# Multichannel analysis of EEG signal applied to sleep stage classification

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## 1. Introduction

The human brain is a complex organ with approximately 100 billion nerve cells (neurons) transmitting electrochemical signals. Regardless of what state we are in, whether asleep or awake, our brain produces brainwaves that can be observed and used for clinical and study applications. German psychiatrist named Hans Berger was the first to measure this electrical activity in humans in 1924, he called it electroencephalogram (EEG). It is a non-invasive method of measuring electrical activity of the brain by recording the brainwaves with electrodes placed on the scalp. Ever since his discovery, EEG has been used to diagnose many medical conditions, identifying the location of a suspected brain tumor, or a disease in the brain such as epilepsy and Parkinson's disease.

In this research the EEG method was employed for sleep disorders study. Most of us refer to sleep as a passive process; in fact the opposite is the truth, sleeping is an extremely active process. Sleep complexity is poorly understood during daily lives, our brain is more active during sleep than it is during the normal waking state. There is a distinct "architecture" of sleep, which includes five stages; four are defined as Non-Rapid Eye Movement and one as Rapid Eye Movement. These sleep stages patterns can be observed in EEG signal by change of waveforms, frequency and magnitude.

EEG is a widespread method for sleep disorders diagnostic and research. The challenge of EEG is the interpolation of recorded signals. This difficult and time consuming task is performed mainly manually by an EEG expert (technician or physiologist). In order to simplify this manual process, an automatic sleep stage detection and classification method should be analyzed. In this chapter a new method for automatic detection and classification of sleep stages using a multichannel signal analysis is proposed.

### 1.1 Previous Work

The idea of an automatic classification system for EEG signals in general and for sleep stages in particular, is not novel. There have been several researches utilizing various methods to achieve high results for automatic classification of EEG signals into sleep stages. One of the most common methods is the neural network and fuzzy rule method. Researches (Kerkeni et al., 2005), (Pinero et al., 2004), (Heiss et al., 2001), (Shimada & Shiina, 1999) and (Schaiboldet al., 2003) examined such methods along with some EEG signals featuring

extraction algorithm achieved acceptable results. Work (Kerkeni et al., 2005) and (Pinero et al., 2004) had accuracy of around 70%, (Shimada & Shiina, 1999) achieved 83% of classification using in addition multichannel information, (Schaiboldet al., 2003) reached 84.4% and (Heiss et al., 2001) succeeded in reaching 86.7% of accuracy. In (Gerla & Lhotska, 2006), the authors used Hidden Markov Model (HMM) for multichannel EEG signal analysis and principal component analysis (PCA) for dimension reduction; they accomplished only 70%-80% accuracy. Furthermore, the author of (Ghosh & Zhong, 2002) used the HMM method, however with an AR model for vector feature extraction, reaching nearly 80% of accuracy. In (Wanli & Luo, 2007) the authors used the conditional random field (CRF) method which is similar to the HMM method and attained merely 70% accuracy. Another analysis method is wavelet transform, used in (Qianli et al., 2005) and (Song et al., 2007), yielded no suitable result. In (Masaaki et al., 2002), for sleep stage classification, a waveform recognition method and decision–tree learning was used with hardly 70% accuracy. Clustering by k-mean is also a useable method, e.g. in (Agerwal & Gotman, 2001) the classification accuracy was 80.6%.

In this section we presented the recent researches in the sleep stage classification of EEG signals. Some of these researchers achieved quite good results, an accuracy of 80-86%. In spite of that it is still not good enough for clinical application, and more research needs to be done. In this research we try to achieve higher accuracy rate and we set as a wishful thinking target to cross the 90%.

## 2. Problem Definition

### 2.1 Physiological Background

#### 2.1.1 ElectroEncephaloGraphy (EEG)

EEG is a non-invasive neurophysiologic measurement of an electrical activity produced by the brain. Usually, EEG measurement is performed during a physical task that stimulates the brain cells, such as blinking, talking, sleeping, etc. The measurement involves a set of electrodes placed on different areas of the outside surface of the scalp. Electrodes are sensors that sense the electrical activity of the drain through the scalp. The electrical activity is expressed as analog signals, which is being sampled and convert into a digital signal by an analog to digital converter. The digital data is collected and stored for further analysis.

The recorded EEG signals are characterized by frequency, patterns, and amplitude. Traditionally, EEG is defined by 5 frequency bands and 5 different wave forms (Zumsteg et al., 2004): Delta waves with frequency range of 0-4Hz / 0.5-3Hz, Theta waves with frequency range of 4-8Hz/3-7Hz, Alpha waves with  frequency range between 8-11/12Hz, Beta waves  with  frequency range of 12/13-26Hz and Gamma waves with frequency of approximately 26-100Hz.

The expanded form of the EEG is the Video EEG. Video EEG consists of recording an electrical activity of the brain along with a simultaneous recording of audio and video of patient's environment. It can help physician to determine if there is a correlation between movement and abnormal brain activity. In this research, a video EEG has been used mostly for artifacts reduction.

### 2.1.2 Sleep Stages

Since the early 20th century, human sleep has been described as a succession of five recurring stages, (or sixth including awakening). Sleep stage transition is characterized by abrupt changes in frequencies and amplitudes of the EEG signal.

The first four stages are defined as a "Non-Rapid Eye Movement" (NREM) sleep and the fifth stage is defined as a "Rapid Eye Movement" (REM) sleep (Zumsteg et al., 2004), (Kelly, 1991), (Pace-Schot & Hobson,2002).

The NREM and REM sleep alternate in 90-110 minute cycles. A normal sleep pattern begins at about 80-90 minutes of NREM sleep, followed by an REM period for about 10-20 minutes. This NREM-REM cycle repeats about 4-6 times during the sleep.

**REM**
**stag**

Fig. 1. Hypnogram - Typical sleep cycle.

The five stage cycle of sleep repeats itself throughout 7-8 hours during the sleep. Stage 1 starts by shutting the eyes, and cycles through stages 2, 3 and 4. From stage 4 the processes goes recursively back, when stage 1 is replaced by the REM sleep (Fig. 1). In the successive cycles of the night, the amount of stages 3 and 4 decreases, and the proportion of the cycles occupied by REM sleep tends to increase.

**Wakefulness:** At the wakefulness state the EEG pattern alternates between two main wave forms. One is the beta wave that has fast activity of 13-26 Hz and low voltage of 10-30 µV. The second wave form is the alpha wave that has higher voltage of 20-40 µV and slower activity of 8-12 Hz.

