DIGITAL FILTERS

Edited by **Fausto Pedro García Márquez**

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Contents

Preface

The new technologies and communications systems are being set up in all areas. It leads to treating data from different sources and for several proposes. But it is necessary to obtain only the information that is required. Digital filters, together with analogue filters, are used for these objectives. The main advantage of the digital filters is that they can be applied at zero cost and with a great flexibility. The mathematical models where they are created have different complexity and computational cost. In this book the most relevant filters are described, and with different applications. The material covered in this text is crucial for getting a general idea about digital filters. This book also presents some best options for each case study considered.

In spite of the mathematical complexity of the digital filters, the text is presented for any reader with a motivation for learning about digital filters. The high level contents are shown with an exhaust introduction, where the most important works in the literature are referenced and it completed with various examples.

A discrete filter is presented within a well-known and common framework, namely the State Space with the help of the Kalman Filter (KF) and/or complementary Fixed Interval Smoother (FIS) algorithms. It is presented in several case studies for detecting faults where these models can be adapted to external and internal conditions to the mechanism. All of these models are developed within a well-known common framework, namely the State Space (SS). The KF is a powerful algorithm, because it supports estimations of past, present and future states. In this case, it is used for filtering with Integrated Random Walks by setting up a bivariate model composed of two time series, i.e. the reference curve on one hand and each one of the empirical curves obtained on line on the other hand. Other options are to use a model VARMA (Vector autoregressive moving-average) class or a local level plus noise but set up in continuous time. Finally, due to the nature of the data, a pertinent class is a Dynamic Harmonic Regression, similar to a Fourier analysis, but with advanced features included to incorporate a time varying period observed in the data.

In the case of a linear circuit and frequency filter analysis for sinusoidal and periodical input signals, the spectral representations employing Fourier transform are studied. In that case, Laplace's transformations are employed in order to consider a complex frequency. The compound finite signal representations are done in the form of the set of damped oscillatory components. It is an efficient method for filtering and it can work with a complex coordinate. In the case of Infinite Impulse Response (IIR) filter impulse functions the representation uses this set of damped oscillatory components. Impulse functions of Finite Impulse Response (FIR) filters representation are also based on this set of damped oscillatory components, but with the difference of a finite duration of the impulse functions. It considers the stationary and non stationary modes, where it can be calculated easily in the spectral representation context. It is possible considering the application of spectral representations in complex frequency coordinates. It leads to consider both spectral approach and the state space method for frequency filter analysis and synthesis. The filter synthesis problem comes to dependence composition for filter transfer function on complex frequencies of input signal components.

Complex filters can be namely digital filters with complex coefficients. They are employed in complex signal processing compared to the real signal processing (e.g. telecommunications). This can imply real and imaginary inputs and outputs, and these signals need to be separated into real and imaginary parts for being studied as complex signals. The first- and second-order IIR orthogonal complex sections are synthesized as filters in designing cascade structures or as single filter structures. It leads delay-free loops and has a canonical number of elements. The low-sensitivity 1 and 2 variable complex sections can be used in narrowband band-pass / band-stop structures. The main advantages of these models are the higher freedom of tuning, reduced complexity and lower stop-band sensitivity.

The response dela in digital circuits should be adjusted to a fraction of the sampling interval and it should be fixed or variable in order to control the fractional delay (FD). These circuits are used in telecommunications applications that require speech synthesis and processing, image interpolation, sigma-delta modulators, time-delay estimation, in some biomedical applications and for modeling of musical instruments. Considering the phase-sensitivity minimization of each individual first- and second-order allpass section in the filter cascade realization, fixed and variable allpass-based fractional delay filters are developed and adjusted through sensitivity minimizations. The real and complex-conjugate poles combinations for different values of the FD parameter D and of the transfer function (TF) order N are analyzed trying to minimize the overall sensitivity.

