Sequential Design of Optimum Sized and Geometric Tolerances

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1. Introduction

Tolerancing has great impact on the cost and quality of a product. Dimensional and geometric tolerancing are designed to ensure that products meet both designed functionality and minimum cost. The task of dimensioning and tolerancing in process planning stage is to determine the working dimensions and tolerances of the machined parts by given blueprint (B/P) specifications.

A lot of research work has been carried out in dimensioning and tolerancing. In earlier studies, optimal solutions to tolerance charts have been developed to meet B/P specifications. Most researches concentrated on dimensioning and tolerancing with optimal objectives to maximize the total working tolerances based on the constraints of tolerance accumulation and machining accuracy. Linear or nonlinear programming models have been applied to obtain the optimal tolerances (Ngoi, 1992; Ngoi & Ong, 1993; Ji, 1993a; Ji, 1993b; Wei & Lee, 1995; Lee & Wei, 1998; Ngoi & Cheong, 1998a; Lee et al., 1999; Chang et al., 2000; Huang et al., 2002; Chen et al., 2003; Gao & Huang, 2003; Huang et al., 2005). Optimal methods have also been presented to allocate B/P tolerances in product design using tolerance chart in process planning (Ngoi & Cheong, 1998; Ngoi & Ong, 1999; Swift et al., 1999) but the generation of dimensional and tolerance chains being one of the most important problems. In one-dimensional (1D) cases, the apparent path tracing and tree approach were commonly used to tolerance chart for manual treatment (Ngoi & Ong, 1993; Ji, 1993a; Ji, 1993b; Wang & Ozsoy, 1993; Ngoi & Cheong, 1998b). Automatic generation of dimensional chains in assembly based on the data structure has been presented (Treacy et al., 1991; Wang & Ozsoy, 1993). Using an Expert System, assembly tolerances analysis and allocation have been implemented by appropriate algorithm in CAD system (Ramani et al., 1998). An intelligent dimensioning method for mechanical parts based on feature extraction was also introduced (Chen et al., 2001). This method could generate the dimensions of

mechanical parts for two-dimensional (2D) drawing from three-dimensional (3D) models. Recently, more valuable and attractive approaches to deal with dimensional and geometric tolerances have been developed (He & Gibson, 1992; Ngoi & Tan, 1995; Ngoi & Seow, 1996). He and Gibbon in 1992 made a significant development in geometric tolerance charting and they presented useful concepts to treat geometric dimensions and tolerances simultaneously. A computerized trace method has been extended to determine the relation-ships between geometrical tolerances and related manufacturing dimensions and tolerances. A new method for treating geometrical tolerances in tolerance chart has been presented (Ngoi & Tan, 1995; Ngoi & Seow, 1996; Tseng & Kung, 1999). The method identified the geometrics that exhibited characteris-tics similar to linear dimensions. These geometrics were first treated as equiva-lent dimensions and tolerances and then applied to tolerance chart directly. Tolerance zones have been utilized to analyze tolerance accumulation includ-ing geometric tolerances. The formulae for bonus and shift tolerances due to position callout have been presented (Ngoi, et al., 1999; Ngoi et al., 2000). In complex 2D cases when both angular and geometric tolerances are concerned, graphic method has been used to implement tolerances allocation (Huang et al., 2002; Zhao, 1987). In conventional tolerancing, fixed working dimensions and tolerances were designed in process planning phrase. Though this method was suitable for mass production in automatic lines, it had limitations to pro-duce low-volume and high-value-added parts such as those found in aircraft, nuclear, or precision instrument manufacturing industry (Fraticelli et al., 1997). To increase the acceptable rate of a machined part, a method named sequential tolerance control (STC) for design and manufacturing has been presented (Fraticelli et al., 1997; Fraticelli et al., 1999; Wheeler et al., 1999; Cavalier & Le-htihet, 2000; Mcgarvey et al., 2001). This method essentially used real-time measurement information of the complete operations to dynamically re-calculate the working dimensions and feasible tolerances for remaining opera-tions. Using acquired measurement information, tool-wear effect compensa-tion under STC has been realized (Fraticelli et al., 1999). An implicit enumera-tion approach to select an optimum subset of technological processes to execute a process planning under STC strategy has been presented (Wheeler et al., 1999). When measurements and working dimension adjustments would be taken to facilitate machining process and reduce manufacturing cost has also been investigated (Mcgarvey et al., 2001).

In spite of the achievement mentioned above, some issues still need further re-search. The previous researches focused on 1D dimensioning and tolerancing.

Though simple 2D drawings were concerned, they could be converted into 1D dimensioning and tolerancing in two different directions, i.e. in axial and diametrical directions or in axis OX and OY directions (He & Gibson, 1992; Ngoi & Tan, 1995; Ngoi & Seow, 1996; Tseng & Kung, 1999). When incline features of 3D parts are machined, complicated dimensioning and tolerancing will occur since angular tolerance will be included in tolerance chains. In addition, the relationships between orientational and angular tolerances need further investigation. Though STC strategy is able to enhance the working tolerances and acceptance rate of manufactured parts (Fraticelli et al., 1997; Cavalier & Lehtihet, 2000), how to extend this method to complex 3D manufacturing is still a new problem when sized, angular, and orientational tolerances are included simultaneously.

Based on the basic principle of STC introduced by Fraticelli et al (Fraticelli et al., 1997), the purpose of this paper is to extend the new methodology to deal with 2D sized, angular, and orientational tolerances of 3D parts. The proposed approach essentially utilizes STC strategies to dynamically recalculate the working dimensions and tolerances for remaining operations. This approach ensures that the working tolerances of a processed part are optimal while satisfying all the functional requirements and constraints of process capabilities. A special relevant graphic (SRG) and vector equation are utilized to formulate the dimensional chains. Tolerance zones are used to express the composite tolerance chains that include sized and angular tolerances to perform tolerances design. With orientational tolerances converted into equivalent sized or angular tolerances, the composite tolerance chains are formulated. Sequential optimal models are presented to obtain optimal working dimensions and tolerances for remaining operations. The working tolerances are amplified gradually and manufacturing capabilities are enhanced.

