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# Industrial Robot Control System Parametric Design on the Base of Methods for Uncertain Systems Robustness

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## 1. Introduction

Industrial robots often operate in conditions of their parameters substantial variation that causes variation of their control systems characteristic equations coefficients values, thus generating the equations families. Analysis of the dynamic systems characteristic polynomial families stability, the stable polynomials and polynomial families synthesis represent complicated and important task (Polyak, 2002, a). Within the parametric approach to the problem the series of the effective methods for analysis have been developed (Bhattaharyya et al., 1995; Polyak, 2002, a). In this way, V. L. Kharitonov (Kharitonov, 1978) proved that for the interval uncertain polynomials family asymptotic stability verification it is necessary and enough to verify only four polynomials of the family with the definite constant coefficients. In the works of Y. Z. Tsypkin and B. T. Polyak the frequency approach to the polynomially described systems robustness was offered (Polyak & Tsypkin, 1990; Polyak & Scherbakov, 2002; Tsypkin & Polyak, 1990; Tsypkin, 1995). This approach comprises the robust stability criteria for linear continuous systems, the methods for calculating the maximal disturbance swing for the nominal stable system on the base of the Tsypkin – Polyak hodograph. These results were generalized to the linear discrete systems (Tsypkin & Polyak, 1990). The robust stability criterion for the relay control systems with the interval linear part was obtained (Tsypkin, 1995). The super-stable linear systems were considered (Polyak & Scherbakov, 2002).
The problem for calculating the polynomial instability radius on the base of the frequency approach is investigated (Kraev & Fursov, 2004). The technique for composing the stability domain in the space of a single parameter or two parameters of the system with the *D*-decomposition approach application is developed (Gryazina & Polyak. 2006).
The method for definition of the nominal polynomial coefficients deviations limit values, ensuring the hurwitz stability, has been offered (Barmish, 1984). The task here is reduced to the single-parameter optimization problem. The similar tasks are solved by A. Bartlett (Bartlett et al., 1987) and C. Soh (Soh et



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al., 1985). Conditions for the generalized stability of polynomials with the linearly dependent coefficients (polytopes) have been obtained (Bartlett et al., 1987; Rantzer, 1992).

One of the most important stages, while calculating dynamic systems with uncertain parameters, is ensuring robust quality. The control process qualitative characteristics are defined by the characteristic equations roots location in the complex plane (the plane of system fundamental frequencies). In this connection, three main groups of tasks being solved can be distinguished: determining the assured roots location domain (region) for the given system, finding conditions of whether roots get into the given region or not (determination of the Λ-stability conditions) and locating roots in the given domain (ensuring Λ-stability).

The frequency stability criteria for the linear systems families and also the method for finding the largest disturbance range of their characteristic equations coefficients, which guarantees the system asymptotic stability, are considered by B. T. Polyak and Y. Z. Tsypkin (Polyak & Tsypkin, 1990). The assured domain of the interval polynomial roots location is found in (Soh et al., 1985). The root locus theory is used in (Gaivoronsky, 2006) for this task solution. Conditions (Vicino, 1989; Shaw & Jayasuriya, 1993) for the interval polynomial roots getting into the given domain of some convex shape are defined.

The parametric approach to robustness, based on the root locus theory (Rimsky, 1972; Rimsky & Taborovetz, 1978; Nesenchuk, 2002; Nesenchuk, 2005), is considered in this chapter in application to the industrial anthropomorphous robot control system parametric design. The developed techniques allow to set up the values of the parameter variation intervals limits for the cases when the stability verification showed, that the given system was unstable, and to ensure the system robust quality by locating the characteristic equations family roots within the given quality domain.

## 2. Industrial robot and its control system description

Most industrial robots are used for transportation of various items (parts), e. g. for installing parts and machine tools in the cutting machines adjustments, for moving parts and units, etc. During the robot operation due to some internal or external reasons its parameters vary, causing variation of the system characteristic equation coefficients. This variation can be rather substantial. In such conditions the system is considered, as the uncertain system.

### 2.1 General description of the anthropomorphous industrial robot

The industrial robot considered here is used for operation as an integrated part of the flexible industrial modules including those for stamping, mechanical as-

sembly, welding, machine cutting, casting production, etc. The industrial robot is shown in fig. 1. It comprises manipulator 1 of anthropomorphous structure, control block 2 including periphery equipment and connecting cables 3. Manipulator has six units (1–8 in fig. 1) and correspondingly is of six degrees of freedom (see fig. 1): column 4 turn, shoulder 5 swing, arm 6 swing, hand 7 swing, turn and rotation. The arm is connected with the joining element 8.
Controlling robots of such a type, belonging to the third generation, is based on the hierarchical principle and features the distributed data processing. It is based on application of special control processors for autonomous control by every degree of freedom (lower executive control level) and central processor coordinating their operation (higher tactical control level).

