distinguished logician, Rudolf
JL Carnap, develops in this book a
new method of semantical meaning
^analysis. After giving a detailed criti-
cal discussion of the traditional meth-
od, according to which any expression
of language (a word, a phrase, or a
sentence) is regarded as a name of one
unique entity (a thing, a property, a
class, a relation, a proposition, a fact,
etc.), Mr. Carnap concludes that the
various forms of this method of the
name-relation lead to numerous diffi-
culties and complications.

He proposes a new approach which he
calls the method of extension and in-
tension. The meaning of any expression
is analyzed into two meaning com-
ponents, the intension, which is ap-
prehended by the understanding of the
expression, and the extension, which is
determined by empirical investigation.
As one important application of this
new semantical method, Mr. Carnap
lays the foundation of a new system of
modal logic, that is, a theory of con-
cepts like necessity and contingency,
possibility and impossibility, which
philosophers and logicians will find
valuable in solving many puzzling]
problems.

MEANING AND NECESSITY

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AND NECESSITY

A Study in Semantics and Modal Logic

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PREFACE

The main purpose of this book is the development of a new method for the semantical analysis of meaning, that is, a new method for analyzing and describing the meanings of linguistic expressions. This method, called the method of extension and intension, is developed by modifying and extending certain customary concepts, especially those of class and property. The method will be contrasted with various other semantical methods used in traditional philosophy or by contemporary authors. These other methods have one characteristic k^corfflHbi^Wrhey all regard an expression in a language as a name of a concrete or abstract entity. In contradistinction, the method here proposed takes an expression, not

as naming anything, but as possessing an intension and an extension.

This book may be regarded as a third volume of the series which I have called "Studies in Semantics", two volumes of which were published earlier. However, the present book does not presuppose the knowledge of its predecessors but is independent. The semantical terms used in the present volume are fully explained in the text. The present method for defining the L- terms (for example, 'L-true', meaning* logically true', ' analytic') differs from the methods discussed in the earlier Introduction to Semantics. I now think that the method used in this volume is more satisfactory for languages of a relatively simple structure.

After meaning analysis, the second main topic discussed in this book is modal logic, that is, the theory of modalities, such as necessity, contingency, possibility, impossibility, etc. Various systems of modal logic have been proposed by various authors. It seems to me, however, that it is not possible to construct a satisfactory system before the meanings of the modalities are sufficiently clarified. I further believe that this clarification can best be achieved by correlating each of the modal concepts with a corresponding semantical concept (for example, necessity with L-truth). It will be seen that this method also leads to a clarification and elimination of certain puzzles which logicians have encountered in connection with modalities. In the Preface to the second volume of "Studies in Semantics," I announced my intention to publish, as the next volume, a book on modal logic containing, among other things, syntactical and semantical systems which combine modalities with quantification. The present book,

however, is not as yet the complete fulfilment of that promise: it contains

only analyses and discussions of modalities, preliminary to the construc-
tion of modal systems* The systems themselves are not given here. In an
article published elsewhere (see Bibliography), I have stated a calculus
and a semantical system combining modalities with quantification, and
have summarized some of the results concerning these systems. A more
comprehensive exhibition of results already found and those yet to be
found must be left for another time.

The investigations of modal logic which led to the methods developed
in this book were made in 1942, and the first version of this book was writ-
ten in 1943, during a leave of absence granted by the University of Chi-
cago and financed by the Rockefeller Foundation. To each of these insti-
tutions I wish to express my gratitude for their help. Professors Alonzo
Church and W. V. Quine reaorhe first version and discussed it with me in
an extensive correspondence. I am very grateful to both for the stimula-
tion and clarification derived from this discussion, and to Quine also for a
statement of his view and, in particular, of his reaction to my method of
modal logic. This statement is quoted in full and discussed in detail in the
penultimate section of this book. I am also indebted to Professors Carl G.
Hempel and J. C. C. McKinsey for some helpful comments. To Miss
Gertrude Jaeger I am grateful for expert help in the preparation of the
manuscript.