This paper is structured as follows. A new method for presenting the dimensional chains from given process planning is discussed in section 2. In section 3, a method for presenting the composite tolerance chains is discussed. In section 4, the optimal mathematical models for sequential tolerances design of 3D processed tolerances are discussed. Section 5 gives a practical example. Finally, section 6 concludes this study.

2. Automatic generation of process tolerance chains with SRG

When a n-operation part is processed by m machine tools in a particular direction, such as axial direction, the apparent path tracing or tree approach methods are usually used to generate the dimensional and tolerance chains for manual treatment (Ngoi & Ong, 1993; Ji, 1993a; Ji, 1993b; Ngoi & Cheong, 1998). If geometric tolerances are involved, only four out of total fourteen geometric tolerance specifications, which exhibit the characteristics similar to linear dimensions, are treated as equivalent dimensions and tolerances and then applied directly to tolerance chart. These four specifications are position, symmetry, concentricity, and profile of a line (surface) (He & Gibson, 1992; Ngoi & Tan, 1995; Ngoi & Seow, 1996; Tseng & Kung, 1999). In 1D case, the following dimensional and tolerance chains must be satisfied (Ji, 1993b):

$$
\begin{aligned}
[A]\{X\} &= \{C\} \\
[B]\{T_X\} &\leq \{T_D\}
\end{aligned}
\tag{1}
$$

Where $A = [a_{ij}]$ is a $m \times n$ coefficient matrix, $a_{ij} = 1$ and -1 for an increasing and decreasing constituent link of u_{di}, respectively. $a_{ij} = 0$ for otherwise. $X = [u_1, u_2, \ldots, u_n]^T$ is a $n \times 1$ vector of the mean working dimensions. $C = [u_{d1}, u_{d2}, \ldots, u_{dm}]^T$ is a $m \times 1$ vector of mean values of B/P dimensions. $B = [b_{ij}]$ is a $m \times n$ coefficient matrix. $b_{ij} = 1$ for an increasing and decreasing constituent link of u_{di}. $b_{ij} = 0$ for otherwise. $T_X = [T_{u1}, T_{u2}, \ldots, T_{un}]^T$ is a $n \times 1$ vector of the working tolerances. $T_D = [T_{d1}, T_{d2}, \ldots, T_{dm}]^T$ is a $m \times 1$ vector of B/P tolerances.

When a complex part is machined, typically a number of operations are involved. Each B/P tolerance is usually expressed as a number of pertinent process tolerances. In previous researches, tremendous efforts have been contributed to 1D dimensional tolerances. Geometric tolerances as well as the interactions between them have not been investigated extensively when complex 3D parts are manufactured. When we machine a complex 3D part, two dimensions components are included to determine the position of a processed feature in 2D drawing in the given view plane. For example, for the part shown in Figure 1 (Zhao, 1987), the position of pin-hole $\Phi15.009\pm0.009$ in the plane XOY is determined by coordinate dimensions and tolerances $-25\pm\frac{1}{2}T_{N'x}$ and $28\pm\frac{1}{2}T_{N'y}$. Similarly the position of incline plane B is determined by $L_{N'E}$ $\pm\frac{1}{2}T_{N'E}$ and $60°\pm\frac{1}{2}T_{\alpha1}$, where $L_{N'E}$ and $T_{N'E}$ be nominal distance and its tolerance from the axis of pin-hole to incline plane, respectively. $\alpha=60°$ and $T_{\alpha1}$ be nomi-

nal angle and its tolerance formed by axis OX and the normal line of incline plane, respectively.

The series of orderly processing operations of a part is generalized as the set A_p = $\{O_{p1}, O_{p2}, \ldots, O_{pn}\}$, $i = 1, 2, \ldots, n$ is the number of machining operations including turning, milling, boring, and grinding etc. The set of working dimensions and tolerances in the view plane is denoted as Ψ = $\{u_1\pm\frac{1}{2}T_1, u_2\pm\frac{1}{2}T_2, \ldots, u_{2n}\pm\frac{1}{2}T_{2n}\}$, where $u_i\pm\frac{1}{2}T_i$, $i = 1, 2, \ldots, 2n$ are the working dimension and tolerance components assigned to the part. Since the working dimensions include sized and angular dimensions, the corresponding tolerance can be sized or angular ones. The constraint set of B/P dimensions and tolerances is denoted as D_{st} = $\{u_{d1}\pm\frac{1}{2}T_{d1}, u_{d2}\pm\frac{1}{2}T_{d2}, \ldots, u_{d2m}\pm\frac{1}{2}T_{d2m}\}$, $i = 1, 2, \ldots, 2m$ denotes 2m B/P sized and angular dimensions and tolerances of the part. The set of B/P orientational tolerances is denoted as T_G = $\{T_{G1}, T_{G2}, \ldots, T_{Gk}\}$, $i = 1, 2, \ldots, k$ are B/P geometric tolerances. In order to establish the required tolerance equations between B/P and pertinent working tolerances, dimensional chains must be derived from process planning to represent the relations between B/P and working dimensions.