A two-dimensional (2D) digital filter is employed to attain the desired cut-off frequency and the stable monotonic amplitude-frequency responses of this filter. It is developed in accordance with monotonic amplitude-frequency responses employing Darlington-type gyrator networks and doubly-terminated RLC-networks by the application of Generalized Bilinear Transformation (GBT). The doubly terminated RLC networks are adjusted as second-order Butterworth and Gargour & Ramachandran. It leads low-pass, high-pass, band-pass and band-elimination filters. The transformation between these filters is done by the value and sign of the parameter called g and GBT. It is useful in digital image (video and audio), and for enhancement and restoration in different fields, as medical science, geographical science and environment, space and robotic engineering, etc.

From a 1D filter (low-pass and maximally-flat or very selective), a 2D filter can be developed. These are essentially spectral transformations (frequency transformations) via bilinear or Euler transformations followed by mappings. This book analyzes the case of recursive filter approaches in the frequency domain applied in image processing: directional selective filters, oriented wedge filters, fan filters, diamond-shaped filters, etc. The zero-phase case is also considered. All the models are mainly analytical, and in some cases, numerical optimization is employed, in particular - rational approximations. The reason to choose the analytical approach is that the 2D parameters can be controlled by adjusting the prototype. An analytical design method in polar coordinates is proposed and defined by a periodic function expressed in polar coordinates in the frequency plane. It can yield selective two or multi-directional filters, and also fan and diamond filters. Finally, two-lobe filters are analysed, selective four-lobe filters with an arbitrary orientation angle, fan filters and diamond filters.

Single correction filters or ensembles of correction filters, sensitivity filters, lumbar spine filter, banks of vehicle filters, and road texture filters are presented. They are studied in two examples on safety of traffic: road hump analysis and determination of road texture. Digital filters are recommended for low robustness, and this originates from the definition of the feature and/or its incomplete specification instead of a feature which is not robust and questionable. The digital filters employed fit into the above mentioned standard linear-in-response finite/infinite impulse response (FIR/IIR) form for direct implementation. In this case any filter may be transferred to a state-space form for generalization into a KF.

Carry-Save Arithmetic is employed in order to achieve an optimal design of single constant multipliers for coefficients with up to 19 bits wordlength. The non-redundant representation is also considered. The proposed techniques are useful when a high-speed realization is required. It is demonstrated in the multiple constant multiplication problems suitable for transposed direct form FIR filters using carry-save representation of intermediate results but non-redundant input.

Lattice wave digital (LWD) filter (parallel connections of all-pass filters) is a structure implemented in the recursive digital filters. Three cases are considered in this book: primarily the overall filter, constructed as a cascade of low-order LWD filters. Secondly, approximately linear-phase LWD filters are constructed as a single block. The reason for this is the lack of benefits for the direct-form LWD filter design in the usage of a cascade of several filter blocks. Finally, it is focused on the design of special recursive single-stage and multistage Nth-band decimators and interpolators. The coefficient optimization is performed with following steps: an initial infinite-precision filter is designed such that it exceeds the given criteria in order to provide some tolerance for coefficient quantization; then, a nonlinear optimization algorithm is employed for determining a parameter space of the infinite-precision coefficients including the feasible space where the filter meets the given criteria; and finally, the filter parameters are found in this space so that the resulting filter meets the given criteria with the simplest coefficient representation forms. The realization of these filters does not require the use of a costly general multiplier element. It leads to the fact that the filters are goods in very large-scale integration (VLSI).

The sampled-data and digital filters (i.e. "memory transistor" or "memory transconductor" approaches) are both studied for their effectivity. This case is about biquadratic sections used in cascade design. The switched-current (SI) circuits are also one of the case studies employed, where it can be extended to cases as digital VLSI-CMOS technologies, lower supply voltage and wide dynamic range, considering an SI as "analog counterpart" of the digital filters. The biquadratic realization structures are developed from the first and second direct forms of the 2nd-order digital filter. The continuous-time biquadratic sections design is also considered. Finally, the optimization of sampled-data and digital filters design is solved by using the heuristic algorithm as the differential evolutionary algorithm.

