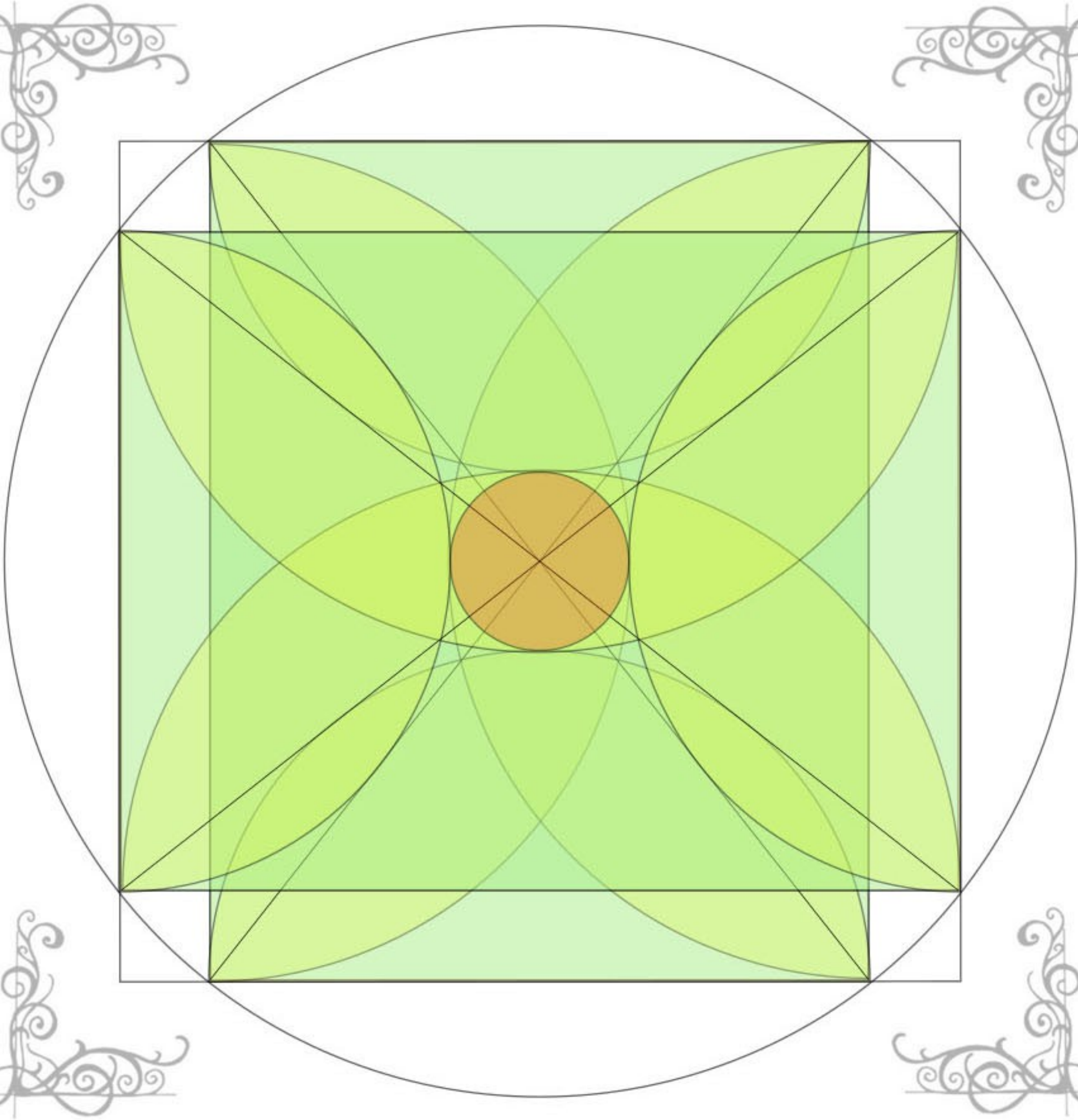
The amazing world of the Golden ratio or phi

(part 2 - the golden thread)



By Folding Φ Circles

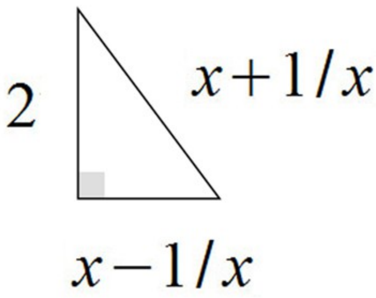
Published by In-Rainbow's 2022

All glory goes to the great spirit, nameless endless.

Cosmic Geometry (±)

A method based on a simple equation that gives valuable insight into the nature of geometry and its hidden beauty.

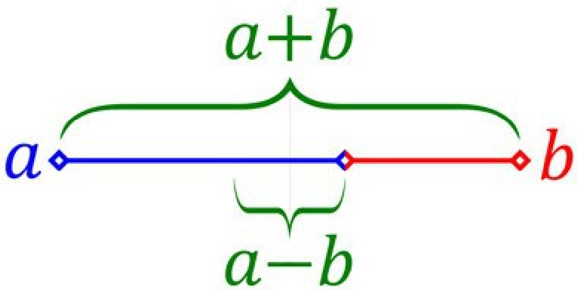
Using this method, we can detail the geometry of a triangle without using trigonometry or Pythagoras's theorem.



