrical, computer and electronic engineering, and material science.

At least since Confucius and Sophocles, i.e. for the past 2,500 years, teachers have known that students can master a new concept or method only if they have seen its application to at least a few particular situations. This is why in my notes, the range of theoretical physics methods is limited to the approaches that are indeed necessary for the solution of the problems I had time to discuss, and the introduction of every new technique is always accompanied by an application example or two. Additional exercise problems are listed at the end of each chapter of the lecture notes; they may be used for homework assignments. Individual readers are strongly encouraged to solve as many of these problems as possible.[5]

Detailed model solutions of the exercise problems (some with additional expansion of the lecture material), and several shorter problems suitable for tests (also with model solutions), are gathered in six separate files – one per semester. These files are available for both university instructors and individual readers – free of charge, but in return for a signed commitment to avoid unlimited distribution of the solutions – see p. vii below. For instructors, these files are available not only in the Adobe Systems' Portable Document Format (*.pdf) but also in the Microsoft Office 1997-2003 format (*.doc) free of macros, so that the problem assignments and solutions may be readily grouped, edited, etc., before their distribution to students, using either virtually any version of Microsoft Word or independent software tools – e.g., the public-domain OpenOffice.org.

I know that my texts are far from perfection. In particular, some sacrifices made at the topic selection, always very subjective, were extremely painful. (Most regretfully, I could not find time for even a brief introduction to general relativity.[6]) Moreover, it is almost certain that despite all my effort and the great help from SBU students and teaching assistants, not all typos/errors have been weeded out. This is why all remarks (however candid) and suggestions by the readers would be highly appreciated; they may be sent to *klikharev@gmail.com*. All significant contributions will be gratefully acknowledged in future editions of the series.

Bose-Einstein condensates, Bloch oscillations and Landau-Zener tunneling, cavity QED, and the Density Functional Theory (DFT). All these topics are discussed, if only concisely, in these notes.

[4] Recently, several high-quality graduate-level teaching materials became available online, including M. Fowler's *Graduate Quantum Mechanics Lectures* (http://galileo.phys.virginia.edu/classes/751.mf1i.fall02/home.html), R. Fitzpatrick's text on *Classical Electromagnetism* (farside.ph.utexas.edu/teaching/jk1/Electromagnetism.pdf), B. Simons' lecture notes on *Advanced Quantum Mechanics* (www.tcm.phy.cam.ac.uk/~bds10/aqp.html), and D. Tong's lecture notes on several topics (www.damtp.cam.ac.uk/user/tong/teaching.html).

[5] The problems that require either longer calculations or more creative approaches (or both) are marked by asterisks.

[6] For an introduction to that subject, I can recommend either its review by S. Carroll, *Spacetime and Geometry*, Addison-Wesley, 2003, or a longer text by A. Zee, *Einstein Gravity in a Nutshell*, Princeton U. Press, 2013.

Disclaimer

Since these materials are available free of charge, it is hard to imagine somebody blaming their author for deceiving "customers" for his commercial gain. Still, I would like to go a little bit beyond the usual litigation-avoiding claims,[7] and offer a word of caution to potential readers, to preempt their possible later disappointment.

This is NOT a course of theoretical physics – at least in the contemporary sense of the term

Though much of the included material is similar to that in textbooks on "theoretical physics" (most notably in the famous series by L. Landau and E. Lifshitz), this lecture note series is different from them by its focus on the basic concepts and ideas of physics, their relation to experimental data, and most important applications – rather than on sophisticated theoretical techniques. Indeed, the set of theoretical methods discussed in the notes is limited to the minimum necessary for quantitative understanding of the key notions of physics and for solving a few (or rather about a thousand :-) core problems. Moreover, because of the notes' shortness, I have not been able to cover some key fields of theoretical physics, most notably the general relativity and the quantum field theory – beyond some introductory elements of quantum electrodynamics in QM Chapter 9. If you want to work in modern theoretical physics, you need to know much more than what this series teaches!

Moreover, this is NOT a textbook – at least not the usual one

A usual textbook tries (though most commonly fails) to cover virtually all aspects of the addressed field. As a result, it is typically way too long for being fully read and understood by students during the time allocated for the corresponding course, so that the instructors are forced to pick up selected chapters and sections, frequently losing the narrative's logic lines. In contrast, these notes are much shorter (about 200 pages per semester), enabling their thorough reading – perhaps with just a few later sections dropped, depending on the reader's interests. I have tried to mitigate the losses due to this minimalistic approach by providing extensive further reading recommendations on the topics I had no time to cover. The reader is highly encouraged to use these sources (and/or the corresponding chapters of more detailed textbooks) on any topics of their special interest.