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Digital Filters for Maintenance Management

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1. Abstract

Faults in mechanisms must be detected quickly and reliably in order to avoid important losses. Detection systems should be developed to minimize maintenance costs and are generally based on consistent models, but as simple as possible. Also, the models for detecting faults must adapt to external and internal conditions to the mechanism. The present chapter deals with three particular maintenance algorithms for turnouts in railway infrastructure by means of discrete filters that comply with these general objectives. All of them have the virtue of being developed within a well-known and common framework, namely the State Space with the help of the Kalman Filter (KF) and/or complementary Fixed Interval Smoother (FIS) algorithms. The algorithms are tested on real applications and thorough results are shown.

2. Introduction

Faults in any important mechanisms must be detected quickly and reliably if the information is to be useful. Generally such mechanisms may be modeled as discrete dynamic systems, where data must be processed on line. When feasible, the detection system should use a model as simple as possible for detecting faults quickly by analyzing data in real time. The models for detecting faults must adapt to external and internal conditions to the mechanism, since both of them may affect the system as a whole.

The present chapter deals with maintenance systems for turnouts in railway infrastructure by means of discrete filters. Turnouts are assembled from switches and a crossing where the moving parts are often described as the "points" move by the point mechanism. The standard railway point mechanism is a complex electro-mechanical device with many potential failure modes.

Several approaches for maintenance of such devices are shown in this chapter and briefly described in this introduction. All of them have the virtue of being developed within a well-known common framework, namely the State Space (SS) with the help of the Kalman Filter (KF) and/or complementary Fixed Interval Smoother (FIS) algorithms, exposed in general terms in the following section.

Based on this common framework, the following subsections in this introduction show the particular applications shown in later sections of the chapter.

2.1. Filtering with Integrated Random Walks (IRW)

One possible way to analyze faults on line is to work with a reference dynamic system for their analysis. If the absolute value of the difference between the actual data and the reference data (i.e. the profile without any fault) is analyzed, the majority of faults may be detected by means of a simplified univariate dynamic system, like the one explored in [9]. The dynamic system and the use of the SS framework and the KF in this study allow increasing the reliability of the model presented that is the basic input to a rule-based decision mechanism. When applied to the linear discrete data filtering problem, the KF is a powerful algorithm, because it supports estimations of past, present and, most importantly, future states. It can therefore be used in predictive maintenance applications where data collected from sensors is affected by measurement and transmission line noise [12].

The previous approach may be exploited by setting up a bivariate model composed of two time series, i.e. the reference curve on one hand and each one of the empirical curves obtained on line on the other hand. More specifically (see section 4.2 below) a tentative model consists of a bivariate trend plus noise structure. The correlation between either trends or signals free from noise is considered as an indication of similarity between the curves and therefore the inexistence of failures. As long as the new incoming data is free from fault, the correlation parameter is close to one, but as a failure starts to develop this parameter tends to differ from one. The cut-off value of the correlation coefficient relevant to discern 'good' and 'bad' curves is selected on practical grounds based on past experience with this kind of data, but refined formal statistical criteria may be used as well [19]. Even forecasts of the curve that is being studied may be produced at any point in time, based on the current parameter values and the future data of the reference curve [14]. Therefore the fault may be detected ahead of time.

2.2. Random Walks and smoothing

Similar measurement data were collected from sensors mounted on a UK type M63 point machine at the Carillion Rail (formerly GTRM) Training Centre in Stafford (UK). It is difficult to compare the measurements taken during induced failure conditions with those from the fault-free condition because of noise in the measurements. The measurement data needed to be filtered in order to reduce the noise before comparisons may be made. Filtering using a SS model and the KF was an option (like in [9], [19] and [20]). Assuming the noisy data is a signal plus noise model, the KF reduces the power of the 100 and 200 Hz interfering signals. Rather than augmenting the SS models to express the additional knowledge of the interfering signals, a much simpler smoothing seems more convenient because of the relationship between the sample rate and the frequencies of the interfering signals, and provides excellent results for the data collected during this series of experiments [10].

2.3. Advance Dynamic Harmonic Regression (DHR)

A different case study was based on data collected from point mechanisms at Abbotswood Junction (UK). Three electro-mechanical and four electro-hydraulic point machines were monitored by a RCM system. Processed information was sent remotely from the trackside data-collection units to a personal computer located in a local relay room.