Cosmic (±) geometry

$$2^2 = (x+1/x)^2 - (x-1/x)^2$$

It adds a level of sophistication to the normal method.



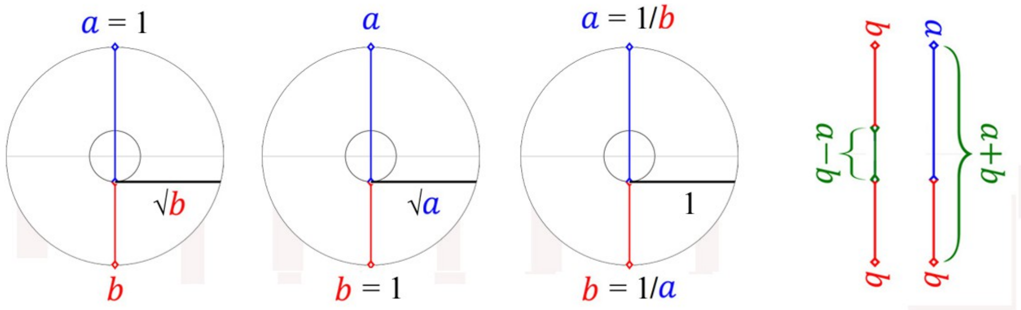
Taking into account the negative principle is a bit of a mind-bender, so, patience is a virtue.

The most interesting cosmic number is (**phi**) golden ratio.

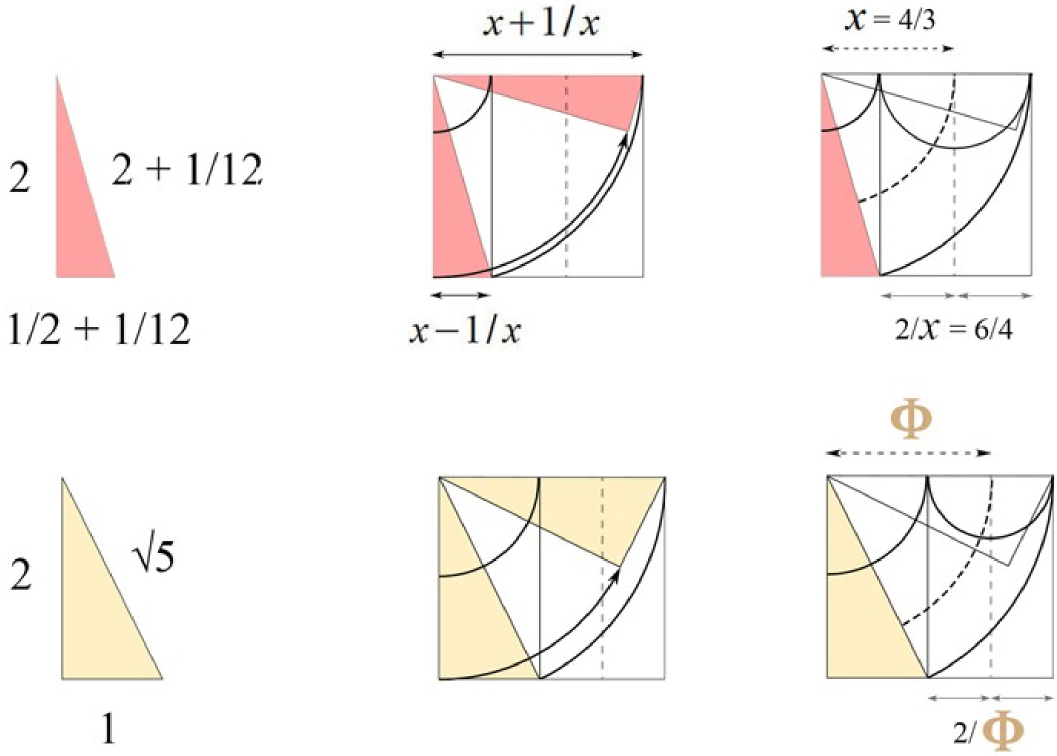
$$\Phi = \sqrt{5/4} + 1/2 \qquad 1/\Phi = \sqrt{5/4} - 1/2$$

As we will see, the **golden ratio** is the most amazing number.

The basic geometry of a **circle**, showing roots and reciprocals.

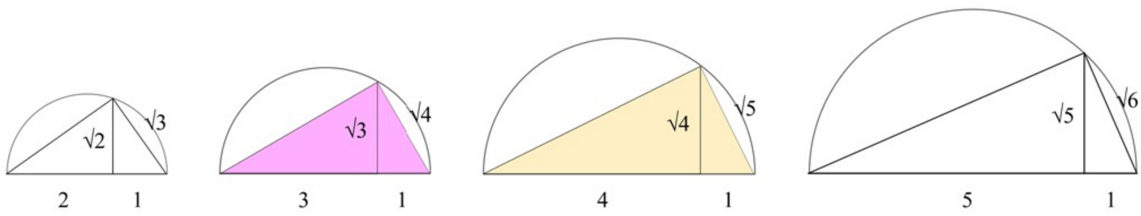


Some basic examples using cosmic mathematics, shown below.