In order to discuss further this issue, we introduce a practical example shown in Figure 1(Zhao, 1987). For simplicity, only the finishing operations are taken into account. The inclined hole ($\Phi25.0105 \pm 0.0105$) and inclined plane (B) of the example have high positional precision requirements. Thus the finish operations on incline hole and incline plane are executed with jig boring and grinding machine, respectively. Point D denotes the intersection of the axis of cylinder $\Phi89.974\pm0.011$ with horizontal plane W. Point C is the intersection of the axis of incline hole with plane W. Let coordinates origin O lie at the intersection point of the axis of cylinder $\Phi89.974\pm0.011$ with plane A. Axis OX lies in plane A and is parallel with plane S. Axis OY is perpendicular to plane A. Axis OZ is perpendicular to plane S. The functional requirements of this part are as such: The distance from point C to D is x_{Cd} = 8±0.07. The functional distance between plane A and W is y_{Cd} = 25.075±0.075. The functional distance between incline plane B and point C is L_{CFd} = 54±0.12. The other requirements are shown in Figure 1. Because functional dimension x_{Cd} and L_{CFd} cannot be measured directly, the finish machining processes involved are assigned as bellows:

1. Set plane A to vertical position to guarantee that plane S is parallel with horizontal plane. Choose plane A and axial line of the shaft $\Phi89.974\pm0.011$ as references. Move the table of jig boring machine to due position and process the pin hole $\Phi15.009 \pm 0.009$ and ensure that the coordinates and

tolerances of axial line of the pin hole as $x_{N'} \pm T_{N'x}/2 = -25 \pm T_{N'x}/2$, $y_{N'} \pm T_{N'y}/2 = 28 \pm T_{N'y}/2$.

2. When a measurement pin is plugged into the pin hole, it is desire that parallelism between axial line of the pin to plane A be not more than TN⊚y and perpendicularity of axial line of the pin to plane S along OX axis be not more than TN⊥x.

3. Take a measurement of the related complete sized dimensions xN', yN', and yC.

4. Turn plane A to horizontal direction in the table of jig boring machine. Then plane A is rotated an angle of 30°. Ensure that the distance between axial line of the pin to that of incline hole is LNB± TLNB/2. Where LNB is nominal dimension of the distance form axial line of the pin to that of incline hole. TNB is the tolerance of LNB. Bore incline hole Φ25.0105 ± 0.0105 and ensure that its axial line and that of Φ89.974±0.011 is in the same plane. The angle of axial line of incline hole is α1 = 60° and its tolerance Tα1 is directly controlled.

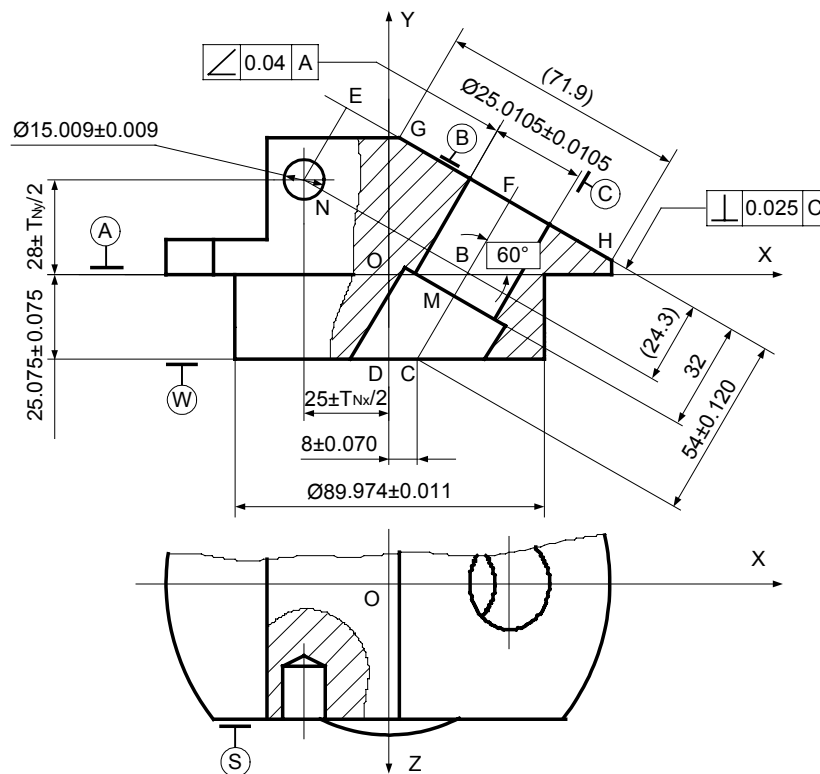5. Take a measurement of the related complete sized dimension LNB.

Figure 1. The 2D mechanical drawing of a 3D machined part

6. Grind incline plane B in grinding machine and guarantee that the distance between axial line of the pin to incline plane B with following dimensions and tolerances: $L_{NE} \pm T_{NE}/2$ and $30° \pm T\alpha_2/2$. Where L_{NE} and T_{NE} is nominal dimension and tolerance of the distance from axial line of the pin to incline plane B, respectively. $\alpha_2 = 30°$ and $T\alpha_2$ are nominal angle value and tolerance of inline plane B to OX axis, respectively.

In term of the above process processing, it is necessary that incline hole and incline plane of the example work piece are thus be processed economically within their dimension and tolerance ranges. The problem needs to be solved is: Establish pertinent dimensional chains in terms of the above manufacturing procedures, give the optimal model to the tolerance allocation problem, and find the optimal solutions. The finish machining process plan is generalized in table 1.