**NREM sleep:** The NREM sleep occurs for 75-80 % of total sleep time and it is characterized by low frequency and high voltage wave activity that correspond to increasing depths of sleep. According to the Academy of Sleep Medicine (ASM) the NREM sleep can be divided into four separate stages, stage 1 to stage 4.

**Stage 1:** The duration of stage 1 is about 5 to 10 minutes, it can be defined as a gateway state between the awake state and sleep state. This stage is characterized by relative low EEG voltage and slow movements of eye rolling. Alpha waves (8-13 Hz), seen in the awake state, disappear in the first stage and are replaced by theta waves (4-7 Hz).

**Stage 2:** Stage 2 takes approximately 45-55% of the total sleep. This stage is characterized by a lack of eye movements, sleep spindles, and K-complexes . Sleep spindles and K-complexes are two distinct brain wave forms appearing on the background of theta waves.

A "Sleep spindle" is a burst of brain activity visible on EEG, it consists of 11-15 Hz waves that occur for 0.5 to 1.5 seconds. A "K-complex" is a sudden, brief, high amplitude

waveform of EEG. It consists of a brief high-voltage peak, and lasts for longer than 0.5 seconds.

**Stage 3:** This stage refers to a deep sleep and happens for 35-45 minutes after falling asleep. Stage 3 takes approximately 12% of the NREM sleep. This stage is characterized by 20-40% of delta (slow) wave and high amplitude (>75 µV). Additionally, a "K-complex" and "Sleep spindle" can also appear at this stage.

**Stage 4:** Stage 4 is very similar to stage 3, in some cases both are regarded as one. Stage 4 refers to a very deep sleep. This stage presents around 13% of the NREM sleep and more than 50% of it is characterized by delta waves.

**REM sleep:** Most dreaming occurs during the REM sleep, therefore a burst of prominent rapid eye movement appears in the EEG at this stage. Adults spend about 20-25% of their sleep cycle in the REM sleep (approximately 10 out of 90 minutes of one cycle). The EEG in this period is aroused and it is very similar to stage 1, it exhibits a mixed frequency and low voltage with occasional bursts of "saw-tooth" waves.


## 2.2 Motivation

Sleep is absolutely essential for a normal, healthy activity. Studies have shown that for normal functionality of the immune system, sleep is a necessity. It is also essential for maintaining a normal operation of the nervous system and the ability to perform both physically and mentally. In addition, sleep is essential for learning and for normal healthy cell growth. About third of the population suffers from chronic long-term disorders and occasional sleep problems. There are more than 70 different sleep disorders that are classified into three categories: lack of sleep (e.g. insomnia), disturbed sleep (e.g. obstructive sleep apnea) and excessive sleep (e.g. narcolepsy). These disorders can have a very significant effect on our daily life, such as chronic tiredness, difficulty to wake up and fall asleep, unwanted numbing, and even heart diseases.

In most cases, sleep disorders can be easily managed once they are properly diagnosed. One of the modern tools for sleep disorder diagnosis is the EEG test. The test provides a record of the patient's brain wave pattern through the whole night (7-9 hours of data). The EEG monitors various stages of sleep, which are later interpreted by a visual analysis specialist. Such analysis can be difficult, time-consuming, tiresome procedure, and not necessarily accurate. In order to assist in this toilsome process and to achieve a better diagnosis, automatic classifications of EEG sleep pattern must be developed.

The aim of this research is to create a novel method for automatic sleep stage classification, using a multichannel EEG signal. Automatic classification will help the specialists to interpret the EEG signal and to conclude a suitable diagnosis.


## 2.3 Problem Definition

As mentioned above this research deals with definition and classification of EEG signals. One of the biggest difficulties of neurologists is the interpretation of an EEG signal. Most of the neurologic world still processes the EEG signals manually, by scanning the EEG records visually.

The goal of this research is to solve this problem by offering a method for an automatic EEG signal classification. The specific difficulty that this research tries to deal with is the

detection and classification of different sleep stages in patients who suffer from sleep disorders.



Fig. 2. System's block diagram.

Numerous researches have been done in this field, however most of them are still not sufficient for clinical use. Consequently, this research aims to achieve higher classification accuracy for future clinical use in sleep EEG and in other EEG applications, by using the multichannel analysis approach.

## 3. Theoretical Overview

### 3.1 Single and Multichannel Analysis

Signal processing can be divided into two main analyses; single channel analysis and multichannel analysis. The single-channel analysis is very common in the signal processing world. The use of single channel analysis is found in various fields; Medicine (EEG, ECG, EMG etc.), Geophysics and Speech processing. Although the use of single channel analysis for some systems description can produce incorrect system model and get false results, this analysis simplifies the signal processing part within a complicated systems, when the input signal is represented by a scalar $s(t)$. On the other hand the multichannel analysis complicates the computations and the system model. Nevertheless, for several processes the multichannel analysis may offer a much more accurate model. In case of multichannel, the input signal is represented by a $d$ dimensional vector $[s_1(t)\ s_2(t)\ s_3(t)\ldots s_d(t)]^T$ , where $d$ represent the number of channels.

The research on sleep stage classification is vastly wide and variant, but almost in all researches the single signal analysis approach is used, (Kerkeni et al., 2005), (Masaaki et al., 2002), (Agerwal & Gotman, 2001), (Estrada et al., 2004), (Krajca et al., 2005), (Van Hese et al., 2001), (Shimada et al., 1998), (Sun et al., 1993). Although the classification system receives an input of more than one EEG channel, the analysis is made per single channel only. Hence, the main goal of this research is to examine sleep stage classification by multichannel analysis of multichannel EEG signal. Fig. 3 & 4 present block diagrams of the discussed signal analysis, fig. 3 demonstrates the single signal analysis and fig. 4 exhibits the multichannel analysis.