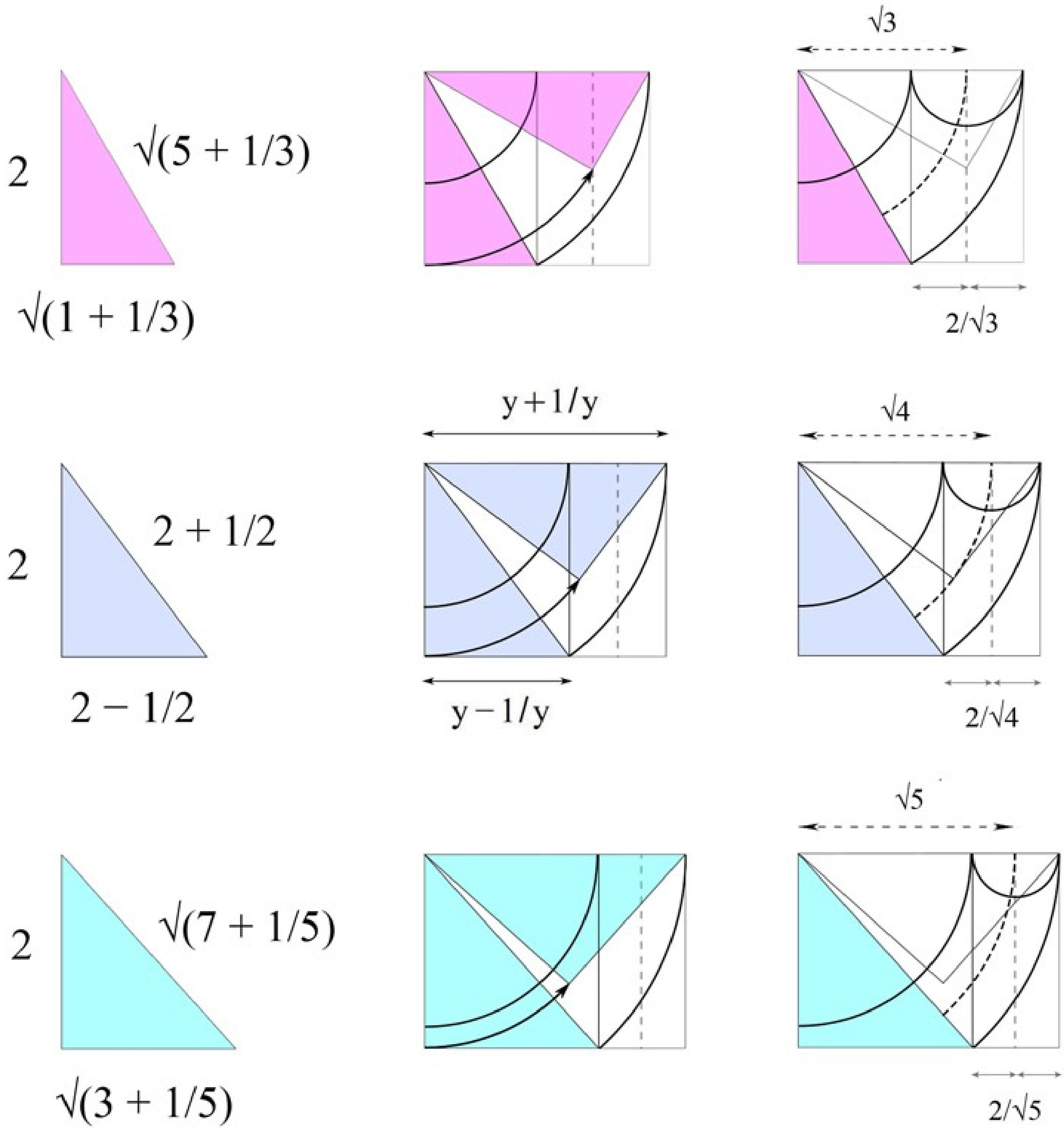


This yellow triangle is a special golden ratio triangle.

Some basic examples of using **circles** to calculate square-roots.



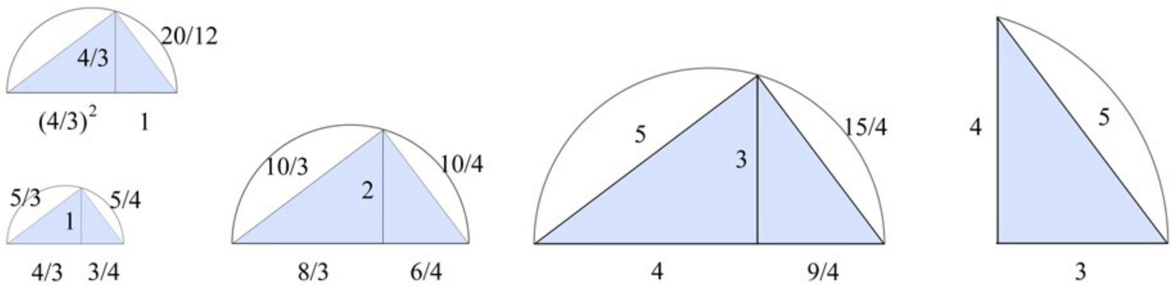
This method uses <u>both</u> the left and right sides of your brain, so expect to struggle and take your time with these examples.