### 2.2 Industrial robot manipulator unit control system, its structure and mathematical model

Executive control of every manipulator unit is usually executed in coordinates of this unit (Nof, 1989) and is of the positional type. It is the closed-loop servo-control system not depending on the other control levels. Although real unit control is executed by a digital device (microprocessor, controller) in a discrete way, the effect of digitization is usually neglected, as the digitization frequency is high enough to consider the unit and the controller as the analog (continuous) systems. As for the structure, the unit control loops are almost similar and differ only in the parameter values. Therefore, any unit of the industrial robot can be considered for investigating the dynamic properties.

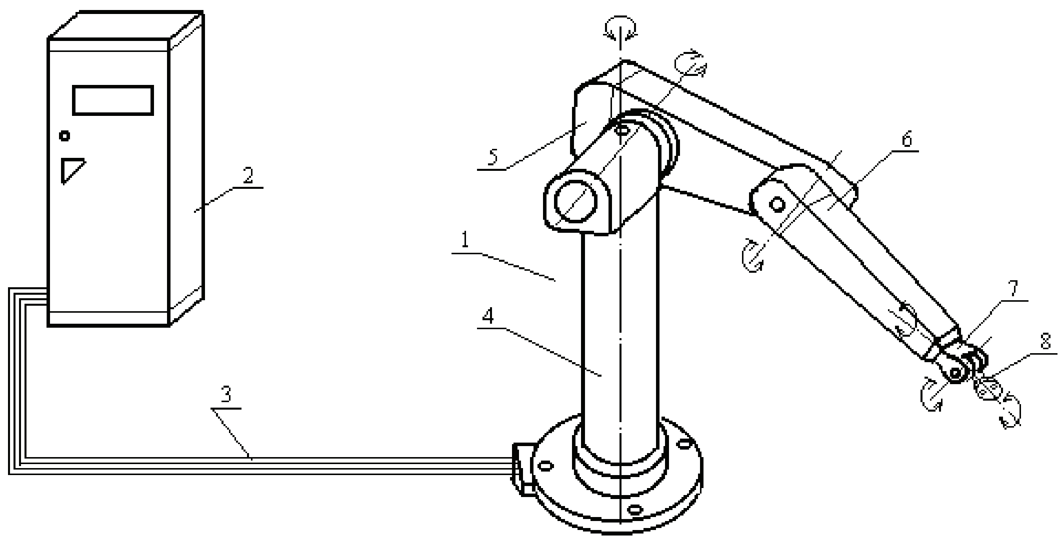


Figure 1. Anthropomorphous industrial robot

The structure of the manipulator unit subordinate control is shown in fig. 2. The simplified version of the structure is presented in fig 3.
In fig. 2 the plant is represented by elements 1–4 (a DC motor); 5 is the sensor transforming the analog speed signal into the speed code (photo-pulse sensor), 6 is the element combining the speed PI regulator, code-pulse width transformer and capacity amplifier, 7 is the transformer of analog position signal into the position code (photo-pulse sensor), 8 is the proportional regulator of the manipulator shoulder position, 9 is the transfer mechanism (reducer). In fig. 3 the transfer function

$$W_p^{'}(s) = W_p(s)s$$

where $W_p(s)$ is the plant transfer function.

Substitute corresponding parameters and express the plant transfer function as follows:

$$W_p(s) = \frac{\varphi}{U_g} = \frac{1}{(j_m + j_l)\frac{L_A}{C_M}s^3 + (j_m + j_l)\frac{R_A}{C_M}s^2 + C_e s}, \quad (1)$$

where $U_g$ is the input voltage, φ is the object shaft angle of rotation.

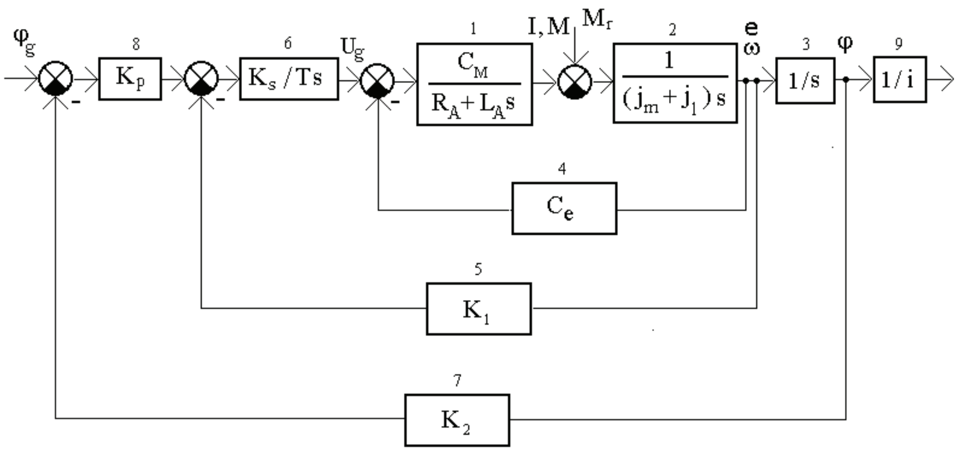


Figure 2. Control system for the industrial robot manipulator shoulder unit

On the basis of (1) write the manipulator unit control system characteristic equation