The EEG is a digital record of a biological signal that describes the electrical activity of the human brain. The recording is performed by using multiple electrodes (4-128 channels) placed on the scalp, during the sleep. Each electrode that records electrical brain activity contains important information about the neurological activity of the patient during the sleep. The spread of several electrodes on the scalp causes an overlap between multi channel recorded data due to electrodes neighborhood, which in many cases redundant. This neighborhood causes by definition an inter relations between the different sensors. Therefore, when taking this kind of data under consideration, it is much more appropriate to use the multichannel analysis, which considers the relations between the channels and produces a more accurate assumption about the sleep mechanism.
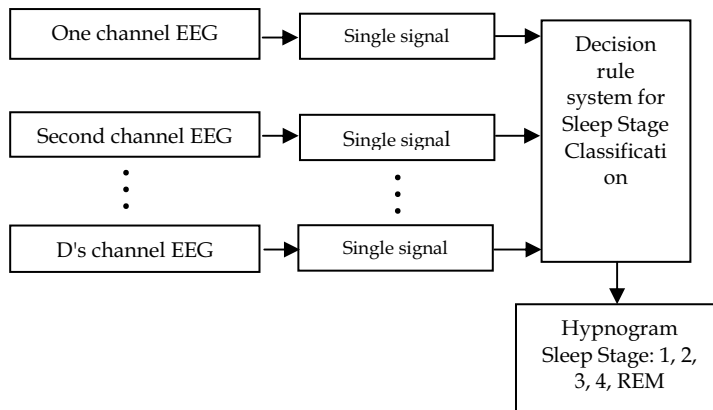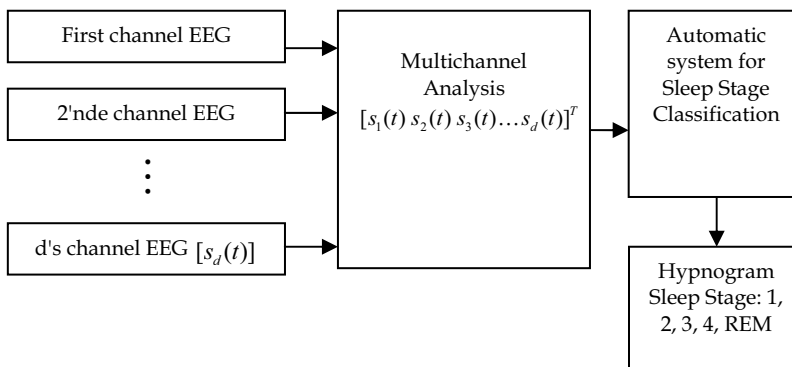
Fig. 3. Block diagram of traditional single analysis.



Fig. 4. Block diagram of multichannel analysis.

## 3.2 Multichannel Analysis

In the previous section the single channel analysis and the necessity of multichannel analysis for EEG signal processing was discussed. The classification method that is presented in this work is based on the multichannel analysis, which will be described in details in the following section. In addition review on other researches in the field will be presented in this section.

## 3.2.1 Overview

The objective of this work is to classify the EEG signal into the correct sleep stage. For this purpose the EEG signal has to be described by some mathematical model. The most common mathematical model approach in the EEG signal research is the parametrical approach which represents the EEG signal by a specific set of parameters.

There are three main types of parametrical models; the all-pole model known as the Autoregressive (AR) model, the all-zero model known as Moving Average (MA) model and the pole-zero model known as Autoregressive Moving Average (ARMA) model (a mix of AR and MA models) (Makhoul, 1975). These models are in fact filters, the analyzed signal is assumed to be the output of the filter when the input is a white noise.

The most extensively used model, in biomedical signal processing, is the scalar AR model (Makhoul, 1975), (Kay, 1988) and (Priestley, 1989). Several researches in the EEG field showed that the use of scalar AR model can describe the EEG signal in a proper way, yielding a feasible classification. More than 27 years ago, the potential use of the parametrical model for EEG signal analysis, and particularly the scalar AR model was forecasted by (Isaksson et al., 1981) and (Jansen et al., 1981). In (Isaksson et al., 1981) work, discussed the potential that EEG research has and presented information about the parametrical and not parametrical signal analysis in EEG signal. The (Jansen et al., 1981) work is focused on AR model and reviews methods for parameters and model order estimation. E. Estrada and H. Nazeran in their works (Estrada et al., 2004), (Estrada et al., 2005) and (Ebrahimi et al., 2007), attempt to classify the EEG signal into right sleep stages by scalar AR models.

The mentioned studies demonstrate a successful use of scalar AR model for EEG signal in different applications. In the PhD thesis from 1990 (Flomen, 1990),   Felix A.Flomen demonstrated the use of AR model for EEG signals and the developing of the multichannel approach for EEG signals analysis, drawing a comparison between them. This work (Flomen, 1990), was one of the pioneers in the MAR model using General Log Likelihood (GLLR) distortion measure, for multichannel analysis signal. Nonetheless, the use of Multichannel AR model (MAR) (Flomen, 1990), (Kay, 1988), (Priestley, 1989) for EEG signal is extremely rare. In (Andreson et al., 1998) work, the multichannel analysis for modeling the EEG signal is used. By modeling the multichannel EEG signal using MAR model, (Andreson et al., 1998) tried to find a satisfying solution for the "Mental Tasks Model and Classification" problem. Furthermore, (Andreson et al., 1998) proved that MAR model for EEG signal provides not only satisfying classification results, but better results than provided by the scalar AR model. The use of MAR model for EEG signal is still not extensive, however, based on the mentioned researches it is clear that the use of MAR model can help with the multichannel classification problem for the EEG signal. Therefore, this research examines sleep stages classification problem, by using the MAR model as a basic EEG signal model.

### 3.2.2 Multichannel AR Model

The following paragraph will explain in details the MAR model chosen for the multichannel EEG signal in this research.

The basic assumption for the MAR model analysis is that the analyzed signal is assumed to be stationary. Therefore, the MAR model is defined for each EEG signal $\underline{s}(n)$, of duration $T$, which is assumed to be stationary.

The $d$  dimensional EEG signal  $\underline{s}(n)$ is defined as:

$$\underline{s}(n) = [s_1(n)\ s_2(n)\ldots s_d(n)]^T, n = 1\ldots N \tag{1}$$

Where $N$ is the number of samples per signal duration $T$.