No	Operation	Reference(s)	Processing feature	Coordinates/ dimensions	tolerance
05	Boring	Plane A and axis of Φ89.974±0.011	Hole N′ Φ15.009±0.009	$x_{N'} = -25$ $y_{N'} = 28$	$T_{N'x}$ $T_{N'y}$
10	Pinning	No	Hole N′	$x_N = -25$ $y_N = 28$	$T_{N\perp x}$ $T_{N//y}$
15	Measure the complete sized dimensions x_N, y_N, and y_C				
20	Boring	Plane A and axis of Φ89.974±0.011	Incline hole Φ25.0105±0.0105	L_{NB} $\alpha_1 = 60°$	T_{NB} $T_{\alpha 1}$
25	Measure the complete sized dimensions L_{NB}				
30	Grinding	Plane A and axis of Φ89.974±0.011	Incline plane B	L_{NE} $\alpha_2 = 60°$	T_{NE} $T_{\alpha 2}$

Table 1. Finishing process plan of the part (Huang et al., 2002)

Unlike previous 1D case in conventional tolerance chart, the methods for generating dimensional chains are two-dimensional related. In other words, because every feature in the view plane has two dimension components, each link of a dimensional chain should contain two dimension components. Therefore we can use vector equation to present dimensional chains in the given 2D view plane.

In Figure 2, when incline hole is bored, the position of point C is indirectly obtained by controlling the position of pin, the distance from pin axis to that of incline hole, and angle α formed by axis OX and the axis of incline hole. Line segment NE is perpendicular to incline plane and point E is the intersection. Point F is the intersection of the axis of incline hole with incline plane. The line segment NB is perpendicular to the axis of incline hole and point B is the intersection.

To generate process tolerance chains correctly, we make use of a special relevant graph (SRG), which can be constructed directly from the process planning of the component, to express the interconnection and interdependence of the processed elements in their dimensions and tolerances in a more comprehensive way. In SRG, there are two kinds of nodes, one for the relevant elements of the component and another for their dimensions and tolerances. By searching through the SRG and coupled with the unique algorithm, dimension and tolerance chains needed relevant to the sequences of the processing plans are generated automatically.

Consider the pertinent point O, N, B, C, E, and F shown in Figure 2, the SRG model is constructed directly form the processing plan as show in Figure 3, where the dimension nodes and the element nodes are used. Dimension nodes are used to describe the dimensions relative to two pertinent elements of the work piece. Element nodes, however, are used to present the geometric elements of the work piece. The geometric elements refer to a point, a center line, or a plane of the work piece. In the graphical representation of the work piece under consideration, a block represents a dimensional node, while a circle corresponds to an element. The block drawn by slender lines is a component dimension node and the block drawn by dotted lines is a resultant one. Because two pertinent dimensions and tolerances must be included to determine the position and variation ranges of an element to origin O or the relative position to its pertinent reference(s), it is reasonable to introduce two dimension nodes to represent its two relative dimensions and tolerance components for an element. The link lines between dimension and element node indicate the interconnection and interdependence among them.

The process tolerance chains can be automatic generated through searching of the SRG coupled with the unique algorithm. The procedure is generalized as follows.

1. For each two selected resultant dimensions, choose any one of the elements relevant to them as the starting element node. Find two correspon-

ding pertinent component dimension nodes linked to it and get to another element node(s). Verify if these two component dimension nodes are linked to the same element node. If this is true, the ending element node obtained is used again as the starting element node and repeat the above process. Otherwise get two different element nodes. The two different element nodes obtained are used respectively again as the starting element node and repeat the above process until intersection element node is acquired. The searching direction is chosen to go along the SRG in a loop with the ending element node coming back to the starting element node, while the searching routes without duplicating the same element and dimension node more than once.

2. Every dimension chain can only contain two resultant dimensions and the minimum numbers of relative dimensions, otherwise, give up this loop and go to step (1).

3. Every resultant dimension is placed on the left side of equation and the other relative dimensions are placed on the right side. With these steps, it is easily to find that the four points O, N, B, and C and the five points O, N, E, F and C shown in Figure 1 and Figure 2 compose respectively a planar dimensional chain.
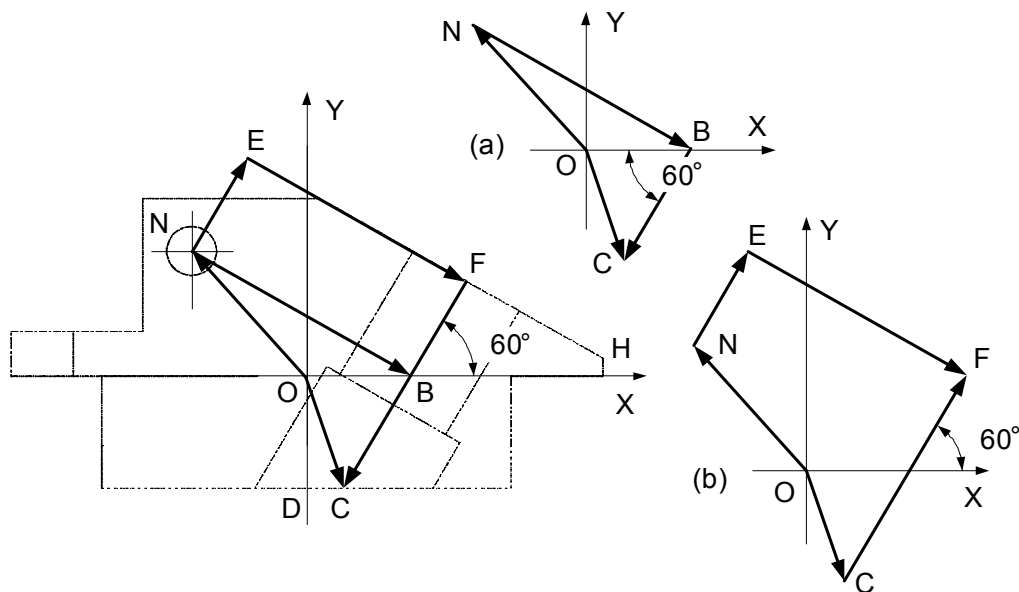

Figure 2. The vector relations between pertinent features

When incline hole is machined, the vector equation of the position of point C is:

$$\overrightarrow{OC} = \overrightarrow{ON} + \overrightarrow{NB} + \overrightarrow{BC} \tag{2}$$

Where $\overrightarrow{OC}$ is position vector of point C, $\overrightarrow{ON}$ is position vector of point N, $\overrightarrow{NB}$ and $\overrightarrow{BC}$ are relative vector from point N to point B and from point B to point C, respectively. When Equation (2) is expressed as algebraic equations, we have

$$x_N + L_{NB} \cos 30° - L_{BC} \sin 30° = x_{Cd}$$
$$y_N - L_{NB} \sin 30° - L_{BC} \cos 30° = y_{Cd} \tag{3}$$

Where x_N and y_N are coordinate component of the axis of the pin. L_{NB} and L_{BC} are nominal length between point N and B, point B and C, respectively. x_{Cd} and y_{Cd} are the B/P coordinates of point C.