R. C.

CHICAGO
November 1946

CONTENTS

CHAPTER I
THE METHOD OF EXTENSION AND INTENSION

A method of semantical meaning analysis is developed in this chapter. It is applied to those expressions of a semantical system S which we call designators; they include (declarative) sentences, individual expressions (i.e., individual constants or individual descriptions) and predicators (i.e., predicate constants or compound predicate expressions, including abstraction expressions) . We start with the semantical concepts of truth and L-truth (logical truth) of sentences (i, 2). It is seen from the definition of L-truth that it holds for a sentence if its truth follows from the semantical rules alone without reference to (extra-

linguistic) facts (2). Two sentences are called (materially) equivalent if both are true or both are not true. The use of this concept of equivalence is then extended to designators other than sentences. Two individual expressions are equivalent if they stand for the same individual. Two predicators (of degree one) are equivalent if they hold for the same individuals. L-equivalence (logical equivalence) is denned both for sentences and for other designators in such a manner that it holds for two designators if and only if their equivalence follows from the semantical rules alone. The concepts of equivalence and L-equivalence in their extended use are fundamental for our method (3).

If two designators are equivalent, we say also that they have the same extension. If they are, moreover, L-equivalent, we say that they have also the same intension (5). Then we look around for entities which might be taken as extensions or as intensions for the various kinds of designators. We find that the following choices are in accord with the two identity conditions just stated.
We take as the extension of a predicator the class of those individuals to which it applies and, as its intension, the property which it expresses; this is in accord with customary conceptions (4) . As the extension of a sentence we take its truth- value (truth or falsity); as its intension, the proposition expressed by it (6). Finally, the extension of an individual expression is the individual to which it refers; its intension is a concept of a new kind expressed by it, which we call an individual concept (7-9). These conceptions of extensions and intensions are justified by their fruitfulness; further definitions and theorems apply equally to extensions of all types or to intensions of all types.

A sentence is said to be extensional with respect to a

designator occurring in it if the extension of the sentence is a function of the extension of the designator, that is to say, if the replacement of the designator by an equivalent one transforms the whole sentence into an equivalent one. A sentence is said to be intensional with respect to a designator occurring hi it if it is not extensional and if its intension is a function of the intension of the designator, that is to say, if the replacement of this designator by an L-equivalent one transforms the whole sentence into an L-equivalent one. A modal sentence (for example, 'it is necessary that . . .') is intensional with respect to its subsentence (n). A psychological sentence like 'John believes that it is raining now* is neither extensional nor intensional with respect to its subsentence (13). The problem of the semantical analysis of these belief-sentences is solved with the help of the concept of intensional structure (14, 15).

2 I. THE METHOD OF EXTENSION AND INTENSION

1. Preliminary Explanations

This section contains explanations of a symbolic language system Si, which will later serve as an object language for the illustrative application of the semantical methods to be discussed in this book. Further, some semantical concepts are explained for later use; they belong to the semantical metalanguage M, which is a part of English. Among them are the concepts of truth Jalsity, and (material) equivalence, applied to sentences. The term 'designator* is introduced for all those expressions to which a semantical meaning analysis is applied, the term will be used here especially for sentences, predicators (i.e., predicate expressions), and individual expressions.

The chief task of this book will be to find a suitable method for the semantical analysis of meaning, that is, to find concepts suitable as tools for this analysis. The concepts of the intension and the extension of an expression in language will be proposed for this purpose. They are analogous to the customary concepts of property and class but will be applied in a more general way to various types of expressions, including sentences and individual expressions. The two concepts will be explained and discussed in chapters i and ii.

The customary concept of name-relation and the distinction sometimes made since Frege between the entity named by an expression and the sense of the expression will be discussed in detail in chapter iii. The pair of concepts, extension-intension, is in some respects similar to the pair of Frege's concepts; but it will be shown that the latter pair has serious disadvantages which the former avoids. The chief disadvantage of the method applying the latter pair is that, in order to speak about, say, a property and the corresponding class, two different expressions are used. The method of extension and intension needs only one expression to speak about both the property and the class and, generally, one expression only to speak about an intension and the corresponding extension.

In chapter iv, a metalanguage will be constructed which is neutral with regard to extension and intension, in the sense that it speaks not about a property and the corresponding class as two entities but, instead, about one entity only; and analogously, in general, for any pair of an intension and the corresponding extension. The possibility of this neutral language

shows that our distinction between extension and intension does not pre-
suppose a duplication of entities.

In chapter v, some questions concerning modal logic are discussed on
the basis of the method of extension and intension.