By the $d$ - dimensional MAR model, the signal $\underline{s}(n)$ is given as a linear combination of past

observations and some random input $\underline{u}(n)$ ,as presented in fig. 5:

$$\underline{s}(n) = -\sum_{k=1}^{p} \underline{A}(k)\underline{s}(n-k) + \underline{G}\underline{u}(n) \tag{2}$$

Where $\underline{G}$ is the gain factor, $p$ is the model order and $\underline{A}(k), k = 1,.., p$ are the $d \times d$ matrices coefficients of the MAR model.
The matrices coefficients $\underline{A}(k), k = 1,.., p$ defined as:

$$\underline{A}(k) = \begin{bmatrix} a_{11}(k) & a_{12}(k) & \ldots & a_{1d}(k) \\ a_{21}(k) & a_{22}(k) & \ldots & a_{2d}(k) \\ \vdots & \vdots & \ddots & \vdots \\ a_{d1}(k) & a_{d2}(k) & \ldots & a_{dd}(k) \end{bmatrix}, k = 1,\ldots,d \tag{3}$$

This model is an all-pole model that can be presented in the z plan as the transfer function $H(z)$ :

$$H(z) = \frac{G}{1 + \sum_{k=1}^{p} \underline{A}(k)z^{-k}} \tag{4}$$

From Eq. (4), fig. 5 and fig. 6, it is evident that MAR is a filter, when the input $\underline{u}(n)$ is a white noise signal and the output signal is $\underline{s}(n)$ .
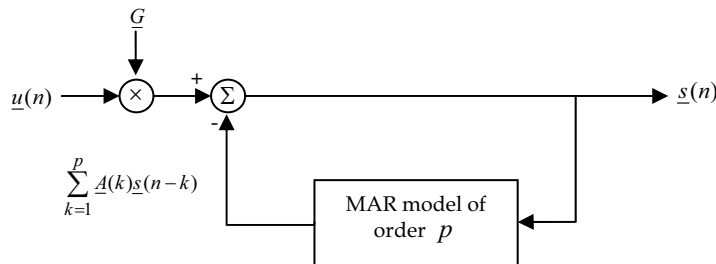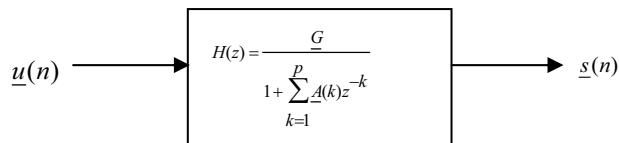


Fig. 5. All-pole model in the time domain



Fig. 6. All-pole model in the frequency domain

The input signal $\underline{u}(n)$ is a totally unknown biological signal, actually it is considered as inaccessible signal, therefore the signal $\underline{s}(n)$ can be linearly predicted only approximately

by (2) and it is defined as:

$$\tilde{\underline{s}}(n) = -\sum_{k=1}^{p} \underline{A}(k)\underline{s}(n-k) \tag{5}$$

Then the error between the actual value $\underline{s}(n)$ and the predicted value $\tilde{\underline{s}}(n)$ is given by:

$$\underline{\varepsilon}(n) = \underline{s}(n) - \tilde{\underline{s}}(n) = \underline{s}(n) + \sum_{k=1}^{p} \underline{A}(k)\underline{s}(n-k) \tag{6}$$

Since the assumption that the input $\underline{u}(n)$ is inaccessible, the gain $\underline{G}$ does not participate in the linear prediction of the signal. And so, it is irrelevant to determine a value for $\underline{G}$. However (6) can be rewritten as:

$$\underline{s}(n) = -\sum_{k=1}^{p} \underline{A}(k)\underline{s}(n-k) + \underline{\varepsilon}(n) \tag{7}$$

From (2) and (7) the following can be seen:

$$\underline{G}\underline{u}(n) = \underline{\varepsilon}(n) \tag{8}$$

Meaning, the input signal is proportional to the error signal.
From comparing (6) with (8), we get:

$$\underline{G}\underline{u}(n) = \underline{\varepsilon}(n) = \underline{s}(n) - \tilde{\underline{s}}(n) \tag{9}$$

By squared Eq. (9) and taking the expectation, we receive:

$$E\left\{ (\underline{G}\underline{u}(n))^2 \right\} = \underline{G}^2 E\left\{ \underline{u}^2(n) \right\} = E\left\{ \underline{\varepsilon}^2(n) \right\} = E\left\{ (\underline{s}(n) - \tilde{\underline{s}}(n))^2 \right\} \tag{10}$$

The input $\underline{u}(n)$ is assumed to be a sequence of uncorrelated samples with zero mean and unit variance, i.e. $E\left\{ \underline{u}(n) \right\} = 0$, for all $n$, and $Var\left\{ \underline{u}(n) \right\} = 1$. The derived equation is:

$$E\left\{ \underline{u}^2(n) \right\} = 1 \tag{11}$$

By placing (11) into (10), we receive:

$$\underline{G}^2 = E\left\{ \underline{\varepsilon}^2(n) \right\} = E\left\{ (\underline{s}(n) - \tilde{\underline{s}}(n))^2 \right\} \tag{12}$$

When (12) can be written as:

$$\underline{G}^2 = E\left\{ \underline{\varepsilon}^2(n) \right\} = E\left\{ (\underline{s}(n) - \tilde{\underline{s}}(n))^2 \right\} = E\left\{ (\underline{s}(n) - \tilde{\underline{s}}(n))(\underline{s}(n) - \tilde{\underline{s}}(n))^T \right\}$$

$$\Rightarrow E\left\{ (\underline{s}(n) - \tilde{\underline{s}}(n))(\underline{s}(n) - \tilde{\underline{s}}(n))^T \right\} = \tag{13}$$

$$E\left\{ \underline{s}(n)(\underline{s}(n) - \tilde{s}(n))^T \right\} - E\left\{ \tilde{\underline{s}}(n)(\underline{s}(n) - \tilde{\underline{s}}(n))^T \right\}$$

From (13) and (9) we get:

$$E\left\{ \underline{s}(n)(\underline{s}(n) - \tilde{\underline{s}}(n))^T \right\} - E\left\{ \tilde{s}(n)(s(n) - \tilde{s}(n))^T \right\} =$$

$$E\left\{ \underline{s}(n)(\underline{s}(n) - \tilde{\underline{s}}(n))^T \right\} - E\left\{ \tilde{\underline{s}}(n)\underline{\varepsilon}^T(n) \right\} \tag{14}$$

By the orthogonality principle, the next expression is valid:

$$E\left\{\underline{\tilde{s}}(n)\underline{\varepsilon}^T(n)\right\} = 0 \tag{15}$$

The (12), (14) and (15) yields:

$$\underline{G}^2 = E\left\{\underline{\varepsilon}^2(n)\right\} = E\left\{\underline{s}(n)(\underline{s}(n) - \underline{\tilde{s}}(n))^T\right\}$$
$$\Rightarrow E\left\{\underline{s}(n)(\underline{s}(n) - \underline{\tilde{s}}(n))^T\right\} = E\left\{\underline{s}(n)\underline{s}^T(n)\right\} - E\left\{\underline{s}(n)\underline{\tilde{s}}^T(n)\right\} \tag{16}$$

Now, by placing (5) into (16) we receive:

$$\underline{G}^2 = E\left\{\underline{\varepsilon}^2(n)\right\} = E\left\{\underline{s}(n)\underline{s}^T(n)\right\} + \sum_{k=1}^{p} \underline{A}(k)E\left\{\underline{s}(n)\underline{s}^T(n-k)\right\} \tag{17}$$

When the autocorrelation matrix of lag $i$ is defined as:

$$\underline{R}(i) = E\left\{\underline{s}(n)\underline{s}^T(n-i)\right\} \tag{18}$$

Where every $R(i), i = 1, \ldots, p$ is a $d \times d$ matrix.