Then, what these notes ARE and why you may like to use them – I think

By tradition, graduate physics education consists of two main components: research experience and advanced physics courses. Unfortunately, the latter component is currently under pressure in many physics departments, apparently because of two reasons. On one hand, the average knowledge level of the students entering graduate school is not improving, so that bringing them up to the level of contemporary research becomes increasingly difficult. On the other hand, the research itself is becoming more fragmented, so that the students frequently do not feel an immediate need for a broad physics

[7] Yes Virginia, these notes represent only my personal opinions, not necessarily those of the Department of Physics and Astronomy of Stony Brook University, the SBU at large, the SUNY system as a whole, the Empire State of New York, the federal agencies and private companies that funded my group's research, etc. No dear, I cannot be held responsible for any harm, either bodily or mental, their reading may (?) cause.

knowledge base for their PhD project success. Some thesis advisors, trying to maximize the time they could use their students as a cheap laboratory workforce, do not help.

I believe that this trend toward the reduction of broad physics education in graduate school is irresponsible. Experience shows that during their future research career, a typical current student will change their research fields several times. Starting from scratch in a new field is hard – terribly hard in advanced age (believe me :-). However, physics is fortunate to have a stable *hard core* of knowledge, which many other sciences lack. With this knowledge, students will always feel in physics at home, while without it, they may not be able even to understand research literature in the new field, and would risk being reduced to auxiliary work roles – if any at all.

I have seen the main objective of my Stony Brook courses to give an introduction to this core of physics, at the same time trying to convey my own enchantment by the unparalleled beauty of the concepts and ideas of this science, and the remarkable logic of their fusion into a wonderful single construct. Let me hope that these notes relay not only the knowledge as such but also at least a part of this enchantment.

Acknowledgments

I am extremely grateful to my faculty colleagues and other readers who commented on certain sections of the notes; here is their list (in the alphabetic order):[8]

A. Abanov, P. Allen, D. Averin, S. Berkovich, P.-T. de Boer, M. Fernandez-Serra, R. F. Hernandez, P. Johnson, T. Konstantinova, A. Korotkov, V. Semenov, F. Sheldon, E. Tikhonov, O. Tikhonova, X. Wang.

(Evidently, these kind people are not responsible for the remaining deficiencies.)

The Department of Physics and Astronomy of Stony Brook University was very responsive to my kind requests of certain time ordering of my teaching assignments, that was beneficial for note writing and editing. The department, and the university as a whole, also provided a very friendly general environment for my work there for almost three decades.

A large part of my scientific background and experience, reflected in these materials, came from my education (and then research work) in the Department of Physics of Moscow State University.

And last but not least, I would like to thank my wife Lioudmila for several good bits of advice on aesthetic aspects of note typesetting, and more importantly for all her love, care, and patience – without them, this writing project would be impossible.

Konstantin K. Likharev
https://you.stonybrook.edu/likharev/

[8] I am very much sorry that I have not kept proper records from the beginning of my lectures at Stony Brook, so I cannot list all the numerous students and TAs who had kindly attracted my attention to typos in earlier versions of these notes. Needless to say, I am very grateful to them all as well.

Problem Solution Request Templates

Requests should be sent to either *klikharev@gmail.com* or *konstantin.likharev@stonybrook.edu* in either of the following forms:

- an email from a valid university address,
- a scanned copy of a signed letter – as an email attachment.

Approximate contents:

A. Request from a Prospective Instructor

Dear Dr. Likharev,

I plan to use your lecture notes and problems of the *Essential Graduate Physics* series, part(s) <select: CM, EM, QM, SM>, in my course <title> during <semester, year> in the <department, university>. I would appreciate sending me the file(s) *Exercise and Test Problems with Model Solutions* of that part(s) of the series in the <select: .pdf, both .doc and .pdf> format(s).

I will avoid unlimited distribution of the solutions, in particular their posting on externally searchable websites. If I distribute the solutions among my students, I will ask them to adhere to the same restraint.

I will let you know of any significant typos/deficiencies I may find.