$$s^4 + \frac{R_A}{L_A} s^3 + \frac{C_e C_M}{j_m L_A} s^2 + \frac{C_M K_1 K_s}{j_m L_A T} s + \frac{C_M K_2 K_p K_s}{j_m L_A T} = 0$$

or as

$$a_0 s^4 + a_1 s^3 + a_2 s^2 + a_3 s + a_4 = 0\,, \tag{2}$$

where

$$a_0 = 1;\ a_1 = {R_A}/{L_A};\ a_2 = \frac{C_e C_M}{(j_m + j_{l'}) L_A};\ a_3 = \frac{C_M K_1 K_s}{(j_m + j_l) L_A T};\ a_4 = \frac{C_M K_2 K_p K_s}{(j_m + j_l) L_A T};$$

- $R_A$ is the motor anchor resistance;
- $L_A$ is the anchor inductance;
- $j_l$ is the load inertia moment;
- $j_m$ is the anchor inertia moment;
- $C_e$ is the electric-mechanical ratio of the motor;
- $C_M$ is the constructive constant of the motor;
- $T$ is the time constant of the PI regulator;
- $K_1$ and $K_2$ are photo-electric sensor coefficients;
- $K_s$ and $K_p$ are gains of regulators by speed and position correspondingly.

Suppose the robot unit has the following nominal parameters:

- $R_A$ = 0,63 Ω;
- $L_A$ = 0,0014 henry;
- $j_l$ = $2{,}04 \cdot 10^{-5}\, kg/m^2$
- $j_m$ = $40{,}8 \cdot 10^{-5}\, kg/m^2$;
- $C_e$ = $0{,}16 \dfrac{V \cdot s}{rad}$;
- $C_M = C_e$;
- $T$= 0,23 $s$;
- $K_1$ = 66,7, $K_2$ = 250;
- $K_s = 0{,}078$, $K_p = 2{,}5$.

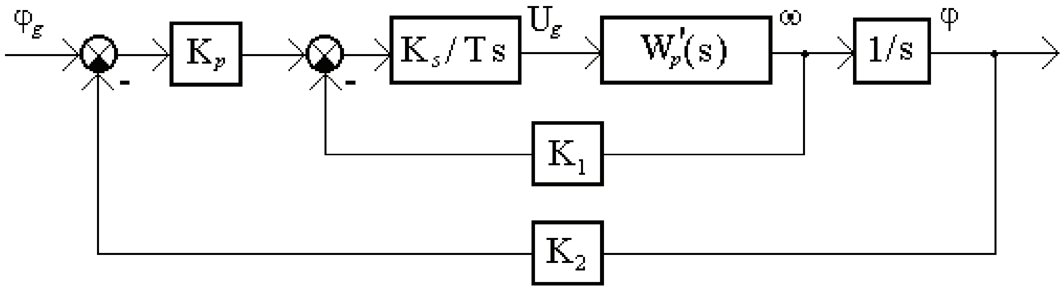


Figure 3. Structure of the position control system loop for the manipulator shoulder unit

After substitution of the nominal values into (2) rewrite the unit characteristic equation as

$$s^4 + 0{,}5 \cdot 10^3 s^3 + 0{,}427 \cdot 10^5 s^2 + 0{,}6 \cdot 10^7 s + 0{,}56 \cdot 10^8 = 0 \quad (3)$$

The coefficients of (3) are the nominal ones and while robot operation they often vary within the enough wide intervals. For this reason when calculating the robot control system it is necessary to consider the parameters uncertainty and ensure the control system robustness.

## 3. The techniques for robust stability of systems with parametric uncertainty

The method is described for synthesis of the interval dynamic system (IDS) stable characteristic polynomials family from the given unstable one, based on the system model in the form of the free root locus portrait. This method allows to set up the given interval polynomial for ensuring its stability in cases, when it was found, that this polynomial was unstable. The distance, measured along the root locus portrait trajectories, is defined as the setting up criterion, in particular, the new polynomial can be selected as the nearest to the given one with consideration of the system quality requirements. The synthesis is carried on by calculating new boundaries of the polynomial constant term variation interval (stability interval), that allows to ensure stability without the system root locus portrait configuration modification

### 3.1 The task description

While investigating uncertain control systems for getting more complete representation of the processes, which occur in them, it seems substantial to discover correlation between algebraic, frequency and root locus methods of investigation. Such correlation exists and can be applied for finding dependence between the system characteristic equation coefficients values (parameters) and its dynamic properties to determine how and what coefficients should be changed for ensuring stability. One of the ways for establishing the above mentioned correlation can be investigation of the systems root locus portraits and Kharitonov's polynomials root loci (Kharitonov, 1978).