By placing (18) into (17) we receive the estimation of residual error covariance matrix, as follow:

$$\underline{G}^2 = E\left\{\underline{\varepsilon}^2(n)\right\} = \underline{R}(0) + \sum_{k=1}^{p} \underline{A}(k)\underline{R}^T(k) \tag{19}$$

This expression will assist us in the forward MAR parameters and order estimation.

The accuracy of the MAR model depends mainly on the $\underline{A}(k)$ coefficients estimation and the model order $p$ definition; therefore it is critical to estimate it as accurate as possible.

There are several ways to estimate the coefficients and the model's order. To estimate the coefficients, the Yule-Walker (YW) equations (Kay, 1988), (Wiggins & Robinson, 1965) should be solved. These equations can be solved by the Levinson, Wiggens, Robinson (LWR) algorithm (Wiggins & Robinson, 1965). The optimum order was estimated by Akaike's Information Criterion (AIC) (Kay, 1988), (Priestley, 1989).

### 3.2.2.1 Yule-Walker equation for coefficients estimation

The $\underline{A}(k)$ coefficients estimation is an extremely important phase at the MAR model creation. The aim is to minimize the prediction error given by (6). By assuming stationarity of the signal $\underline{s}(n)$ and multiplying both sides of (6) by $\underline{s}^T(n-i)$ from the right, we obtain:

$$\underline{\varepsilon}(n)\underline{s}^T(n-i) = \underline{s}(n)\underline{s}^T(n-i) - \underline{\tilde{s}}(n)\underline{s}^T(n-i) \tag{20}$$

Taking expectation from both sides of (20), yields:

$$E\left\{\underline{\varepsilon}(n)\underline{s}^T(n-i)\right\} = E\left\{\underline{s}(n)\underline{s}^T(n-i) - \underline{\tilde{s}}(n)\underline{s}^T(n-i)\right\} \tag{21}$$

By the orthogonality principle, the left side of (21) equals to zero:

$$0 = E\left\{\underline{s}(n)\underline{s}^T(n-i)\right\} - E\left\{\underline{\tilde{s}}(n)\underline{s}^T(n-i)\right\} \tag{22}$$

From (5) and (22) we receive the following:

$$0 = E\left\{\underline{s}(n)\underline{s}^T(n-i)\right\} + \sum_{k=1}^{p}\underline{A}(k)E\left\{\underline{s}(n-k)\underline{s}^T(n-i)\right\} \qquad (23)$$

The autocorrelation matrix of lag $i$ was defined by Eq. (18) as:

$$\underline{R}(i) = E\left\{\underline{s}(n)\underline{s}^T(n-i)\right\}$$

Therefore, (18) and (23) lead to a set of linear equations known as Yule-Walker equations:

$$0 = \sum_{k=1}^{p}\underline{A}(k)\underline{R}(i-k) + \underline{R}(i) \qquad (24)$$

It may be written in the following matrix form:

$$\begin{bmatrix} \underline{R}(0) & \underline{R}(-1) & \dots & \underline{R}(1-p) \\ \underline{R}(1) & \underline{R}(0) & \dots & \underline{R}(2-p) \\ \vdots & \vdots & \ddots & \vdots \\ \underline{R}(p-1) & \underline{R}(p-2) & \dots & \underline{R}(0) \end{bmatrix}\begin{bmatrix} \underline{A}(1) \\ \underline{A}(2) \\ \vdots \\ \underline{A}(p) \end{bmatrix} = -\begin{bmatrix} \underline{R}(1) \\ \underline{R}(2) \\ \vdots \\ \underline{R}(p) \end{bmatrix} \qquad (25)$$

We use the fact that $\underline{R}(-i) = \underline{R}^T(i)$, which can be proven by:

$$\underline{R}^T(i) = E^T\left\{\underline{s}(n)\underline{s}^T(n-i)\right\} = E\left\{\underline{s}(n-i)\underline{s}^T(n)\right\}$$

$$\Rightarrow E\left\{\underline{s}(n-i)\underline{s}^T(n)\right\} = E\left\{\underline{s}(n)\underline{s}^T(n+i)\right\} = \underline{R}(-i) \qquad (26)$$

$$\Rightarrow \underline{R}^T(i) = \underline{R}(-i)$$

When (25) can be written in the matrix form as:

$$\begin{bmatrix} \underline{R}(0) & \underline{R}^T(1) & \dots & \underline{R}^T(p-1) \\ \underline{R}(1) & \underline{R}(0) & \dots & \underline{R}^T(p-2) \\ \vdots & \vdots & \ddots & \vdots \\ \underline{R}(p-1) & \underline{R}(p-2) & \dots & \underline{R}(0) \end{bmatrix}\begin{bmatrix} \underline{A}(1) \\ \underline{A}(2) \\ \vdots \\ \underline{A}(p) \end{bmatrix} = -\begin{bmatrix} \underline{R}(1) \\ \underline{R}(2) \\ \vdots \\ \underline{R}(p) \end{bmatrix} \qquad (27)$$

Where the $\begin{bmatrix} R \end{bmatrix}$ matrix is a Toeplitz block matrix.

For coefficients estimation, the YW equations should be solved. The most efficient and known way to do it is by applying the LWR recursive algorithm (Wiggins & Robinson, 1965). The LWR algorithm is a generalized form of Levinson's single channel case (Makhoul, 1975). At the end of this process we get $p$ autoregressive coefficients $\underline{A}(k)$ matrices of $d \times d$ dimensions, for every recorded EEG signal.