My interest was first directed toward the problems here discussed when
I was working on systems of modal logic and found it necessary to clarify
the concepts which will be discussed here under the terms of 'extension'

and 'intension' and related concepts which have to do with what is usual-
ly called the values of a variable. Further stimulation came from some
recent publications by Quine 1 and Church, 3 whose discussions are valu-
able contributions to a clarification of the concepts of naming and
meaning.

Before we start the discussion of the problems indicated, some explana-
tions will be given in this section concerning the object languages and the
metalanguage to be used. We shall take as object languages mostly sym-
bolic languages, chiefly three semantical language systems, S x , S 2 , and S 3 ,
and occasionally also the English word language. For the sake of brevity,
not all the rules of these symbolic systems will be given, but only those of
their features will be described which are relevant to our discussion. S r
will now be described; S 2 is an extension of it that will be explained later
(41); S 3 will be described in 18.

The system 5 X contains the customary connectives of negation '~*

('not'), disjunction 'V' Cor'), conjunction '' ('and'), conditional (or material implication) ' 3 ' ('if ... then ...'), and biconditional (or material equivalence) ' =' ('if and only if). The only variables occurring are individual variables '#', l y\ V, etc. For these variables the customary universal and existential quantifiers are used: '(#)(..#..)' ('for every x, : . x . .') and '(3#)(. . x . .)' ('there is an x such that . . * . .'). All sentences in S x and the other systems are closed (that is, they do not contain free variables). In addition to the two quantifiers, two other kinds of operators occur: the iota-operator for individual descriptions ('(t#)(..#..)', 'the one individual x such that . . x . .') and the lambda-operator for abstraction expressions ('(X#)(. . x . .)', 'the property (or class) of those x which are such that . . x . .') . If a sentence consists of an abstraction expression followed by an individual constant, it says that the individual has the property in question. Therefore, '(X#)(. . x . .)a' means the same as ' . . a . .', that is, the sentence formed from '. . x . .' by substituting 'a' for '#'. The rules of our system will permit the transformation of '(X#)(. . x . .)a' into '. . a . .' and vice versa; these transformations are called conversions.

Si contains descriptive constants (that is, nonlogical constants) of indi-

1 [Notes] (see Bibliography at the end of this book). Quine's views concerning the name-relation (designation) will be discussed in chap, iii; and the conclusions which he draws from them for the problem of quantification in modal sentences will be discussed in chap. v.

[Review C.] and [Review QJ. Church's conceptions will be discussed in chap, iii, in connection with those of Frege. Church's contributions are

more important than is indicated by the form of their publication as reviews. It is to be hoped that he will soon find the opportunity for presenting his conception in a more comprehensive and systematic form.

vidual and predicate types. The number of predicates in S r is supposed to be finite, that of individual constants may be infinite. For some of these constants, which we shall use in examples, we state here their meanings by semantical rules which translate them into English.

1-1. Rules of designation for individual constants

V is a symbolic translation of 'Walter Scott',

'w' (the book) Waverley'.

1-2. Rules of designation for predicates

(H#' 'x is human (a human being)',

'RA#' 'x is a rational animal',

'Fx' (x is (naturally) featherless',

'Bx' # is a biped',

' Axy y e x is an author of y\

The English words here used are supposed to be understood in such a way that * human being' and ' rational animal' mean the same. Further, we shall use 'a', 'b', V, as individual constants, and *P', 'Q', as predicator constants (of level one and degree one) ; the interpretation of these signs will be specified in each case, or left unspecified if not relevant for the discussion.

In order to speak about any object language here the symbolic language

systems S x , etc. we need a metalanguage. We shall use as our metalan-
guage M a suitable part of the English language which contains transla-
tions of the sentences and other expressions of our object languages (for
example, the translations stated in i-i and 1-2), names (descriptions) of
those expressions, and special semantical terms. For the sake of simplicity,
we shall usually construct a name of an expression in the customary way
by including it in single quotation marks. In order to speak about expres-
sions in a general way, we often use *!['," 8/, etc -> for expressions of any
kind and**', '/, etc., for sentences, sometimes also blanks like '...',
'- -', etc., and blanks with a variable, e.g., '. . x . .', for an expression in
which that variable occurs freely. If a German letter occurs in an expres-
sion together with symbols of the object language, then the latter ones are
used autonymously, i.e., as names for themselves. 3 Thus, we may write in
M, for instance, 1 U% 33 a/; this is meant as referring to that expression
of the object language which consists of the expression H (whatever this
may be, e.g., 'Hs') followed by the sign ' ^\ followed by the expression
a/. (In symbolic formulas both in the object languages and in M, paren-
theses will often be omitted under the customary conditions.) The term

* See [Syntax], 42.