Consider the IDS, described by the family of characteristic polynomials

$$P(s) = \sum_{j=0}^{n} a_j s^{n-j} = 0, \tag{4}$$

where $a_j \in [\underline{a}_j, \overline{a}_j]$, $\underline{a}_0 > 0$, $j = 0, \ldots, n$; $\underline{a}_j$ and $\overline{a}_j$ are correspondingly the lower and upper boundaries of the closed interval of uncertainty, $[\underline{a}_j, \overline{a}_j]$; $s = \sigma + i\omega$. The coefficients of polynomial (4) are in fact the uncertain parameters.

The task consists in synthesis of the stable interval family of polynomials (4) on the basis of the initial (given) unstable one, i. e., when the initial system stability verification by application of Kharitonov's polynomials gave the negative result. Calculation of new parameter variation intervals boundaries is made on the base of the initial boundaries in correspondence with the required dynamic characteristics of the system. The new boundaries values definition criteria can be different, in particular they can be selected the nearest to the given ones. In this case the distance, measured along the system roots trajectories, is accepted to be the criterion of such proximity.

### 3.2 The interval system model in the form of the root locus portrait

Introduce the series of definitions.

*Definition 1.* Name the root locus of the dynamic system characteristic equation (polynomial), *as the dynamic system root locus*.

*Definition 2.* Name the family (the set) of the interval dynamic system root loci, *as the root locus portrait of the interval dynamic system*.

*Definition 3.* The algebraic equation coefficient or the parameter of the dynamic system, described by this equation, being varied in a definite way for generating the root locus, when it is assumed, that all the rest coefficients (parameters) are constant, name as the algebraic equation root locus free parameter or simply the root locus parameter.

*Definition 4.* The root locus, which parameter is the coefficient $a_k$, name as the algebraic equation *root locus relative to the coefficient* $a_k$.

*Definition 5.* The root locus relative to the dynamic system characteristic equation constant term name as the *free root locus of the dynamic system*.

*Definition 6.* The points, where the root locus branches begin and the root locus parameter is equal to zero, name as the *root locus initial points.*

*Remark 1.* One of the free root locus initial points is always located at the origin of the roots complex plane.
The above remark correctness follows from the form of equation (4).

*Remark 2.* The free root locus positive real branch portion, adjacent to the initial point, located at the origin, is directed along the negative real half-branch $\sigma$ of the complex plane to the left half-plane.

*Remark 2* is correct due to the root loci properties (Uderman, 1972) and because real roots of equations with positive coefficients are always negative (see fig. 4).

The peculiarity of the free root loci, which distinguishes them from another types of root loci, consists in the fact, that all their branches strive to infinity, approaching to the corresponding asymptotes.
For carrying on investigation apply the Teodorchik – Evans free root loci (TEFRL) (Rimsky, 1972), i. e. the term "root locus" within this section will mean the TEFRL, which parameter is the system characteristic equation constant term.
To generate the IDS root locus portrait apply the family of the mapping functions

$$s^n + a_1 s^{n-1} + a_2 s^{n-2} + \ldots + a_{n-2} s^2 + a_{n-1} s = u(\sigma,\omega) + iv(\sigma,\omega) = -a_n, \qquad (5)$$

where $u(\sigma,\omega)$ and $v(\sigma,\omega)$ are harmonic functions of two independent variables $\sigma$ and $\omega$; $a_n$ is the root locus parameter; $s = \sigma + i\omega$. Analytical and graphical root loci are formed using mapping function (5). The root locus equation is as follows:

$$iv(\sigma,\omega) = 0 \qquad (6)$$

and the parameter equation (Rimsky, 1972) as follows:

$$u(\sigma,\omega) = -a_n. \qquad (7)$$

The fragmentary root locus portrait for the IDS of the forth order, which is made up of four Kharitonov's polynomials free root loci, is shown in fig. 4. The Kharitonov's polynomials $h_1$, $h_2$, $h_3$ and $h_4$ in this figure are represented by points (roots), marked with circles, triangles, squares and painted over squares correspondingly. There are the following designations: $\sigma_{h_i}$, $i$ = 1, 2, 3, 4, – the cross centers of asymptotes for the root loci of every polynomial $h_i$, $t_l$, $l$ = 1, 2, 3, – cross points of the root loci branches with the system asymptotic stability boundary, axis $i\omega$. The root loci initial points, which represent zeroes of mapping function (5), are depicted by X-s. Because in fig. 4 all roots of the Kharitonov's polynomials are completely located in the left half-plane, the given interval system is asymptotically stable (Kharitonov, 1978).