Sincerely, <signature, full name, university position, work phone number>

B. Request from an Individual Learner

Dear Dr. Likharev,

I plan to use your lecture notes and problems of the *Essential Graduate Physics* series, part(s) <select: CM, EM, QM, SM>, for my personal education. I would appreciate sending me the file(s) *Exercise and Test Problems with Model Solutions* of that part(s) of the series.

I will not share the material with anyone, and will not use it for passing courses that are officially based on your series.

I will let you know of any significant typos/deficiencies I may find.

Sincerely, <signature, full name, present home address (in English), acting phone number>

Notation

Abbreviations

Eq. any formula (e.g., equality)

Fig. figure

Sec. section

c.c. complex conjugate

h. c. Hermitian conjugate

Fonts

F, $\mathcal{F}$ scalar variables[9]

$\mathbf{F}$, $\boldsymbol{\mathcal{F}}$ vector variables

$\hat{F}$, $\hat{\mathcal{F}}$ scalar operators

$\hat{\mathbf{F}}$, $\hat{\boldsymbol{\mathcal{F}}}$ vector operators

F matrix

$F_{jj'}$ matrix element

Symbols

$\dot{}$ time differentiation operator (d/dt)

∇ spatial differentiation vector (*del*)

$\approx$ approximately equal to

$\sim$ of the same order as

$\propto$ proportional to

$\equiv$ equal to by definition (or evidently)

$\cdot$ scalar ("dot-") product

$\times$ vector ("cross-") product[10]

$\bar{}$ time averaging

$\langle\ \rangle$ statistical averaging

[,] commutator

{ , } anticommutator

$\mathbf{n}$ unit vector

Parts of the series

CM: Classical Mechanics

EM: Classical Electrodynamics

QM: Quantum Mechanics

SM: Statistical Mechanics

Appendices

MA: Selected Mathematical Formulas

CA: Selected Physical Constants

Prime signs

The prime signs (′, ″, etc) are used to distinguish similar variables or indices (such as j and j' in the matrix element above), rather than to denote derivatives.

Formulas

The most general and/or important formulas are highlighted with blue frames and short titles on the margins.

Numbering

Chapter numbers are dropped in all references to formulas, figures, footnotes, and problems within the same chapter.

[9] The same letter, typeset in different fonts, typically denotes different variables.

[10] On a few occasions, the cross sign is used to emphasize the usual multiplication of scalars.

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Beta version

Open online access at

http://commons.library.stonybrook.edu/egp/

and

https://sites.google.com/site/likharevegp/

Part CM:
Classical Mechanics

Last corrections: August 22, 2022

A version of this material was published in 2017 under the title

Classical Mechanics: Lecture notes

IOPP, Essential Advanced Physics – Volume 1, ISBN 978-0-7503-1398-8,

with model solutions of the exercise problems published in 2018 under the title

Classical Mechanics: Problems with solutions

IOPP, Essential Advanced Physics – Volume 2, ISBN 978-0-7503-1401-5

However, by now this online version of the lecture notes, as well as the problem solutions available from the author by request, have been much better corrected

About the author:
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Additional file (available from the author upon request):
Exercise and Test Problems with Model Solutions (198 + 48 = 246 problems; 340 pp.)

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Chapter 1. Review of Fundamentals

After a brief discussion of the title and contents of the course, this introductory chapter reviews the basic notions and facts of the non-relativistic classical mechanics, that are supposed to be known to the reader from their undergraduate studies.[1] *Due to this reason, the discussion is very short.*

1.0. Terminology: Mechanics and dynamics

A more fair title for this course would be *Classical Mechanics and Dynamics*, because the notions of mechanics and dynamics, though much intertwined, are still somewhat different. The term *mechanics*, in its narrow sense, means the derivation of equations of motion of point-like particles and their systems (including solids and fluids), solution of these equations, and interpretation of the results. *Dynamics* is a more ambiguous term; it may mean, in particular:

(i) the part of physics that deals with motion (in contrast to *statics*);
(ii) the part of physics that deals with reasons for motion (in contrast to *kinematics*);
(iii) the part of mechanics that focuses on its two last tasks, i.e. the solution of the equations of motion and discussion of the results.[2]

Because of this ambiguity, after some hesitation, I have opted to use the traditional name *Classical Mechanics*, with the word *Mechanics* in its broad sense that includes (similarly to *Quantum Mechanics* and *Statistical Mechanics*) studies of dynamics of some non-mechanical systems as well.