A fault is detected by comparing the forecasts of the model, considered as the expected signal in the case of no faults, with the actual data coming from the point mechanism when a movement is in progress. If the error is too large, measured by its standard deviation, a fault alarm is issued. The limit at which an error is considered too large is a design parameter that is fixed by experimentation. The system adapts to the changes experienced by the point machine. There are internal alterations (like friction, wear, etc.) and external as well (like environmental conditions, impacts, obstacles, etc.). The adaptability of the system is accomplished by continuous estimation of the models as new information becomes available and by discarding the oldest information. Models are always estimated on fault-free data [13].

The key point in this application is that the expected shape is computed as the forecast of a combination of two models that work interactively on historical data coming from signals free from any fault. The first of the models forecasts the time span a movement would take in case of absence of faults (an appropriate model used in this case was of the VARMA class or a local level plus noise but set up in continuous time). The second model is run to forecast the signal itself (due to the nature of the data a pertinent class is a Dynamic Harmonic Regression, DHR, similar to a Fourier analysis, but with advanced features included to incorporate a time varying period observed in the data).

The outline of the chapter is as follows. Section 3 reports a brief explanation of the general framework on which all the models in this chapter are set up, namely the State Space systems. Section 4 shows the first of the applications, i.e. in the point mechanisms. Finally section 5 shows how a fault detection algorithm may be implemented on seven point machines at Abbotswood junction (UK).

3. State Space systems

The general framework on which all models in this chapter are cast, is the so called State Space systems, that have experienced a remarkable attention during the last decades, as the extended literature about it reveals [3], [7], [13], [15], [16], [17], [21], [24], [26] and [27].

A stochastic discrete-time State Space system (SS) is a model composed of two sets of equations, the *Observation Equations*, and *State Equations*. The former relates the output to the states of the system, while the latter reflects the dynamic behavior of the system by relating the current value of the states to their past values. There are a number of different formulations of these equations, but one fairly general representation is given by equations (1) (see [3] and [21]). In general, much simpler models are sufficient, as later case studies show.

$$\begin{array}{lll} \text{State Equations} & : \ \mathbf{x}_{t+1} = \mathbf{\Phi}_t \mathbf{x}_t + \mathbf{E}_t \mathbf{w}_t & \text{(i)} \\ \text{Observation Equations} & : \ \mathbf{z}_t = \mathbf{H}_t \mathbf{x}_t + \mathbf{C}_t \mathbf{v}_t & \text{(ii)} \end{array} \quad (1)$$

In (1) $\mathbf{z}_t$ is the m dimensional vector of observed variables for $t = 1,2,\ldots,N$; $\mathbf{x}_t$ is an n dimensional stochastic state vector; $\mathbf{w}_t$ is an r dimensional vector of (to be Gaussian) system disturbances, i.e. zero mean white noise inputs with a covariance matrix $\mathbf{Q}_t$; and $\mathbf{v}_t$ is a s dimensional vector of zero mean white noise variables (measurement noise: again assumed to be Gaussian) with a covariance matrix $\mathbf{R}_t$. In general, the vector $\mathbf{v}_t$ is assumed to be independent of $\mathbf{w}_t$ (not necessarily), and these two noise vectors are independent of the initial state vector $\mathbf{x}_0$. $\mathbf{\Phi}_t, \mathbf{E}_t, \mathbf{H}_t, \mathbf{C}_t, \mathbf{Q}_t$, and $\mathbf{R}_t$ are, respectively, the $n \times n$, $n \times r$, $m \times n$, and $m \times s$, $r \times r$ and $s \times s$ system matrices, some elements of which are known and others that need to be estimated in some way.