Do more examples in your own time, there is no rush.

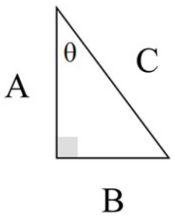
This book has taken <u>many</u> years to write. So remember, this method may be simple but it is also extremely sophisticated.

Some basic examples of scaling with the **345** triangle.

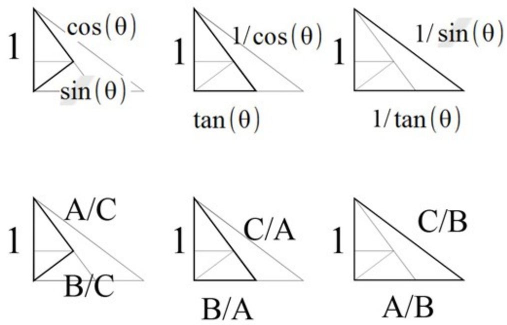


You are going to need a basic knowledge of geometry, a sharp pencil, lots of paper and pocketfuls of patience.

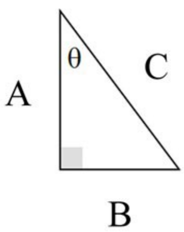
$$A^2+B^2=C^2$$



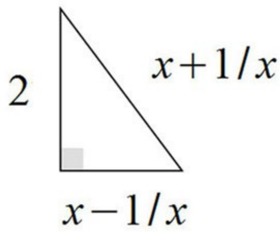
$$A/C=\cos(\theta)$$
$$B/C=\sin(\theta)$$
$$B/A=\tan(\theta)$$



The following method works for <u>all</u> right angled triangles.



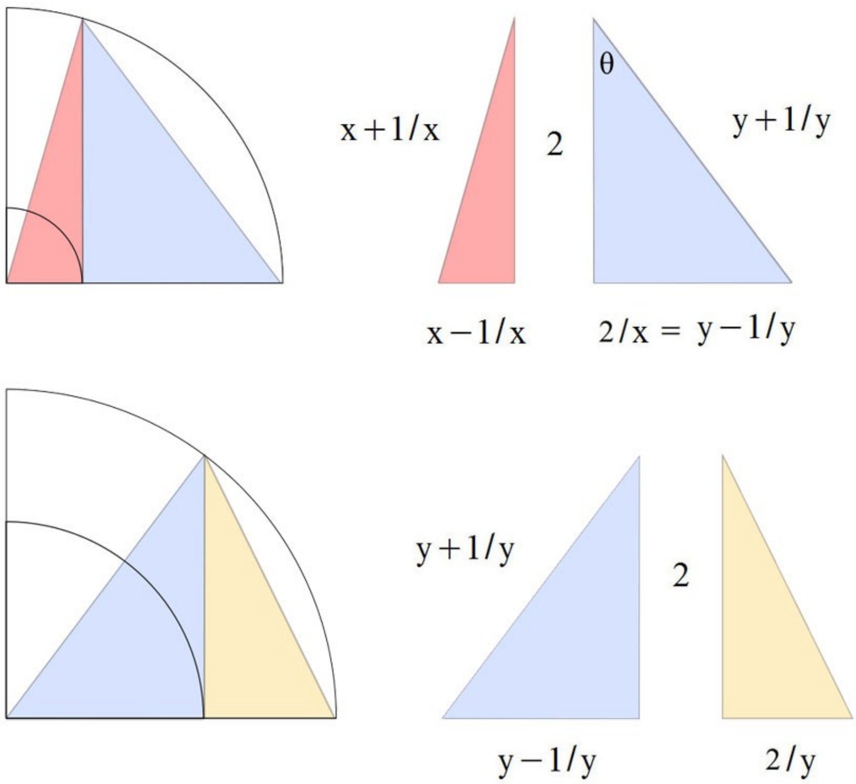
$$\frac{C+B}{A} = x \qquad \frac{C-B}{A} = 1/x$$



For brevity the following substitutions will be used, sometimes.

$$\cos=\cos(\theta) \qquad \tan=\tan(\theta) \qquad \cos.\tan=\sin(\theta)$$

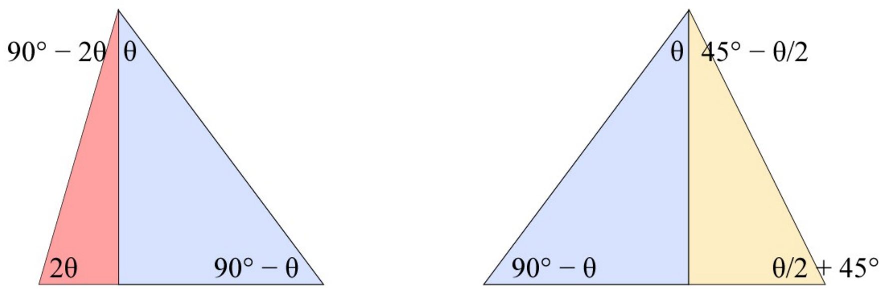
All coloured triangles are derived from a single angle (θ), which is found in the blue triangle as shown below.



The value of the reciprocals x and y for all angles.

$$x = 1/\tan \qquad 1/x = \tan$$
$$y = 1/\cos + \tan \qquad 1/y = 1/\cos - \tan$$

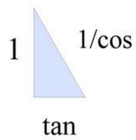
The relationship between the angles is shown below.