Similarly, when incline plane is machined, the distance from point F to point C is indirectly obtained by controlling the position of pin, the distance from pin axis to incline plane, and the angle α formed by axis OX and the normal line of incline plane. The vector equation is:

$$\overrightarrow{CF} = \overrightarrow{ON} + \overrightarrow{NE} + \overrightarrow{EF} - \overrightarrow{OC} \tag{4}$$

Where $\overrightarrow{CF}$ is relative vector from point C to point F, $\overrightarrow{NE}$ and $\overrightarrow{EF}$ are relative vector from point N to point E, and from point E to point F, respectively. It is easy to find in Figure 2 that the length of line segment L_{EF} is equal to the length of line segment L_{NB}, i.e. $L_{EF} = L_{NB}$. Also, when we represent Equation (4) into algebraic equations, we get

$$y_N + L_{NE} \cos 30° - L_{EF} \sin 30° - y_C$$
$$= L_{CFd} \cos 30°, \text{ where } L_{EF} = L_{NB} \tag{5}$$

Where L_{NE} is nominal length between point N and E. x_C and y_C are the coordinates of point C. L_{CFd} is the B/p length between point C and point F.

30°±0.5T α2   LEF±0.5TEF   LCF=54±0.095

(E)   (F)

LNE±0.5TNE   30°±0.5T α2   60°±0.5T α N

N'x±0.5TNX   0±0.5TN⊥X   LNB±0.5TNB

(O)   (N')   (N)   (B)—LBC±0.5LBCT α1—(C)

N'y±0.5TNy   0±0.5TN∥Y   30°±0.5T α1
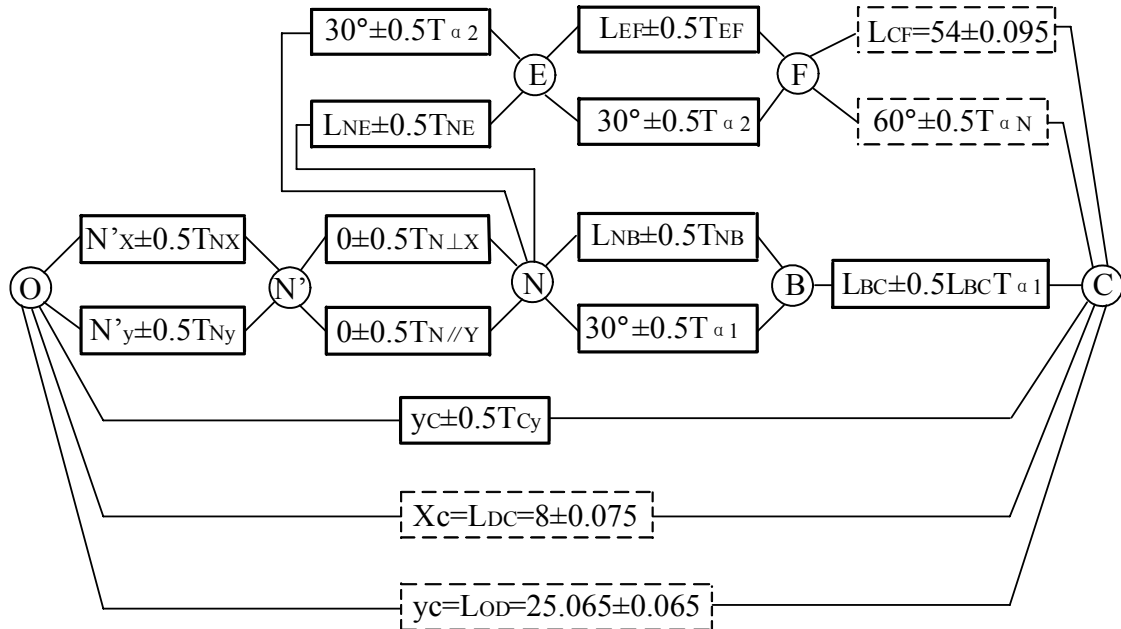
yc±0.5TCy

Xc=LDC=8±0.075

yc=LOD=25.065±0.065

Figure 3. The SRG model of the work piece relevant to the processing plan

4. With resultant dimension chains established, the relative tolerance chain is generated in the graphic way that the resultant tolerance zone should envelope all of the pertinent component tolerance zones and it is also enveloped by design tolerance zone.