'sentence* will be used in the sense of 'declarative sentence'. The term
(sentential matrix' or, for short, 'mtfftix' will be used for expressions
which are either sentences or formed from sentences by replacing indi-

vidual constants with variables. (If a matrix contains any number of free occurrences of n different variables, it is said to be of degree n; for example, ' Axy V Px' is of degree two; the sentences are the matrices of degree zero). A sentence consisting of a predicate of degree n followed by n individual constants is called an atomic sentence (e.g., 'Pa', ' Abe').

A complete construction of the semantical system Si, which cannot be given here, would consist in laying down the following kinds of rules:

(1) rules of formation, determining the admitted forms of sentences;

(2) rules of designation for the descriptive constants (e.g. i-i and 1-2);

(3) rules of truth, which we shall explain now; (4) rules of ranges, to be explained in the next section. Of the rules of truth we shall give here only three examples, for atomic sentences (1-3), for ' V' (1-5), and for ' =' (1-6).

1-3. Rule of truth for the simplest atomic sentences. An atomic sentence in S x consisting of a predicate followed by an individual constant is true if and only if the individual to which the individual constant refers possesses the property to which the predicate refers.

.This rule presupposes the rules of designation. It yields, together with rules i-i and 1-2, the following result as an example:

1-4. The sentence 'Bs' is true if and only if Scott is a biped.

1-5. Rule of truth for ' V'. A sentence * V <S ; - is true in S x if and only if at

least one of the two components is true.

1-6. Rule of truth for ' = '. A sentence @< s @ y is true if and only if either

both components are true or both are not true.

There are some further rules of truth for the other connectives, corresponding to their truth-tables, and for the quantifiers; another example of a rule of truth will be given in 3-3. The rules of truth together constitute a recursive definition for l true in S r ', because they determine, in combination with the rules of designation, for every sentence in S x a sufficient and necessary condition of its truth (as is given for 'Bs' in 1-4). Thereby they give an interpretation for every sentence. Thus, for example, we learn from the rules that the sentence 'Bs' says that (in other words, expresses the proposition that) Scott is a biped. For the purposes of our discussion it is not necessary to give the whole definition of truth. 4 It will suffice to pre-

< The first definition of the semantical concept of truth was given by Tarski [Wahrheitsbegriff]; I have given a slightly different form in [I], 7. For nontechnical discussions of the nature of the semantical concept of truth see Tarski [Truth] and my [Remarks].

suppose that the term 'true' is defined in such a manner that it has its customary meaning as applied to sentences. More specifically, we presuppose that a statement in M saying that a certain sentence in Si is true means the same as the translation of this sentence; 5 for example, 'the sentence 'Hs' is true in S x ' means the same as 'Walter Scott is human'.
On the basis of 'true', some further semantical terms are defined as fol-

lows, with respect to any semantical system S, e.g., Sj, etc.

1-7, Definition. @ is false (in 5) =DI ~ is true (in S).

Thus ' false' has here its ordinary meaning.
1-8. Definition. @ is equivalent to ,- (in 5) =DI @< ** @; is true (in 5).

This definition, together with the rule of truth for ' ==' (1-6), yields this result:

1-9. Two sentences are equivalent if and only if both have the same truth-value, that is to say, both are true or both are false.

It is to be noticed that the term 'equivalent' is here defined in such a manner that it means merely agreement with respect to truth- value (truth or falsity), a relation which is sometimes called 'material equivalence'. The term is here not used, as in ordinary language, in the sense of agreement in meaning, sometimes called 'logical equivalence'; for the latter concept we shall later introduce the term 'L-equivalent' (2-3c).

I propose to use the term 'designator* for all those expressions to which a semantical analysis of meaning is applied, the class of designators thus being narrower or wider according to the method of analysis used. [The word 'meaning' is here always understood in the sense of 'designative meaning', sometimes also called 'cognitive', 'theoretical', 'referential', or 'informative', as distinguished from other meaning components, e.g., emotive or motivative meaning. Thus here we have to do only with declarative sentences and their parts.] Our method takes as designators at least sentences, predicators 6 (i.e., predicate expressions, in a wide sense,