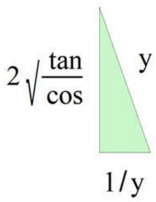
The green triangle is derived using the well know theorem of geometry called the **power of a point**, as shown below.

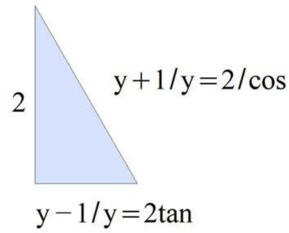
$$1 = 1/\cos^2 - \tan^2 = (\overbrace{1/\cos + \tan}^{y}) \times (\underbrace{1/\cos - \tan}_{1/y}) \qquad y^2 - 1/y^2 = 4(\tan/\cos)$$

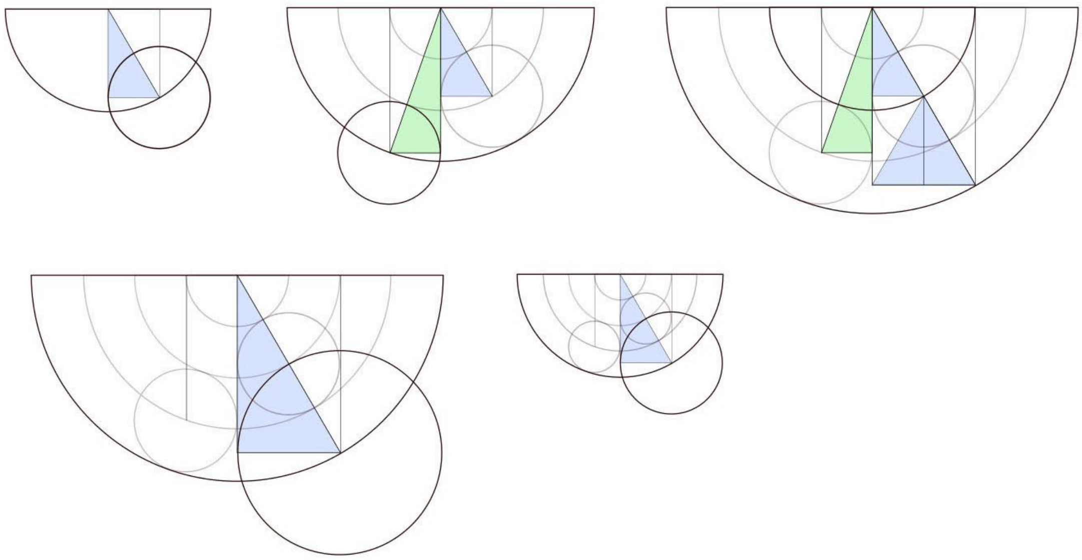
$$y^2 = (1/\cos + \tan)^2 = 1/\cos^2 + 2(\tan/\cos) + \tan^2$$
$$1/y^2 = (1/\cos + \tan)^2 = 1/\cos^2 - 2(\tan/\cos) + \tan^2$$



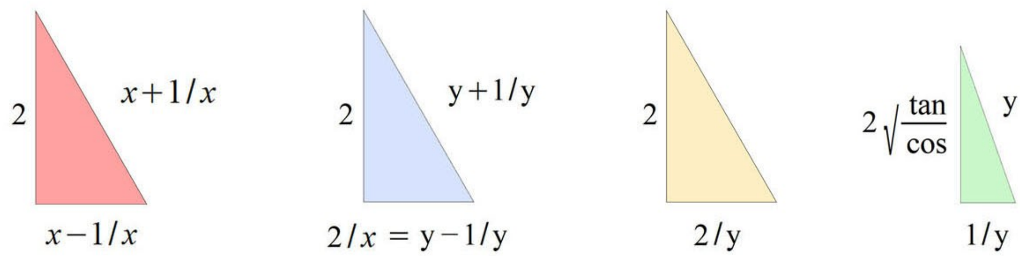
$$y = 1/\cos + \tan$$
$$1/y = 1/\cos - \tan$$