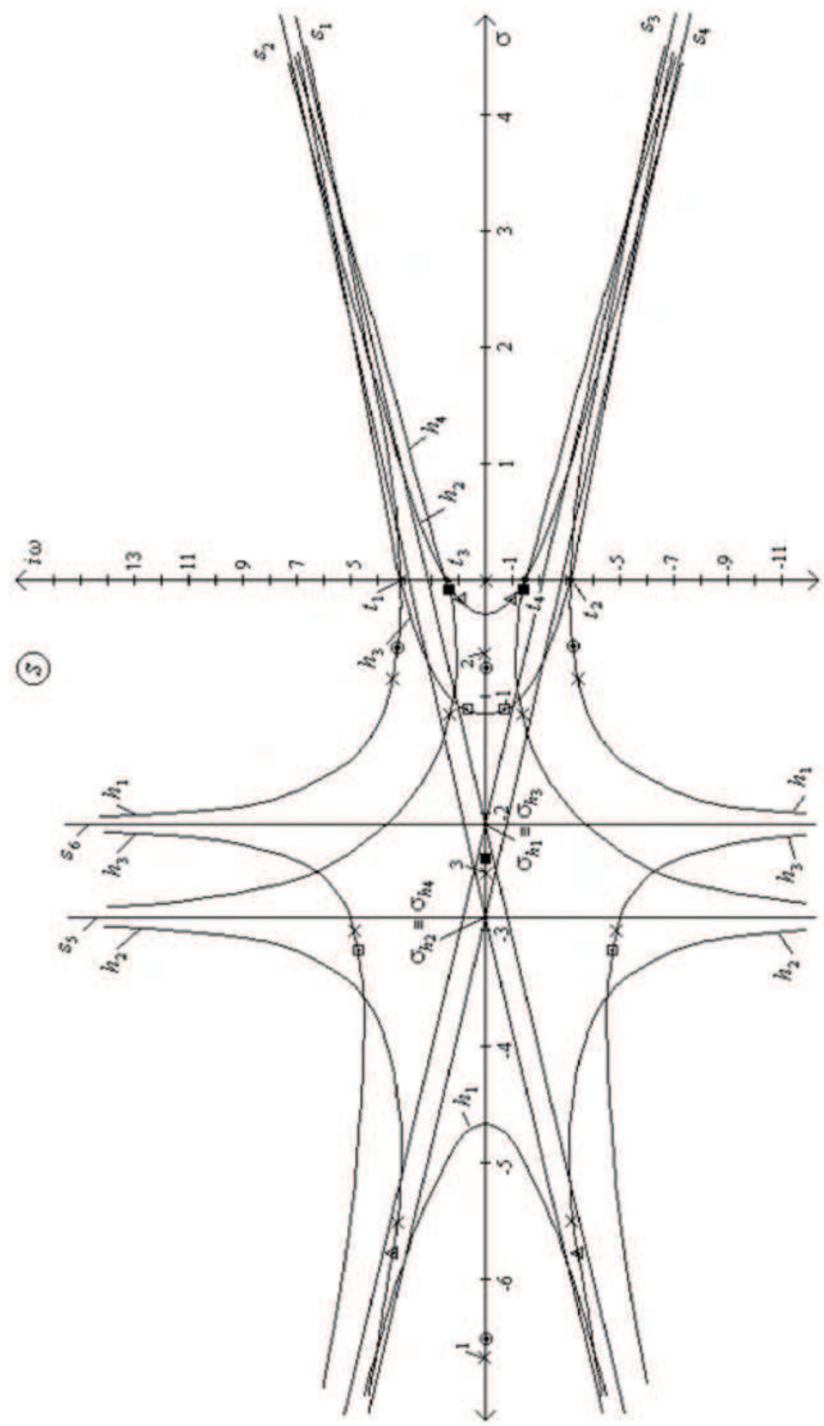

Figure 4. Root loci of the Kharitonov's polynomials for the system of class [4;0]

### 3.3 Investigation of the characteristic polynomial family root loci branches behavior at the asymptotic stability boundary of the system

The branches of the IDS root locus portrait, when crossing the stability boundary, generate on it the region (set) of cross points. Name this region, as *the cross region* and designate it as $R_{\omega}$. According to the theory of the complex variable (Lavrentyev & Shabat, 1987) and due to the complex mapping function (5) continuity property, this region is the many-sheeted one and is composed of the separate sheets with every sheet (continuous subregion), formed by the separate branch while it moves in the complex plane following the parameters variation. The cross region portion, generated by only positive branches of the system root locus portrait, name as *the positive cross region* and designate it as $R_{\omega}^{+}$.

$$R_{\omega}^{+} \subset R_{\omega}. \tag{8}$$

Define also the subregion $r_{\omega}^{+}$ (either continuous or discrete one) of the cross region $R_{\omega}^{+}$ (8) generated by the root loci branches of any arbitrary *subfamily f* of the interval system polynomials family (4), and name it as *the* (*positive*) *cross subregion*, thus,

$$r_{\omega}^{+} \subset R_{\omega}^{+}. \tag{9}$$

Introduce the following sets:

$$W_r^{+} = \left\{\omega_{r\,i}^{+}\right\} \tag{10}$$

$$A_r^{+} = \left\{a_{r\,i}^{+}\right\} \tag{11}$$

where $W_r^{+}$ is the set (family) of the cross subregion $r_{\omega}^{+}$ (9) points coordinates $\omega_{r\,i}^{+}$; $A_r^{+}$ is the set (family) of values $a_{r\,i}^{+}$ of the root locus parameter $a_n$ at the set $W_r^{+}$ points.