There are two methods for coefficients estimation, the covariance and the autocorrelation methods. This research has used the autocorrelation method, since it is a more convenient and a widespread method. The autocorrelation method leads to a solution based on the LWR algorithm (Makhoul, 1975), (Chen & Gersho, 1998).

### 3.2.2.2 Model Order estimation by AIC

An important decision to be made in the MAR model is the determination of an optimal order model. Since the order $p$ of the model is apriori unknown, it is to be determined by minimizing the widespread order criteria AIC (Kay, 1988), (Priestley, 1989). The (Aufrichtig

& Pedersen, 1992), (Herrera et al., 1997), (Akin & Kiymik, 2005) and (Palaniappan, 2006) researches deals with challenging issue of AR model order estimation for EEG signals. The AIC is defined as:

$$AIC(p) = N\ln(\det\sum\nolimits_{p}) + 2d^2 p \tag{28}$$

When the $\sum_{p}$ is the estimation of residual error covariance matrix using a $p^{th}$ order that was defined by Eq. (19), meaning:

$$\sum\nolimits_{p} = \underline{G}^2 = E(\underline{\varepsilon}^2(n)) = \underline{R}(0) + \sum_{k=1}^{p}\underline{A}(k)\underline{R}^T(k) \tag{29}$$

This matrix is a by-product of the LWR algorithm; therefore it is calculated recursively by the algorithm. The aim of AIC is to estimate the optimal order by finding the trade-off between the estimated prediction error matrix and the model order value. The AIC is calculated each time for a range of $p's$ , and the selected $p$ yields the minimum AIC.

## 4. Classification Method

The goal of this research is to classify the EEG signal into different sleep stages, by a multichannel analysis. In this chapter the suggested method will be described. The first section gives a general review about the processes, using a block diagram. The next section broadens the blocks of this diagram (Fig. 7).
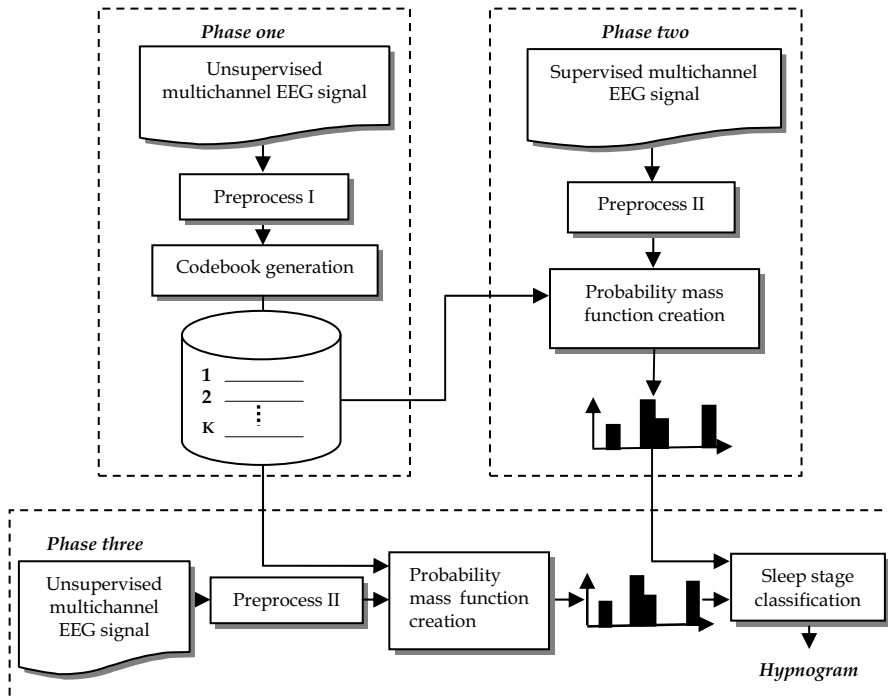


Fig. 7. Block diagram of the proposed classification system.

### 4.1 Classification System - Block Diagram

The block diagram appearing in fig. 7, describes the classification system that was created in this research. The system consists of three main phases; the first and the second phase are presented as the training phases. The first phase creates K size codebook from unsupervised data. The second phase builds histogram for each of the sleep stages, using supervised EEG signals and the codebook's codewords. The final phase is the classification stage that verifies the performance of the system.

### 4.2 Preprocess

The classification system composed of three phases (Fig.7), receives as an input some multichannel EEG signals. Every signal that gets into the classification system has to pass through the preprocess step, described by a block diagram in fig. 8. The preprocess step takes the raw EEG signal and makes it ready for the classification system. There are two kinds of preprocess steps: preprocess I and II. Both preprocesses are very similar, the difference between them will be explained in the following chapters. Preprocess I is used in the first phase and preprocess II is used in the second and third phases (Fig. 7). This section will explain in details preprocess I.
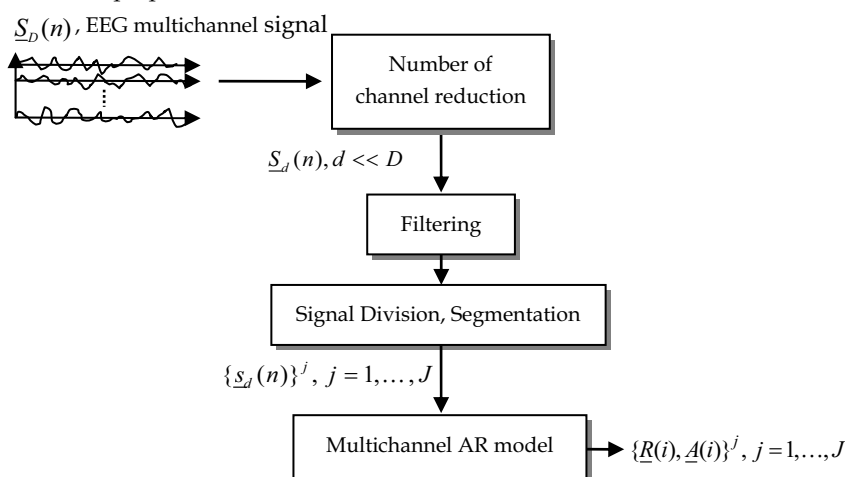


Fig. 8. Preprocess of EEG signal block diagram.

Preprocess I starts with a channel reduction. The raw EEG signal $\underline{S}_D(n)$, can contain up to 128 recorder channels ($D$) that can cause data redundancy. Therefore, according to the recommendation of an expert neurologist, a sub set of few channels ($d, d \ll D$) is chosen to represent the originally recorded EEG signal. Following channel reduction, the signals pass thorough an anti aliasing and if necessary a down sampling filter, for noise reduction.