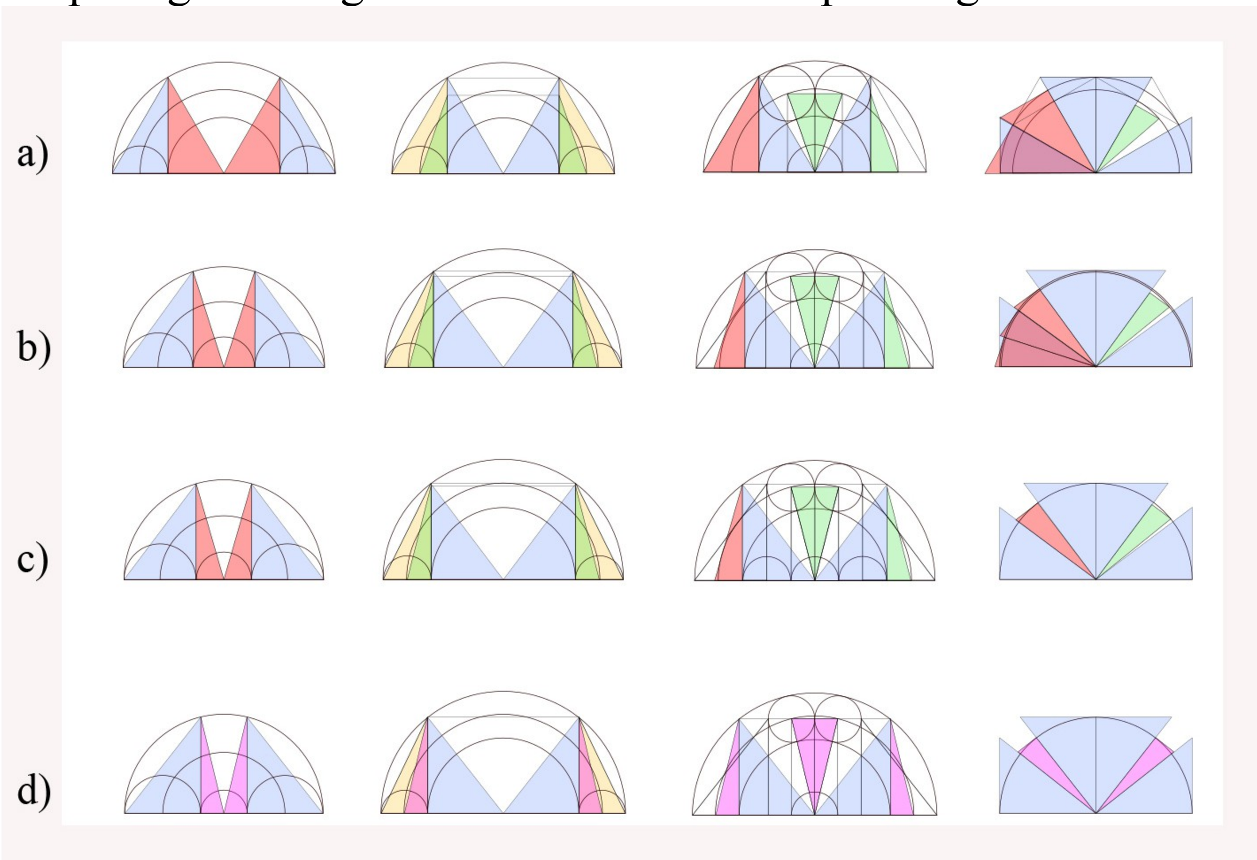


This gives us the following set of four coloured triangles.

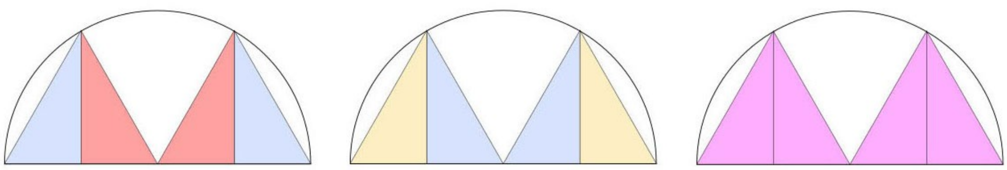


The base of green and yellow triangles always has a (2:1) ratio.

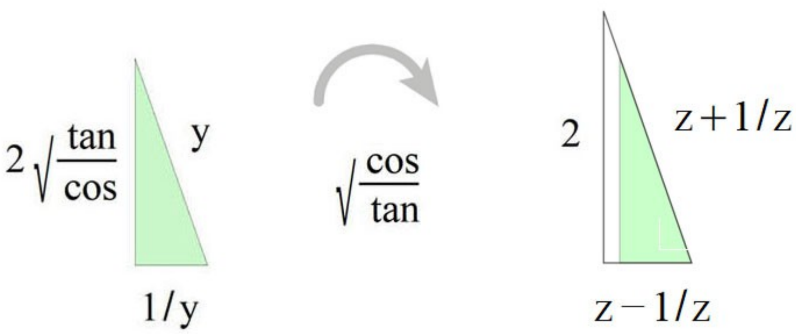
Comparing four angles and there relationships using this method.



Angle (a) is special because the blue, red and yellow triangles are all <u>equal</u>, hence the different colour, as shown below.