Define the minimal positive value $a_{r\,\min}^{+}$ of the root locus parameter within the cross subregion $r_{\omega}^{+}$:

$$a_{r\,\min}^{+} = \inf A_r^{+}. \tag{12}$$

Peculiarities of the IDS root loci initial points location make it possible to draw a conclusion about existence of its characteristic equation coefficients variation intervals, ensuring asymptotic stability of the given system.

**Statement**. If the initial points of the IDS characteristic polynomials arbitrary subfamily *f* free root loci, excluding points always situated at the origin, are located in the left complex half-plane *s,* there exists the interval *d* of the root loci parameter $a_n$ values, ensuring asymptotic stability of the subfamily *f.*

$$d = (0, a^{+}_{r\,\min}), \tag{13}$$

*Proof.* The subfamily *f* free root loci generate at the system stability boundary the cross subregion $r_\omega{}^+$ (9) of cross points, which is formed by the set (10) of the cross points coordinates and corresponding set (11) of the parameters values. If the initial points are located, as it is defined by the statement, on every *i*-th branch of every polynomial root loci there exist an interval $r_i = (\sigma_{l_i}, 0)$ of roots values (starting from the branch initial point with coordinate $\sigma_{l_i}$ until the point, where it crosses the stability boundary, axis $i\omega$ of the complex plane), which is completely located in the left half-plane. Therefore, there exists also the appropriate maximum possible common interval $d_m$ (which is common for all the branches) of the root loci parameter $a_n$ values (beginning from zero up to the definite maximum possible value $a_n = a^{+}_{r\,m}$), corresponding to the values of roots within some interval $r_k = (\sigma_{l_k}, 0)$, which ensures the system stability. Name this interval $d_m$ the *dominating interval* and define it as $d_m = (0, a^{+}_{r\,m})$. Designate the roots $\sigma_i$ coordinates values interval, located on every positive *i*-th branch of the family and corresponding to the dominating interval, as $r_d = (\sigma_{l_i}, \sigma_{r_i})$. It is evident, that $a^{+}_{r\,m}$ will be maximum possible at the stability boundary, i. e. at $\sigma_{r_i} = 0$. Then, $\forall \sigma_{r_i} [a^{+}_{r\,m} = a^{+}_{r\,\min} \rightarrow \sigma_{r_i} \leq 0]$, i. e. the dominating one is the interval $d_m = (0, a^{+}_{r\,\min})$, which represents itself the interval *d* (13). Hence, the statement is correct.

*Definition 7.* The interval of polynomial (4) root loci parameter values name *the polynomial stability interval by this parameter* or simply *the polynomial stability interval*, if the polynomial asymptotic stability property holds within this interval.

In case, if some initial points are located at the stability boundary (excluding the point, which is always located at the origin), and on the assumption, that all the rest points are located in the left half-plane, the additional analysis is required for finding the stability interval existence. For this purpose it is necessary to define the root loci branches direction at their outcome from the initial points, located at the stability boundary, i. e. just to determine what half-plane they are directed to: left one or right one. Obviously, such stability interval exists in the following cases:

a) all the root loci branches with initial points, located at the stability boundary, are directed from these points to the left half-plane;
b) all positive root loci branches with initial points, located at the stability boundary, are directed from these points to the left half-plane.

To determine the above indicated branches direction at the initial points, it is enough to define the root locus sensitivity vector (Nesenchuk, 2005) direction at them.

As a result of the IDS root locus portraits analysis several general regularities have been discovered, being inherent in Kharitonov's polynomials free root loci: paired convergence of the root loci branches at the complex plane imaginary axis (points $t_1$, $t_2$, $t_3$, $t_4$ in fig. 4); paired convergence of the corresponding asymptotes at the real axis of the complex plane (points $\sigma_{h1}$, $\sigma_{h2}$, $\sigma_{h3}$, $\sigma_{h4}$ in fig. 4); the tendency for the system robust properties variation while varying its characteristic polynomial coefficients values. It gives the possibility to fix the fact of existence of the system characteristic equation coefficients variation intervals, ensuring its robust stability and also to determine how the coefficients values should be changed for the system dynamic characteristics correction, if it is unstable.