1.1. Kinematics: Basic notions

The basic notions of kinematics may be defined in various ways, and some mathematicians pay much attention to alternative systems of axioms and the relations between them. In physics, we typically stick to less rigorous ways (in order to proceed faster to solving particular problems) and end debating any definition as soon as "everybody in the room" agrees that we are all speaking about the same thing – at least in the context they are being discussed. Let me hope that the following notions used in classical mechanics do satisfy this criterion in our "room":

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[1] The reader is advised to perform (perhaps after reading this chapter as a reminder) a self-check by solving a few problems of those listed in Sec. 1.6. If the results are not satisfactory, it may make sense to start with some remedial reading. For that, I could recommend, e.g., J. Marion and S. Thornton, *Classical Dynamics of Particles and Systems,* 5th ed., Saunders, 2003; and D. Morin, *Introduction to Classical Mechanics*, Cambridge U., 2008.

[2] The reader may have noticed that the last definition of dynamics is suspiciously close to the part of mathematics devoted to differential equation analysis; what is the difference? An important bit of philosophy: physics may be defined as an art (and a bit of science :-) of describing Mother Nature by mathematical means; hence in many cases the approaches of a mathematician and a physicist to a problem are very similar. The main difference between them is that physicists try to express the results of their analyses in terms of the properties of the *systems* under study*,* rather than the *functions* describing them, and as a result develop a sort of intuition ("gut feeling") about how other similar systems may behave, even if their exact equations of motion are somewhat different – or not known at all. The intuition so developed has an enormous heuristic power, and most discoveries in physics have been made through gut-feeling-based insights rather than by plugging one formula into another one.

(i) All the *Euclidean geometry* notions, including the *point*, the *straight line*, the *plane*, etc.[3]

(ii) *Reference frames*: platforms for observation and mathematical description of physical phenomena. A reference frame includes a *coordinate system* used for measuring the point's position (namely, its *radius vector* **r** that connects the coordinate origin to the point – see Fig. 1) and a clock that measures *time t*. A coordinate system may be understood as a certain method of expressing the radius vector **r** of a point as a set of its *scalar coordinates*. The most important of such systems (but by no means the only one) are the *Cartesian* (orthogonal, linear) *coordinates*[4] r_j of a point, in which its radius vector may be represented as the following sum:

$$\boxed{\mathbf{r} = \sum_{j=1}^{3} \mathbf{n}_j r_j ,} \tag{1.1}$$

Cartesian coordinates

where $\mathbf{n}_1$, $\mathbf{n}_2$, and $\mathbf{n}_3$ are unit vectors directed along the coordinate axis – see Fig. 1.[5]

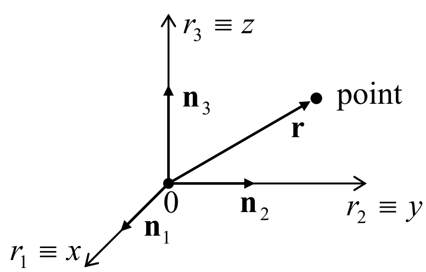

Fig. 1.1. Cartesian coordinates of a point.

(iii) The *absolute* ("Newtonian") *space/time*,[6] which does not depend on the matter distribution. The space is assumed to have the *Euclidean metric*, which may be expressed as the following relation between the length *r* of any radius vector **r** and its Cartesian coordinates:

$$\boxed{r^2 \equiv \left|\mathbf{r}\right|^2 = \sum_{j=1}^{3} r_j^2 ,} \tag{1.2}$$

Euclidean metric

while time *t* is assumed to run similarly in all reference frames. These assumptions are critically revised in the relativity theory (which, in this series, is discussed only starting from EM Chapter 9.)

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[3] All these notions are of course abstractions: *simplified models* of the real objects existing in Nature. But please always remember that *any* quantitative statement made in physics (e.g., a formula) may be strictly valid only for an approximate model of a physical system. (The reader should not be disheartened too much by this fact: experiments show that many models make extremely precise predictions of the behavior of the real systems.)