The algebraic dimensional chains related to Equation 2 and 4 are:

$$
\begin{bmatrix}
1 & 0 & \cos 30° & -\sin 30° & 0 & 0 \\
0 & 1 & -\sin 30° & -\cos 30° & 0 & 0 \\
0 & \dfrac{1}{\cos 30°} & -tg\,30° & 0 & 1 & \dfrac{-1}{\cos 30°}
\end{bmatrix}
\begin{bmatrix}
x_N \\ y_N \\ L_{NB} \\ L_{BC} \\ L_{NE} \\ y_C
\end{bmatrix}
=
\begin{bmatrix}
x_{Cd} \\ y_{Cd} \\ L_{CFd}
\end{bmatrix}
\qquad (6)
$$

3. Tolerance zones and tolerances accumulation

The shapes of tolerance zones in the view plane vary with the dimensions and tolerances specified to the feature. Several cases are given in Figure 4 to illustrate this issue in the view plane XOY. The different shape of parallelogram shown in Figure 4(a)-(c) corresponds to a particular tolerance zone of point A which is controlled by two different dimensions and tolerances. The tolerance zone is center at point A and its position is controlled by $L_{OA} \pm \frac{1}{2} T_{OA}$ and $Y \pm \frac{1}{2} T_Y$, $X \pm \frac{1}{2} T_X$ and $Y \pm \frac{1}{2} T_Y$, and $L_{OA} \pm \frac{1}{2} T_{OA}$ and $60° \pm \frac{1}{2} T_\alpha$, respectively. Figure 4 (d)-(e) corresponds to two different cases of tolerance accumulations.

Figure 4(d) shows the tolerance accumulation case when one-base-point is related. This case is defined when the two dimension and tolerance components of a feature are related to only one reference feature (base point). Assume that parallelogram 1 is tolerance zone of base point A and parallelogram 2 is tolerance zone of point B relative to base point A. Resultant tolerance zone of point B is obtained by adding up the above two tolerance zones geometrically. So we can move parallelogram 2 parallelly along the outline of parallelogram 1 and the zone enveloped by outmost contour of parallelogram 2 forms the resultant tolerance zone of point B. If B/P dimensions and tolerances of point B are specified as $X_B \pm \frac{1}{2} T_{BX}$ and $Y_B \pm \frac{1}{2} T_{BY}$, for acceptable point B, B/P tolerance zone (drawn by dotted lines and measured by Tx and Ty) must envelop resultant tolerance (see right hand side in Figure 4 (d)).

Figure 4(e) shows another case of tolerance accumulation when two-base-point is related. This case is defined when two dimension and tolerance components of a feature are related respectively to two different reference features (two base points). If the smaller parallelogram centers at point C is tolerance zone of point C relative to its two base points i.e. point A and B. The parallelogram center at point A and B are tolerance zone of base point A and B, respectively. Resultant tolerance zone of point C is obtained as such. First, extract tolerance zone of point C which is resultant tolerance zone of point A and B (denote as 1-2-3-4). So draw two parallel lines perpendicular to line segment AC and let the distance between them be the tolerance magnitude of base point A in the direction of line segment AC. Similarly, draw another two parallel lines perpendicular to line segment BC and let the distance between them be the tolerance magnitude of base point B in the direction of ling segment BC. The zone formed by these four lines will construct a bigger parallelogram 1-2-3-4 which centers at point C. It is the resultant tolerance zone of point C resulting from its two-base-point tolerance zones. Then, move the smaller parallelogram center

at point C parallelly along the outline of parallelogram 1-2-3-4. The zone enveloped by outmost contour of the smaller parallelogram forms resultant tolerance zone of point C. If B/P tolerance zone of point C is a rectangle (drawn by dotted lines), the rectangle must envelop resultant tolerance zone when point C is acceptable.

Tolerance accumulation and the relationships between different sorts of tolerance specifications must be solved in presenting composite tolerance chains. For given orientational tolerances shown in Figure 1, tolerance accumulation process is dependent upon the characteristic attributes they exhibit.
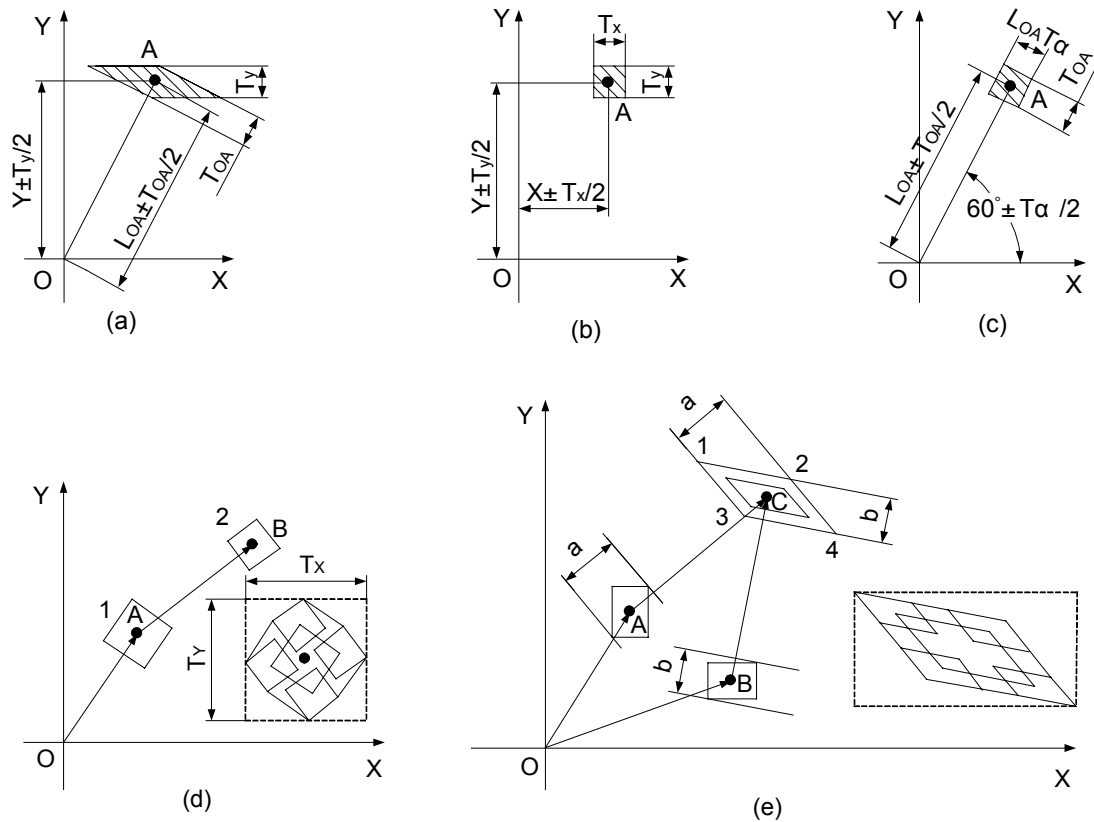