The IDS root locus portraits investigation, which has been carried out, confirms that they can be successfully applied for the in-depth studying robust properties of these systems.

### 3.4 Parametric synthesis of stable uncertain systems

The conditions for existence of the polynomials (4) family coefficients stability intervals were formulated in the previous section. Here we define what these intervals values should be. For this purpose consider the polynomials (4) subfamily $f$, consisting of the system Kharitonov's polynomials, and develop the procedure for synthesis of the stable Kharitonov's polynomials on the base of the unstable ones, which depends on the root loci initial points location in relation to the asymptotic stability boundary. For the synthesis procedure development apply the Kharitonov's polynomials free root loci. Consider the case, when initial points are located in the left half-plane. In this case the algorithm of synthesis can be divided into the following stages.

***Stage 1.*** Obtaining the Teodorchik – Evans free root loci equation (6) for each one of the IDS four Kharitonov's polynomials.
As the Kharitonov's polynomials represent the subfamily of the IDS polynomials family, they generate the above described cross subregion $r_\omega^+$ (9) on the stability boundary, which is formed by the set (10) of the cross points coordinates.

***Stage 2.*** Calculating coordinates $\omega_{r\,i}^+$ of the set (10) by solution of the TEFRL equations, obtained in stage 1, relative to ω in condition, that σ = 0. In this way the set $W_r^+$ (10) is formed.
For every obtained value of $\omega_{r\,i}^+$ from $W_r^+$ the corresponding value of the variable coefficient $a_n$ is calculated by formula (7), thus, forming the set $A_r^+$ (11).

***Stage 3.*** Definition of the stability interval by the coefficient $a_n$.
For this purpose, using (12), define the minimal one, $a_{r\,\min}^+$, of the parameter values at points of the set $A_r^+$. Thus obtain the interval $d$ (13) of the parameter $a_n$ variation, which ensures stability of the Kharitonov's polynomials and, therefore, the system in whole.

Before describing the next stage of synthesis formulate the following theorem.

**Theorem**. For robust stability of the polynomial family (4) it is necessary and enough to ensure the upper limit of the constant term $a_n$ variation interval to satisfy the inequality

$$\overline{a}_n < a_{r\,\min}^+, \tag{14}$$

if the family is stable at $a_n = 0$.

*Proof*. Let the coefficient $a_n$ to be the polynomial (4) root locus parameter. Under the theorem condition family of (4) is stable at $a_n = 0$, i.e. the root loci initial points are located in the left half-plane. Therefore, in view of statement 1 the theorem is valid.

***Stage 4.*** Comparing the obtained stability interval (13) with the given interval $a_n \in [\underline{a}_n, \overline{a}_n]$ of the parameter $a_n$ variation in correspondence with inequality (14).

In case, if condition (14) is not satisfied, the upper limit $\overline{a}_n$ of the parameter variation interval is set up in correspondence with this inequality.
When the power n of the polynomial is less or equal than 3, n ≤ 3, the above given theorem is applied without any conditions, i. e. it is not required to sat-

isfy condition of the Kharitonov's polynomials roots real parts negativity at $a_n = 0$, because in this case the coefficients positivity always guarantees negativity of the roots real parts.

The above described algorithm allows to carry on the parametric synthesis of the stable interval system without modification of its root locus portrait configuration, by simple procedure of setting up the characteristic polynomial constant term variation interval limits.

The *numerical example* , demonstrating the results obtained, is given below

Consider the interval system, described by the initial characteristic polynomial

$$s^4 + 10s^3 + 35s^2 + 50s + 24 = 0, \quad (15)$$

where the real coefficients are: $a_0 = 1$; $8{,}4 \le a_1 \le 11{,}6$; $24 \le a_2 \le 48$; $26{,}5 \le a_3 \le 83{,}1$; $8{,}99 \le a_4 \le 50{,}3$.

Let the coefficient $a_4$ to be the root locus parameter. Then, define the mapping function:

$$\begin{aligned} -a_4 &= a_0\sigma^4 + 4a_0\sigma^3 i\omega - 6a_0\sigma^2\omega^2 - 4a_0\sigma i\omega^3 + a_0\omega^4 + a_1\sigma^3 + 3a_1\sigma^2 i\omega - \\ &\quad - 3a_1\delta\sigma^2 - a_1 i\omega^3 + a_2\sigma^2 + 2a_2\delta i\sigma - a_2\omega^2 + a_3\sigma + a_3 i\omega. \end{aligned}$$

Write correspondingly the TEFRL and the parameter equations::

$$\begin{aligned} &\omega(4a_0\sigma^3 - 4a_0\sigma\omega^2 + 3a_1\sigma^2 - a_1\omega^2 + 2a_2\sigma + a_3) = 0; \\ &a_0\sigma^4 - 6a_0\sigma^2\omega^2 + a_0\omega^4 + a_1\sigma^3 - 3a_1\sigma\omega^2 + a_2\sigma^2 - a_2\sigma = -a_4. \end{aligned}$$