Given the general SS form (1), the estimation problem consists of finding the first and second order moments (mean and covariance) of the state vector, conditional on all the data in a sample. Provided that all disturbances in the model are Gaussian, a *Kalman Filter* (KF) produces the optimal estimates of such moments in the sense of minimizing the Mean Squared Errors (MSE). An algorithm that is used in parallel with the KF and is not so well-known in certain contexts is the *Fixed Interval Smoothing* (FIS) algorithm, which allows for an operation similar to that of the KF but with a different set of information. The KF used in this chapter is:

$$
\begin{aligned}
\mathbf{F}_t &= \left[\mathbf{C}_t\mathbf{R}_t\mathbf{C}_t^T + \mathbf{H}_t\hat{\mathbf{P}}_{t|t-1}\mathbf{H}_t^T\right] \\
\mathbf{K}_t &= \left[\mathbf{\Phi}_{t+1}\hat{\mathbf{P}}_{t|t-1}\mathbf{H}_t^T\right]\mathbf{F}_t^{-1} \\
\hat{\mathbf{x}}_{t+1|t} &= \left[\mathbf{\Phi}_{t+1} - \mathbf{K}_t\mathbf{H}_t^T\right]\hat{\mathbf{x}}_{t|t-1} + \mathbf{K}_t\mathbf{z}_t \\
\hat{\mathbf{P}}_{t+1|t} &= \mathbf{\Phi}_{t+1}\hat{\mathbf{P}}_{t|t-1}\mathbf{\Phi}_{t+1}^T - \mathbf{K}_t\left[\mathbf{\Phi}_{t+1}\hat{\mathbf{P}}_{t|t-1}\mathbf{H}_t^T\right]^T + \mathbf{E}_t\mathbf{Q}_t\mathbf{E}_t^T
\end{aligned}
$$

The backward FIS recursions are:

$$
\begin{aligned}
\hat{\mathbf{x}}_{t|N} &= \hat{\mathbf{x}}_{t|t-1} + \hat{\mathbf{P}}_{t|t-1}\mathbf{s}_{t-1} \\
\hat{\mathbf{P}}_{t|N} &= \hat{\mathbf{P}}_{t|t-1} - \hat{\mathbf{P}}_{t|t-1}\mathbf{S}_{t-1}\hat{\mathbf{P}}_{t|t-1} \\
\mathbf{s}_{t-1} &= \mathbf{H}_t^T\mathbf{F}_t^{-1}\left(\mathbf{z}_t - \mathbf{H}_t\hat{\mathbf{x}}_{t|t-1}\right) + \overline{\mathbf{\Phi}}_t^T\mathbf{s}_t \qquad \text{with } \mathbf{s}_N = \mathbf{0} \\
\mathbf{S}_{t-1} &= \mathbf{H}_t^T\mathbf{F}_t^{-1}\mathbf{H}_t + \overline{\mathbf{\Phi}}_t^T\mathbf{S}_t\overline{\mathbf{\Phi}}_t \qquad \text{with } \mathbf{S}_N = \mathbf{0} \\
\overline{\mathbf{\Phi}}_t &= \mathbf{\Phi}_t - \mathbf{\Phi}_t\hat{\mathbf{P}}_{t|t-1}\mathbf{H}_t^T\mathbf{F}_t^{-1}\mathbf{H}_t
\end{aligned}
$$

This general SS formulation is capable of handling many nonstationary linear dynamical systems; also it can model nonlinear systems but conditionally Gaussian; general heteroscedastic systems; time-varying systems; etc. In addition, many kinds of extensions of model have been proposed in the literature, such as linear approximations of functionally nonlinear dynamic systems; non-Gaussian disturbances; etc. Missing data is not a problem given the recursive nature of the algorithms, because such data are replaced by their expectations based on the model and the data. Then, if such data is at the end of the sample the KF produces forecasts of the signal, while if they are in the middle or at the beginning both algorithms produce interpolation or forecasts from the beginning of the series backwards.

The application of the recursive KF/FIS algorithms requires values for all the system matrices $\mathbf{\Phi}_t, \mathbf{E}_t, \mathbf{H}_t, \mathbf{C}_t, \mathbf{Q}_t$, and $\mathbf{R}_t$. Most of the elements of these matrices must be estimated by efficient methods. The Maximum Likelihood (ML) method in the time domain by means of 'prediction error decomposition' ([24] and [15]) is the most common because of its generality and good theoretical properties.