[4] In this series, the Cartesian coordinates (introduced in 1637 by René Descartes, a.k.a. Cartesius) are denoted either as either $\{r_1, r_2, r_3\}$ or $\{x, y, z\}$, depending on convenience in each particular case. Note that axis numbering is important for operations like the vector ("cross") product; the "correct" (meaning generally accepted) numbering order is such that the rotation $\mathbf{n}_1 \to \mathbf{n}_2 \to \mathbf{n}_3 \to \mathbf{n}_1 \ldots$ looks counterclockwise if watched from a point with all $r_j > 0$ – like the one shown in Fig. 1.

[5] Note that the representation (1) is also possible for locally-orthogonal but *curvilinear* (for example, polar/cylindrical and spherical) coordinates, which will be extensively used in this series. However, such coordinates are not Cartesian, and for them some of the relations given below are invalid – see, e.g., MA Sec. 10.

[6] These notions were formally introduced by Sir Isaac Newton in his main work, the three-volume *Philosophiae Naturalis Principia Mathematica* published in 1686-1687, but are rooted in earlier ideas by Galileo Galilei, published in 1632.

(iv) The (instant) *velocity* of the point,

Velocity

$$\boxed{\mathbf{v}(t) \equiv \frac{d\mathbf{r}}{dt} \equiv \dot{\mathbf{r}}\,,} \tag{1.3}$$

and its *acceleration*:

Acceleration

$$\boxed{\mathbf{a}(t) \equiv \frac{d\mathbf{v}}{dt} \equiv \dot{\mathbf{v}} = \ddot{\mathbf{r}}\,.} \tag{1.4}$$

(v) *Transfer between reference frames*. The above definitions of vectors **r**, **v**, and **a** depend on the chosen reference frame (are "reference-frame-specific"), and we frequently need to relate those vectors as observed in different frames. Within Euclidean geometry, the relation between the radius vectors in two frames with the corresponding axes parallel at the moment of interest (Fig. 2), is very simple:

Radius vector's transformation

$$\boxed{\mathbf{r}\big|_{\text{in }0'} = \mathbf{r}\big|_{\text{in }0} + \mathbf{r}_0\big|_{\text{in }0'}\,.} \tag{1.5}$$

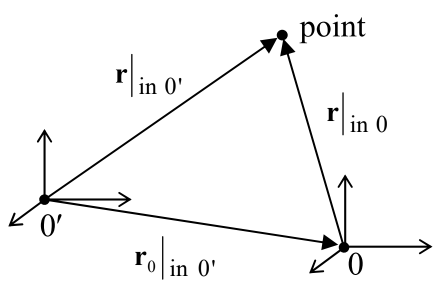

Fig. 1.2. Transfer between two reference frames.

If the frames move versus each other by *translation* only (no mutual rotation!), similar relations are valid for the velocities and accelerations as well:

$$\mathbf{v}\big|_{\text{in }0'} = \mathbf{v}\big|_{\text{in }0} + \mathbf{v}_0\big|_{\text{in }0'}\,, \tag{1.6}$$

$$\mathbf{a}\big|_{\text{in }0'} = \mathbf{a}\big|_{\text{in }0} + \mathbf{a}_0\big|_{\text{in }0'}\,. \tag{1.7}$$

Note that in the case of mutual rotation of the reference frames, the transfer laws for velocities and accelerations are more complex than those given by Eqs. (6) and (7). Indeed, in this case, the notions like $\mathbf{v}_0|_{\text{in }0'}$ are not well defined: different points of an imaginary rigid body connected to frame 0 may have different velocities when observed in frame 0′. It will be more natural for me to discuss these more general relations at the end of Chapter 4 devoted to rigid body motion.

(vi) A *particle* (or "point particle"): a localized physical object whose size is negligible, and the shape is irrelevant *to the given problem*. Note that the last qualification is extremely important. For example, the size and shape of a spaceship are not too important for the discussion of its orbital motion but are paramount when its landing procedures are being developed. Since classical mechanics neglects the quantum mechanical uncertainties,[7] in it, the position of a particle at any particular instant t, may be identified with a single geometrical point, i.e. with a single radius vector $\mathbf{r}(t)$. The formal final goal of classical mechanics is finding the *laws of motion* $\mathbf{r}(t)$ of all particles participating in the given problem.

[7] This approximation is legitimate when the product of the coordinate and momentum scales of the particle motion is much larger than Planck's constant $\hbar \sim 10^{-34}$ J·s. More detailed conditions of the classical mechanics' applicability depend on a particular system – see, e.g., the QM part of this series.