Define the Kharitonov's polynomials for the interval system with the initial characteristic polynomial (15):

$$\begin{aligned} h_1(s) &= s^4 + 8{,}4s^3 + 24s^2 + 83{,}1s + 50{,}3; \\ h_1(s) &= s^4 + 11{,}6s^3 + 48s^2 + 26{,}5s + 8{,}99; \\ h_1(s) &= s^4 + 8{,}4s^3 + 48s^2 + 83{,}1s + 8{,}99; \\ h_1(s) &= s^4 + 11{,}6s^3 + 24s^2 + 26{,}5s + 50{,}3. \end{aligned}$$

The root loci of these polynomials are represented in fig. 4, described above. Number of asymptotes $n_a$ (in fig. 4 they are indicated as $s_1$, $s_2$, …, $s_6$) is constant for every one of Kharitonov's polynomials and is equal to

$$n_a = n - m = 4 - 0 = 4,$$

where $m$ is the number of poles for function (5).

The centers of asymptotes are located on the axis σ and have coordinates: $\sigma_{h1}$ = 2,10; $\sigma_{h2}$ = 2,90; $\sigma_{h3}$ = 2,10; $\sigma_{h4}$ = 2,90 (see fig. 4). The asymptotes centers coordinates coincide in pairs: for the pair $h_1(s)$ and $h_3(s)$, and also for the pair $h_2(s)$ and $h_4(s)$.
The inclination angles of asymptotes for the given root loci are correspondingly the following:

$$\varphi_1 = 0^0; \quad \varphi_3 = 135^0;$$
$$\varphi_2 = 45^0; \quad \varphi_4 = 180^0.$$

According to fig. 4, every pair of the root loci strives to the same asymptotes, i.e. the pairs are formed by those root loci, which asymptotes centers coincide, as it was indicated above.
For definition of equation (15) coefficients intervals, ensuring the system stability, stability condition (14) is applied. Thus, the following values $a^+_{r\,i}$ of the set $A_r^+$ have been defined:

$a^+_{r\,1}$ = 139,67 for the polynomial $h_1$;
$a^+_{r\,2}$ = 116,33 for the polynomial $h_2$;
$a^+_{r\,3}$ = 377,75 for the polynomial $h_3$;
$a^+_{r\,4}$ = 54,89 for the polynomial $h_4$.

The minimal value is

$$a^+_{r\,\min} = a^+_{r\,4} = 54{,}89.$$

Because $\bar{a}_4$ < 54,89, in correspondence with (14) the given interval system is asymptotically stable.

## 4. The method for ensuring uncertain systems quality

In this section the task is solved for locating the uncertain system roots within the trapezoidal domain. The method allows to locate roots of the uncertain system characteristic equations family within the given quality domain, thus ensuring the required system quality (generalized stability). The task is solved by inscribing the system circular root locus field into the given quality domain. The trapezoidal domain, bounded by the arbitrary algebraic curve, is considered. Peculiarity of the method consists in the root locus fields application.

The systems with parametric uncertainty are considered, described by the family of characteristic polynomials

$$p(s) = s^n + a_1 s^{n-1} + \ldots + a_{n-1}s + a_n \qquad (16)$$

where $a_1$, ..., $a_n$ are coefficients, which depend linearly of some uncertain parameter $k$, and can be either real or complex ones.
For selection of the uncertain parameter $k$, transform equation (16) and rewrite it in the following form:

$$\phi(s) + k\psi(s) = 0 \qquad (17)$$

where $\phi(s)$ and $\psi(s)$ are some polynomials of the complex variable $s$; $k$ is the system uncertain parameter.

Based on (17), derive the expression for k in the form

$$k = f(s) = -\frac{\varphi(s)}{\psi(s)} = u(\sigma,\omega) + iv(\sigma,\omega) \qquad (18)$$

where $u(\sigma,\omega)$, $v(\sigma,\omega)$ are harmonic functions of two independent real variables $\sigma$ and $\omega$.

Consider some provisions about the root locus fields.

*Definition 8.* *The root locus field of the control system* is the field with the complex potential

$$\varphi(s) = u(\sigma,\omega) + i\nu(\sigma,\omega),$$

that is defined in every point of the extended free parameter complex plane by setting the *root locus image* existence over the whole plane (Rimsky & Taborovetz, 1978).
Then, set the root locus image by the real function $h = h(u,\nu,t)$, where $t$ is the constant value for every image. Name $t$, as *the image parameter*. Suppose the image is defined over the whole free parameter plane by setting the corresponding boundaries of the parameter $t$. Thus, using mapping function (18), define in the general form the scalar root locus field function

$$f^* = f^*(\sigma,\omega) \qquad (19)$$