Figure 4. Tolerance zone and its stack-up in view plane XOY

In Figure 1, the tolerance zone of angularity of incline hole relative to plane A (datum A) is represented by rectangle area with shadow lines shown in Figure 5(a). Assume that the axis of incline hole has ideal geometric shape and angularity tolerance can be assured by controlling the tolerance of angle α formed by axis OX and the axis of incline hole. The expression is:

$$T_{\alpha d1} = \frac{T_{\angle}}{L_{FM}} \times 2 \qquad\qquad (7)$$

Where $T_{\alpha d1}$ is equivalent design angular tolerance determined by tolerance $T_{\angle}$, which is angularity tolerance of the axis of incline hole relative to plane A. L_{FM} is nominal length of incline hole.

Similarly, in Figure 1 the tolerance zone of perpendicularity of incline plane relative to the axis of incline hole (datum C) is represent by rectangle area with shadow lines shown in Figure 5(b). Assume that incline plane has ideal geometric shape, angularity tolerance can be assured by controlling the tolerance of angle α formed by axis OX and normal line of incline plane. The expression is:

$$T_{\alpha d2} = \frac{T_{\perp}}{L_{GH}} \times 2 \qquad\qquad (8)$$

Where $T_{\alpha d2}$ is equivalent design angular tolerance determined by $T_{\perp}$, which is perpendicularity tolerance of the incline plane relative to the axis of incline hole. L_{GH} is nominal length of incline plane.

Furthermore, according to the functional role of pin, when it is plugged into pin-hole, the following equations should be satisfied:

$$\begin{cases} T_{Nx} = T_{N'x} + T_{N\perp x} \\ T_{Ny} = T_{N'y} + T_{N\parallel y} \end{cases} \qquad (7)$$

Where T_{Nx} and T_{Ny} is composite tolerance component of pin axis, respectively. $T_{N'x}$ and $T_{N'y}$ is tolerance component of the axis of pin-hole, respectively. $T_{N\perp x}$ and $T_{N\parallel y}$ is perpendicularity of pin axis to plane S in the direction of axis OX and parallelism between pin axis and plane A, respectively.

With above discussion, for Equation 2, the tolerance zone of each vector and their accumulation is shown in Figure 6. Where zone 1-2-3-4 is the tolerance zone of pin axis. Zone 5-6-7-8 is the tolerance zone of point B relative to pin axis. Zone 9-10-11-12 is the tolerance zone of point C relative to its two base points i.e. origin O and point B. Where $L_{BC}T_{\alpha 1}$ is the tolerance component perpendicular to line segment BC and T_{Cy} is another tolerance component in the direction of axis OY.

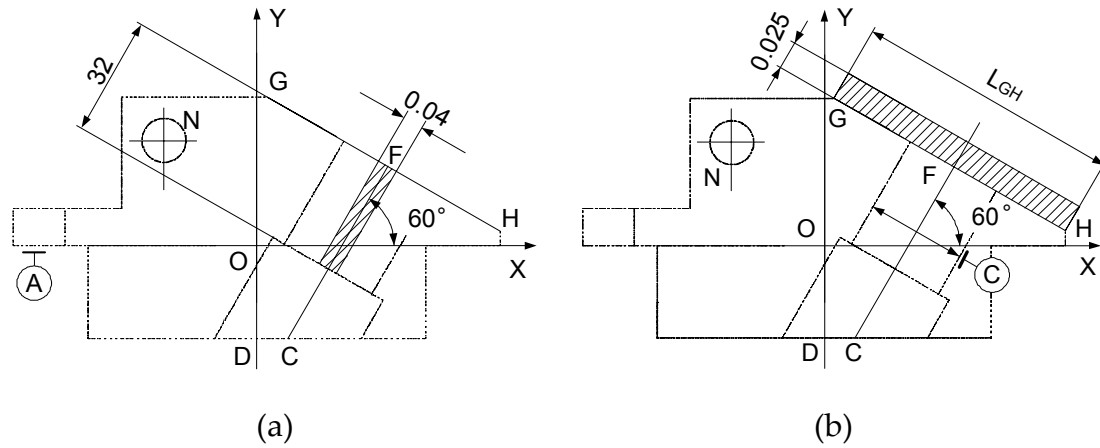(a)   (b)

Figure 5. Relationships between angular tolerance and orientational tolerances

Using the above tolerance accumulation principle discussed in Figure 4(e), the final resultant tolerance zone of point C is obtained through following steps. First, find tolerance zone of point C resulting from its two-base-point tolerance zones and denote as £, which is acquired by adding up its two base point tolerance zones in two due directions. For base point B, its tolerance zone is obtained by adding up the zone 1-2-3-4 and 5-6-7-8 geometrically. Because the direction of tolerance component of point C relative to point B is perpendicular to line segment BC (also along line segment NB) and its magnitude is $L_{BC}T_{\alpha A}$, the tolerance magnitude of point B in this direction is expressed as:

$$T_{B\perp} = T_{N\perp} + T_{NB} \tag{10}$$

Where $T_{B\perp}$ is tolerance component of point B in the direction perpendicular to line segment BC. $T_{N\perp}$ is the tolerance of pin axis in the direction perpendicular to line segment BC. T_{NB} is the tolerance of L_{NB}, which is mean dimension of the distance form pin axis to that of incline hole.