The EEG signal is a stochastic non-stationary multichannel (vector) process. Therefore, the sampled EEG signal has to be divided into fixed length segments, when $j$ is the segment index. For further MAR model creation, the segments length $N$ ($n = 1, \ldots, N$) should be short enough in order to be considered stationary. Nevertheless, $N$ should be long enough

to enable accurate feature estimation, meaning enough samples per one coefficient estimation. The next part is the main part of the preprocess I step, the MAR model parameters estimation. The MAR model is described profoundly in chapter 3.2. In this step, the matrix coefficients $\underline{A}(i)$ are calculated for every one of signal segment $j$. The coefficients are calculated by the LWR recursive algorithm for the MAR model. Each phase in the system receives as an input a set of coefficients $\{\underline{R}(i),\underline{A}(i)\}^j$, where $\underline{R}(i)$ is the autocorrelation matrix and $\underline{A}(i)$ is the matrix coefficients. The autocorrelation matrix $\underline{R}(i)$ is necessary for GLLR (Flomen, 1990) calculation which is part of every phase in the proposed classification system. Therefore, in addition to $\underline{A}(i)$ matrix, the autocorrelation matrix $\underline{R}(i)$ considered as a part of the EEG signal representation.

After the preprocess I, the classification system works only with the coefficient's matrices and has no direct use with the original EEG signal.

The following sections will explain in details the automatic classification system in all three phases.


## 4.3 Codebook Generation - First Phase

The first phase creates from unsupervised EEG signals a *K* codewords codebook, using the Linde, Buzo, Gray (LBG) algorithm (Linde et al., 1980). The LBG algorithm takes the MAR coefficients $\{\underline{R}(i),\underline{A}(i)\}^j$, $j=1,\ldots,J$ that calculated from an unsupervised EEG data in preprocess I, and creates new *K* clusters called codewords. The role of this phase is to present a large amount of data by a reduced amount of representatives called codewords. First phase block diagram:
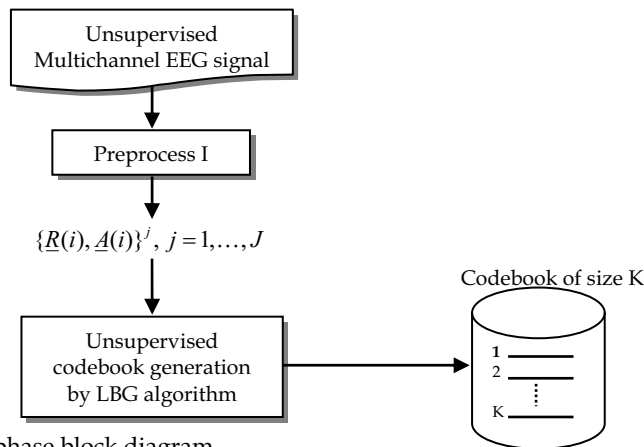


Fig. 9. The first phase block diagram.

As mentioned above, the data used in this phase is an unsupervised data, i.e. the input EEG signal does not pass through visual analysis and is not classified for any sleep stage. The entire unsupervised EEG signals, existing in our data base, pass through the preprocess I step yielding a set of $J$ coefficient's matrices denoted by $\{\underline{R}(i),\underline{A}(i)\}^j$. The $J$ coefficient's

matrices are the input parameters of the clustering algorithm LBG. The aim of the LBG algorithm is to reduce the number of MAR coefficients $J$, eventually creating a codebook with $K$ ($K \ll J$) coefficient's matrices $\{\underline{R}(i), \underline{A}(i)\}^k$ as codewords.

The LBG algorithm, like any cluster algorithm, is based on some distortion measure. We used a Generalized Log Likelihood Ratio (GLLR) distortion measure that was first developed by Felix A.Flomen in 1990 (Flomen, 1990), as part of his thesis work. The Log Likelihood Ratio (LLR) (Itakura, 1975), originally proposed by Itakura, is widely used in speech processing application for measuring the dissimilarity between two AR processes.

The LLR measure was already tested on the EEG signal in the past. In (Estrada et al., 2005) and (Ebrahimi et al., 2007) by means of LLR, a similarity between EEG and electro-oculographic (EOG) is measured during different sleep stages. In (Kong et al., 1997), a change in EEG pattern was detected by the LLR and in (Estrada et al., 2004) a similarity between base line EEG segments (sleep stage) with the rest of EEG was measured. The works (Estrada et al., 2005), (Kong et al., 1997) and (Estrada et al., 2004) showed that LLR may be used as a distortion measure in an AR model for EEG signals. We use the LLR in its generalized form, for the multichannel case and it is defined as:

$$D_{GLLR} = \log\left( \frac{\det(\underline{A}_t \underline{R}_r \underline{A}_t^T)}{\det(\underline{A}_r \underline{R}_r \underline{A}_r^T)} \right) \tag{30}$$

$D_{GLLR}$ is the GLLR distortion, $\underline{A}_r$ and $\underline{R}_r$ are the reference AR coefficients, and $\underline{A}_t$ is the tested AR coefficients.

It is important to mention that without the generalized form of LLR distortion measure of Felix A.Flomen (Flomen, 1990) it would be impossible to use the MAR model for classification of the EEG signal by the proposed system.
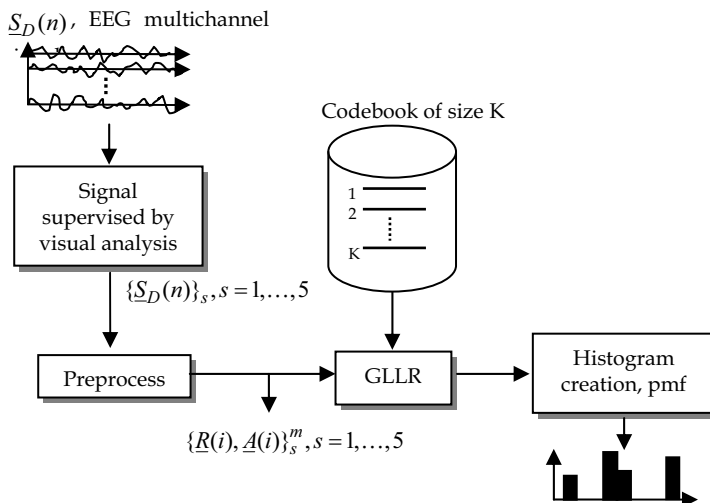
## 4.4 System Training - Second Phase



Fig. 10. The second phase block diagram.