4. Filtering with Integrated Random Walks (IRW)

4.1. Data

Approximately 55 % of railway infrastructure component failures on high speed lines are due to signalling equipment and turnouts. "Signalling equipment" covers signals, track circuits, interlockings, automatic train protection (ATP) or LZB (track loop based ATP), and the traffic control centre. From another point of view, the annual cost of maintaining points is rather high compared to other infrastructure elements, about 3.4 million UKP (United Kingdom Pound) per year for about 1000 km of railway. TC-TCR trade circuits, for example, cost 2.1 million UKP per year for the same area. Of the points expenditure, 1.2 million UKP is for clamp lock type (hydraulic) turnout and 1.4 UPK million for electrically operated turnouts (data provided by a British asset manager). Turnouts can also be used to implement flank protection for a train route allocated to another train. This is achieved by positioning the blades of the turnout in such a way that a train driving through the turnout is not directed into a track segment belonging to the route of another train.

Most standard point machines (see Fig. 1) contain a switch actuating and a locking mechanism which includes a hand-throw lever and a selector lever to allow operation by power or hand. The mechanism is normally divided into three major subsystems: (i) the motor unit which may includes a contactor control arrangement and a terminal area; (ii) a gearbox comprising spur-gears and a worm reduction unit with overload clutch; and (iii) the dual control mechanism as well as a controller subsystem with motor cut-off and detection contacts. Generally, there are also mechanical linkages for the detection and locking of the point. The standard railway point is therefore a complex electro-mechanical device with many potential failure modes.

The circuit controller includes detection switches and a pair of snap-action switches to stop the machine at the end of its stroke and to brake the motor electrically so that the mechanism is not subject to impacts. The detection switches have high pressure wiping contacts made of silver/cadmium oxide or gold and they are operated by both the lockbox and the detection rod. The detection switches have additional contacts to allow mid-stroke short circuiting of the detection relays to avoid wrong indications in the signal box or electronic interlocking.

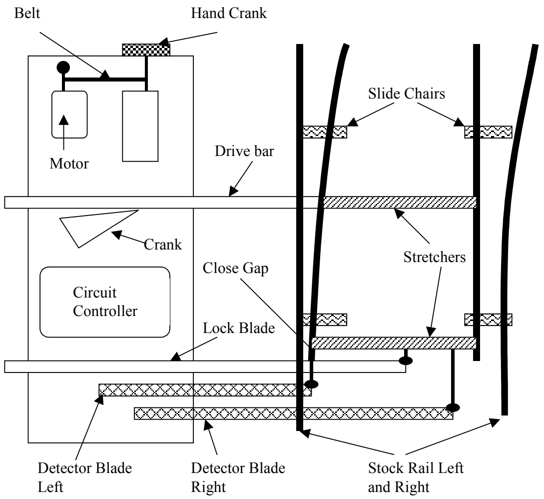

Fig. 1. Point Mechanism

476 experiments (point moves or attempted point moves) were carried out while collecting time, force and operating current data. The data from the point mechanism is initially classified in terms of direction of movement, i.e., either reverse to normal direction or normal to reverse direction. For both directions, faults have been detected with "current (A) vs. time (s)" curves and "force (N) vs. (s)" curves (see some examples in Fig. 2(a) and 2(b)). It was observed that "current (A) vs. time (s)" curves are not the best choice for detecting faults in point mechanisms. The final classification of faults employs only the magnitude and the moment when they change with respect to the reference curves.