On the other hand, another component of £ is in the direction of axial OY. Because the origin is fixed, the distance of £ in the direction of axial OY is nil. So £ is finally obtained as a horizontal line segment mn shown in the right down side in Figure 6. The length of line segment mn is:

$$L_{mn} = T_{Nx} + T_{Ny} tg30° + \frac{T_{NB}}{\cos 30°} \tag{11}$$

Zone 13-14-15-16 is resultant tolerance of point C and finally acquired by mov-

ing parallelly zone 9-10-11-12 along line segment *mn*. B/P tolerance zone *I - II-III-IV* (drawn by dotted lines) should envelop zone 13-14-15-16 (right up side in Figure 6). The algebraic equations are:

$$\begin{cases} T_{Nx} + T_{Ny}tg30° + \dfrac{T_{NB} + L_{BC}T_{\alpha 1}}{\cos 30°} + T_{Cy}tg30° \leq T_{Cxd} \\ T_{Cy} \leq T_{Cyd} \end{cases} \qquad (12)$$

Where T_{Cxd} = 0.140mm and T_{Cyd} = 0.150mm are two components of B/P tolerance of point C. For Equation 4, tolerance zone of each vector and their tolerance accumulation is shown in Figure 7. Zone 1-2-3-4 and 13-14-15-16 have been discussed above. Zone 17-18-19-20 is tolerance zone of point E relative to pin axis, zone 21-22-23-24 is tolerance zone of point F relative to point E, and zone *V- VI-VII- VIII* (drawn by dotted lines) is B/P tolerance zone of point F relative to point C. Resultant tolerance zone is the one of point F relative to point C. It includes four components: zone 1-2-3-4, 17-18-19-20, 21-22-23-24, and 13-14-15-16. It is necessary that B/P tolerance zone contain its resultant tolerance zone for an acceptable part.
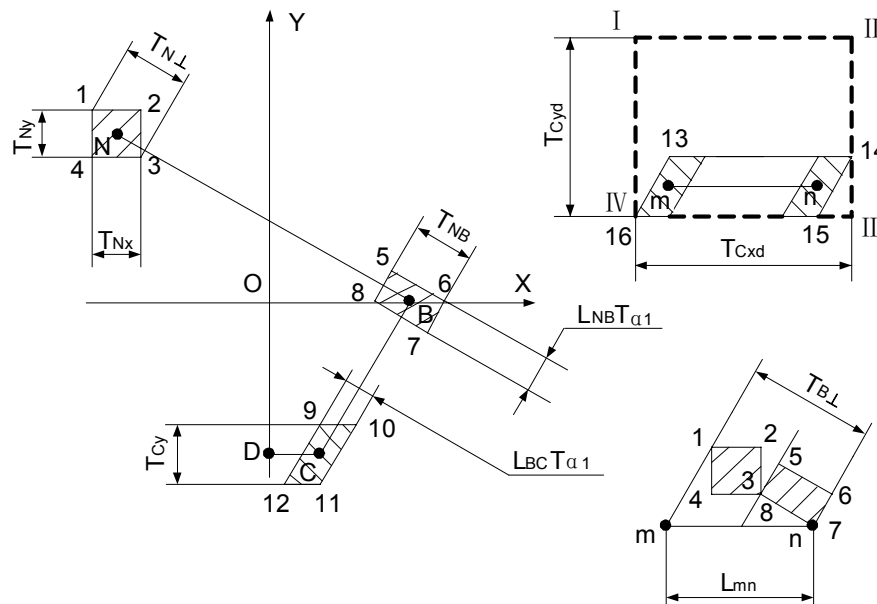

Figure 6. Original tolerance accumulation between point O, N, B, and C

When we change graphic representation into algebraic form, we can only considerer the tolerance component in the direction of line segment FC. For the

acceptable parts, the resultant tolerance component should be less than or equal to its B/P tolerance component. The algebraic equation is:

$$2T_{Nx}\sin 30° + \frac{T_{Ny}}{\cos 30°} + T_{NB}tg30° +$$
$$L_{BC}T_{\alpha 1}tg30° + T_{NE} + L_{EF}T_{\alpha 2} + \frac{T_{Cy}}{\cos 30°} \leq T_{CFd} \tag{13}$$

Where T_{CFd} = 0.240mm is B/P tolerance of L_{CFd}.

4. 3 D sequential tolerance design

In process planning, each machining operation is specified with an appropriate tolerance based on the constraints of B/P specification and process capability. In conventional dimensioning and tolerancing, all working dimensions and process tolerances are fixed. This method, however, is suitable for mass, batch, and automated production. A new method termed STC for production of complex, low-volume, and high-value-added parts was introduced (Fraticelli et al., 1997; Fraticelli et al., 1999; Wheeler et al., 1999; Cavalier & Lehtihet, 2000; Mcgarvey et al., 2001; Huang & Zhong, (in press)). The method essentially used real-time measurement information at any completion stage of operations to exploit available space inside the dynamic feasible zone and recalculate the working dimensions and tolerances for remaining operations. It has been proved that this method can enhance the process tolerances for remaining operations and increase the acceptable rate of manufacturing.

The above researches, however, did not include geometric tolerances and were confined to 1D problem. This paper aims to extend STC method to 3D space when angular and orientational tolerances are also involved. The method essentially utilizes the measurement data of sized dimensions at appropriate completion stage of operations to evaluate the working dimensions and tolerances for remaining operations based on the process capabilities. Let actual working dimension and deviation be set $M = \{u_i^*, \Delta_{ui}, j=1, \ldots, 2n\}$, where u_i^* is acquired measurement value of working dimension u_i, $\Delta_{ui} = u_i^* - u_i$ is actual deviation of working dimension u_i^*. The original dimensional and process tolerance chains are respectively expressed in the following matrix form.

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