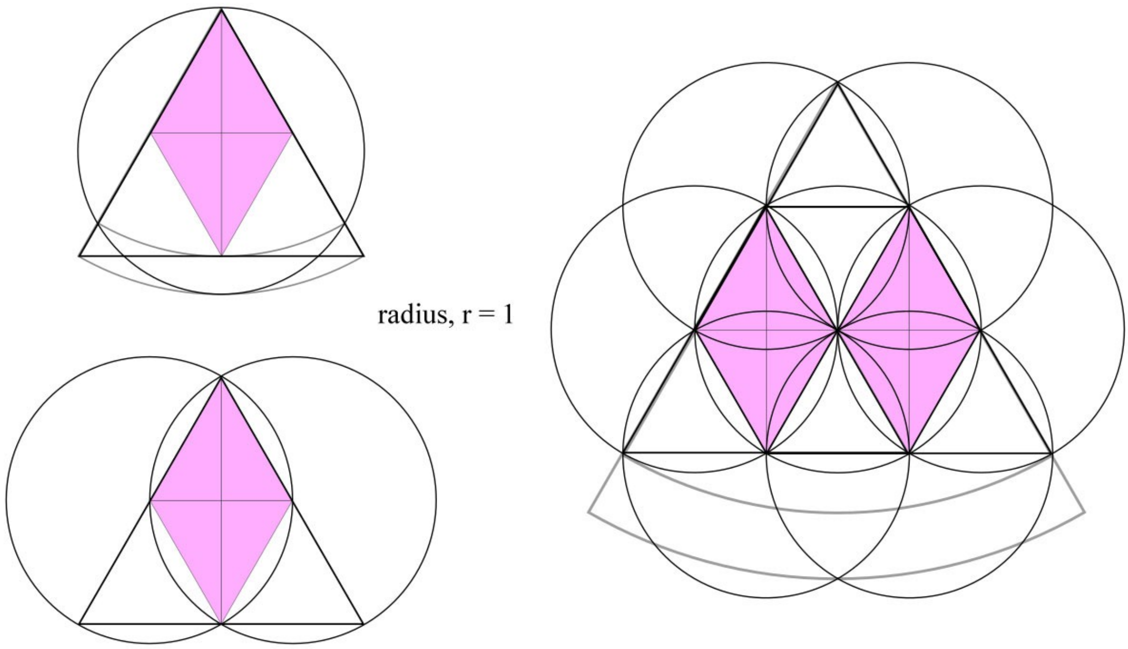
The final triangle is this scaled version of the green triangle.



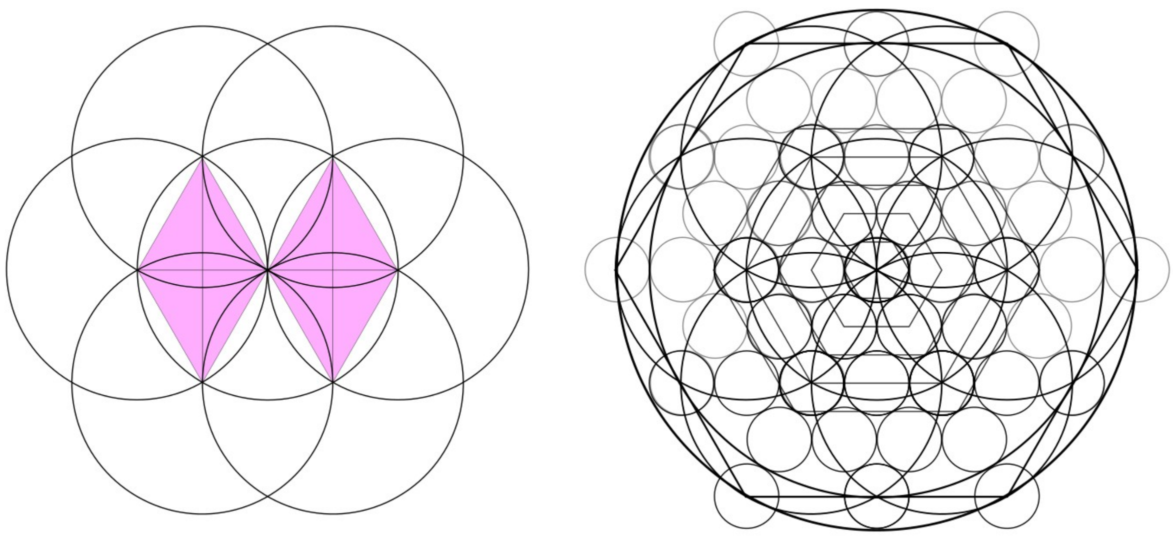
The value of **z** and reciprocal for <u>all</u> angles, shown below.

$$z = \sqrt{(1/\cos.\tan)} \qquad 1/z = \sqrt{(\cos.\tan)}$$

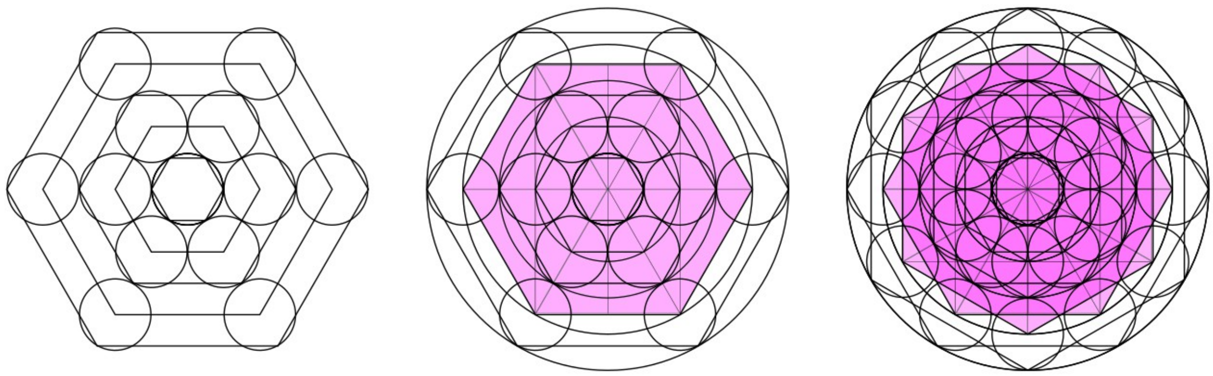
Angle (a) gives us the geometry of the **vesica piscis**, and six circles around a hexagon with perimeter=6, as shown below.