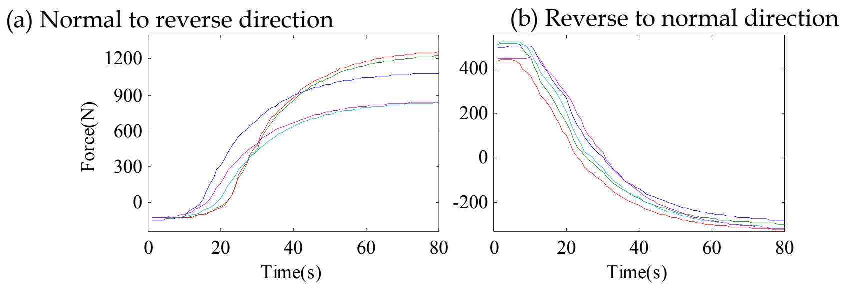

Fig. 2. Operating force curves for a point mechanism

For detecting faults in point mechanisms, a model was employed that can determine the dynamic character of the system. For instance, the reference signals or curves for detecting faults depend on the environmental conditions (temperature, humidity, etc.), and on the in service time of the system, because the friction forces are larger at the beginning than once the system has worn in. The available data consists of 79 curves for the reverse to normal direction, including 4 curves "as commissioned", and 72 curves for the normal to reverse direction, with 3 curves "as commissioned" (some of them may be seen in Fig. 2). A reference dynamic system has to be applied to all of these variables. The data collected refers to force (N) versus time (s). The first conclusion after studying these curves is that we can detect only a few faults by analyzing the signal directly but, if we analyze the differences between the current data x^j and the reference data x^i in the form of absolute values d^j (1), we can detect the majority of faults as they develop.

$$d_t^j = \left| x_t^j - x_t^i \right|, \quad \forall t \tag{1}$$

Some of these curves are shown in Fig. 3(a) and 3(b) for reverse to normal direction and normal to reverse direction respectively. The 'x' axis is time [s] and the 'y' axis is the difference between the dynamic mean geometric and the current curve as an absolute value [N].

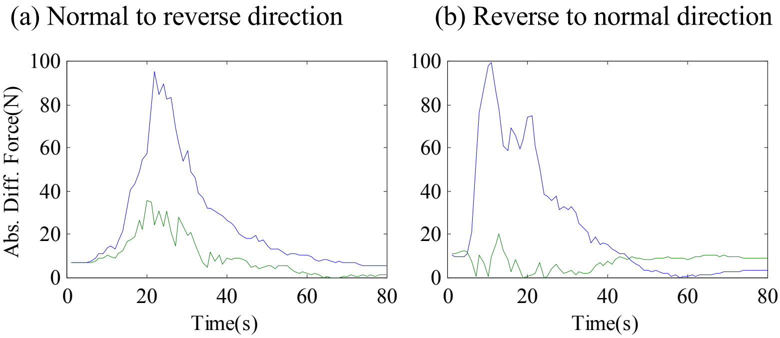

Fig. 3. Difference between the reference signal for the point and the newly acquired data in absolute values

4.2. The model

One feasible model written in SS form (1) for this application is of the type local mean plus noise for two signals simultaneously, where the local means are modeled by the dynamics implied by the state equations, i.e.

$$\begin{gathered}
\mathbf{x}_{t+1} = \begin{pmatrix} \mathbf{I} & \mathbf{I} \\ \mathbf{0} & \mathbf{I} \end{pmatrix} \mathbf{x}_t + \begin{pmatrix} \mathbf{0} \\ \mathbf{I} \end{pmatrix} \begin{pmatrix} w_{1t} \\ w_{2t} \end{pmatrix} \\
\mathbf{z}_t = \text{signal} + \text{noise} = \begin{pmatrix} \mathbf{I} & \mathbf{0} \end{pmatrix} \mathbf{x}_t + \mathbf{v}_t \\
\mathbf{Q} = \begin{pmatrix} \sigma_{w_1}^2 & \rho\sqrt{\sigma_{w_1}^2 \sigma_{w_2}^2} \\ \rho\sqrt{\sigma_{w_1}^2 \sigma_{w_2}^2} & \sigma_{w_2}^2 \end{pmatrix}; \quad \mathbf{R} = \begin{pmatrix} \sigma_{v_1}^2 & \sigma_{v_1 v_2} \\ \sigma_{v_1 v_2} & \sigma_{v_2}^2 \end{pmatrix}
\end{gathered} \tag{2}$$

In model (2) all the system matrices are time invariant: $\mathbf{I}$ is a two dimensional identity matrix; $\mathbf{0}$ is a two by two matrix of zeros; $\sigma_\bullet^2$ are the variances of the noise signals or disturbances either in the state or observation equations; $\sigma_{\bullet\bullet}$ is the covariance between two disturbances; and ρ is the correlation coefficient between the two noise signals in the state equation.