Following the codebook creation, the second phase can be carried out. The intention of this phase is to represent each sleep stage by discrete probability mass function (pmf), of $K$ codewords that estimated by histogram. Fig. 10 provides a general look on the training phase.

At first, a new unused and unsupervised EEG signals visually classifieds into a suitable sleep stage. The manual classification of unsupervised EEG signal is performed by an EEG expert. The manually supervised EEG signals clustered into five groups according to the supervised sleep stage. Every supervised EEG signals group is pass through the preprocess II that generates $M$ MAR coefficients. Preprocess II is slightly different from preprocess I, the channel reduction and the filtering is the same, however the segmentation step has been changed according to the new needs of the second phase. In firs the supervised EEG signal divided into one minute duration fragments. Of course every one minute fragment represents only one specific sleep stage. Subsequently, every minute fragments, 60 seconds, were divided into $Q$ ( $q_1,..,q_Q$ ) segments with 50% overlap. When segment's duration is $T$

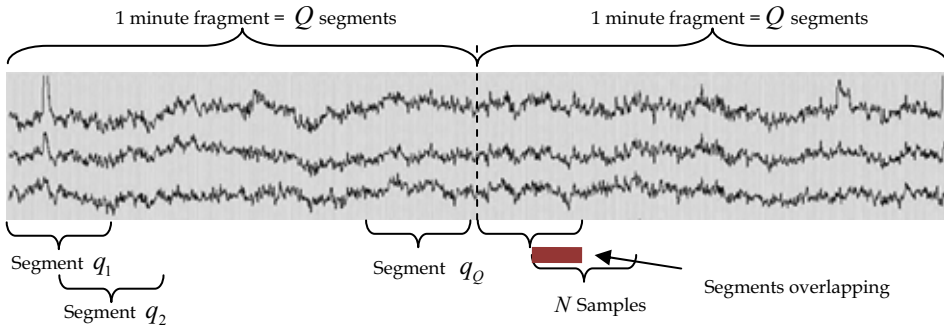and samples number $N$ , as it illustrated in fig. 11.



Fig. 11. Classification for every segment of EEG signal.

Preprocess II yields a $M$ set's of $\{\underline{R}(i), \underline{A}(i)\}_s^m$ coefficients for all the segments, when ' $s$ ' is the sleep stage tag in the range of $s = 1,\ldots,5$ .
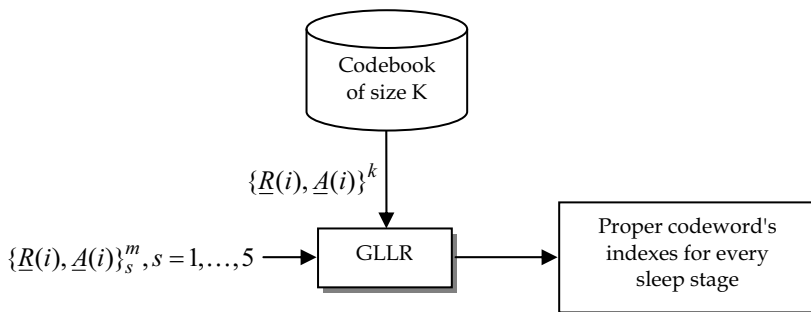


Fig. 12. Block diagram focused on codewords selection for every sleep stage.

After the parameters estimation for all segments of every one minute fragment, the next step can be preformed. The $\{\underline{R}(i), \underline{A}(i)\}_s$ coefficients of the supervised data are compared by the GLLR distortion measure (30) with each of the codeword $\{\underline{R}(i), \underline{A}(i)\}^k$ from the codebook. Fig. 12 illustrates a close-up look of this step in the second phase.

The GLLR distortion measure, $D_{GLLR}$, is calculated between parameters of segment $q$ and every codeword from the codebook. The codeword that produce the minimum $D_{GLLR}^k$, is the chosen codeword index to represent the segment, i.e.:

$$Index_{m,s} = \arg\min_{k\in1,\ldots,K}\{D_{GLLR}^k\}, \ m = 1,..,M, \ s = 1,..,5 \tag{31}$$

In other words, for every segment, a codeword is fitted by minimization of $D_{GLLR}^k$, $k = 1,..K$, resulting its argument i.e. index.

This process repeats for all segments of the supervised signal. At the end of this process we get a set of $Q$ codeword indexes for every minute of the data. Next, for every minute, a histogram from the $Q$ codeword indexes is created and normalized. In fact by this way we define a new random variable $x$ as follow:

Let $k$ be the codeword index from the codebook ($k = 1,..,K$). Let us define a random variable $x$ which indicates which index $k$ has been chosen for some segment $q$ (of duration $T$) by the $\arg\min_{k\in1,\ldots,K}\{D_{GLLR}^k\}$.

The distribution of $x$ is given by:

$$\Pr(x = k) = p(k) \quad k = 1,..,K$$
$$\sum_{k=1}^{K} p(k) = 1 \tag{32}$$

By this action we receive a probability mass function (pmf) $\Pr(x = k)$, for random variable $x$ per every minute of data that is estimated by a histogram.

We locate all the pmfs (histogram) of a certain sleep stage and by averaging them we receive a single pmf which represents the codewords distribution for a certain sleep stage. Eventually, a specific pmf $P_s(x = k), s = 1,..,5$ is estimated for every sleep stage. Fig. 13 exhibits the averaging of all pmfs (represented by histograms) ascribed to one sleep stage, and create the pmf of codebook indexes.
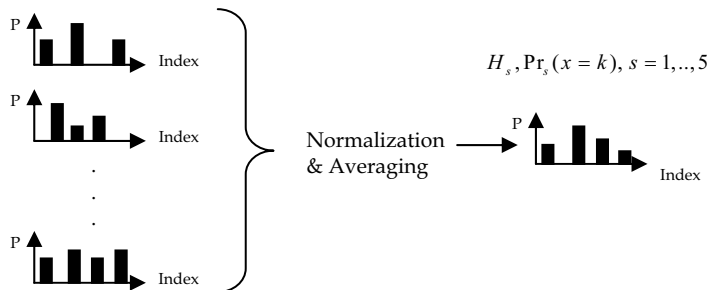


Fig. 13. Histogram for each sleep stage.

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