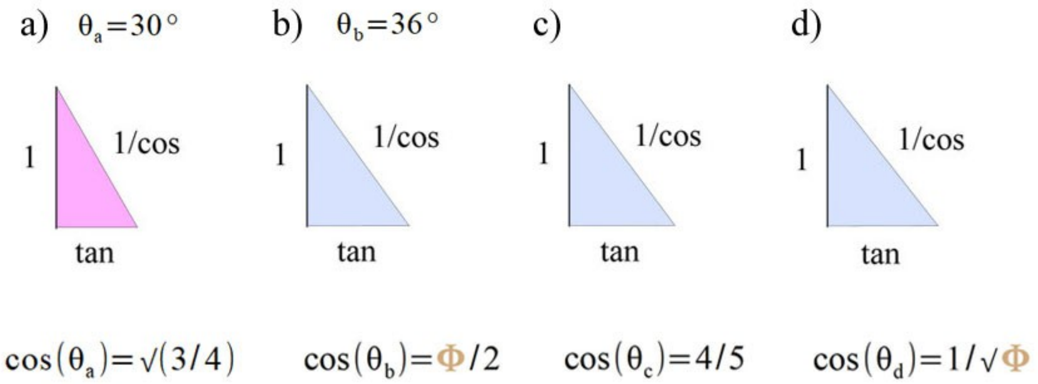
This following beautiful geometry is known as the **flower of life**.



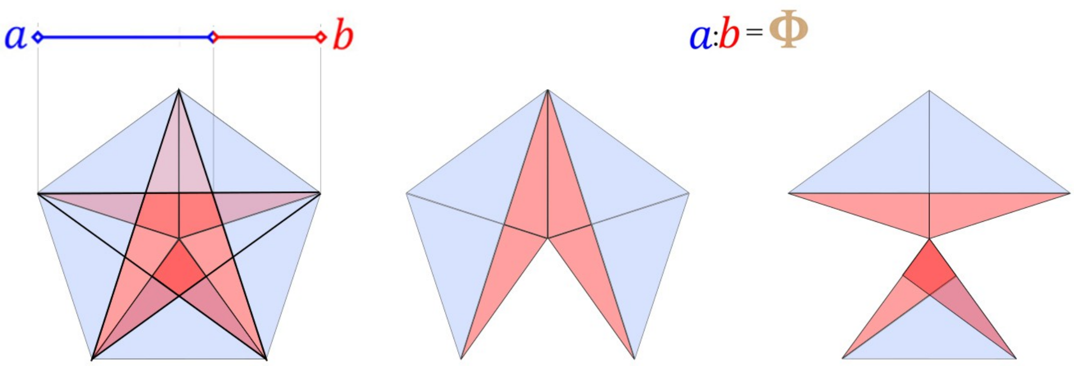
The **fruit of life** with nested circles and hexagons, shown below.



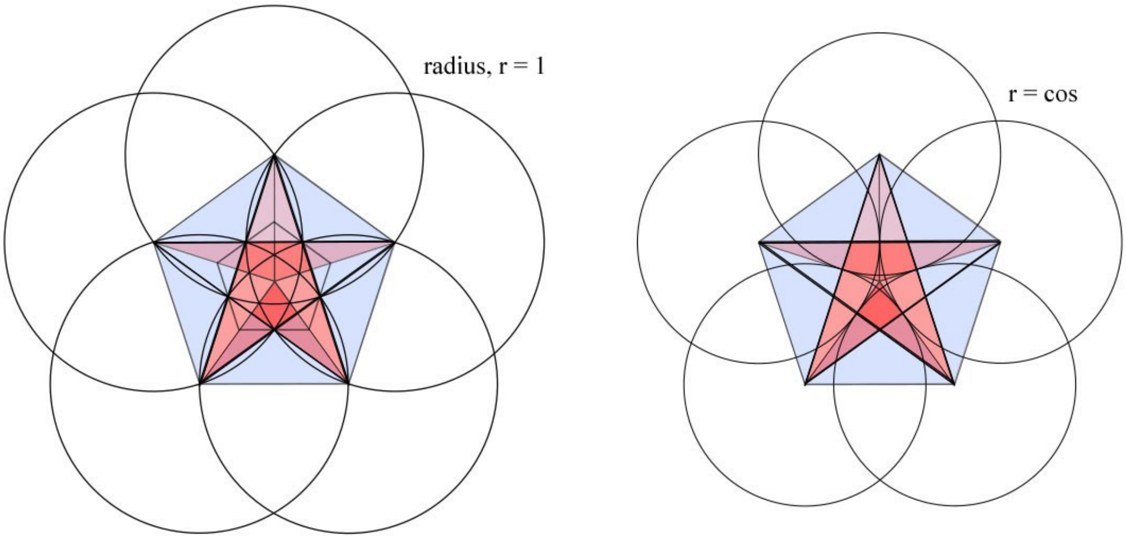
The next three angles have an interesting **phi** relationship.



Angle (b) gives us the geometry of the pentagon, which we can construct using the red and blue triangles, as shown below.



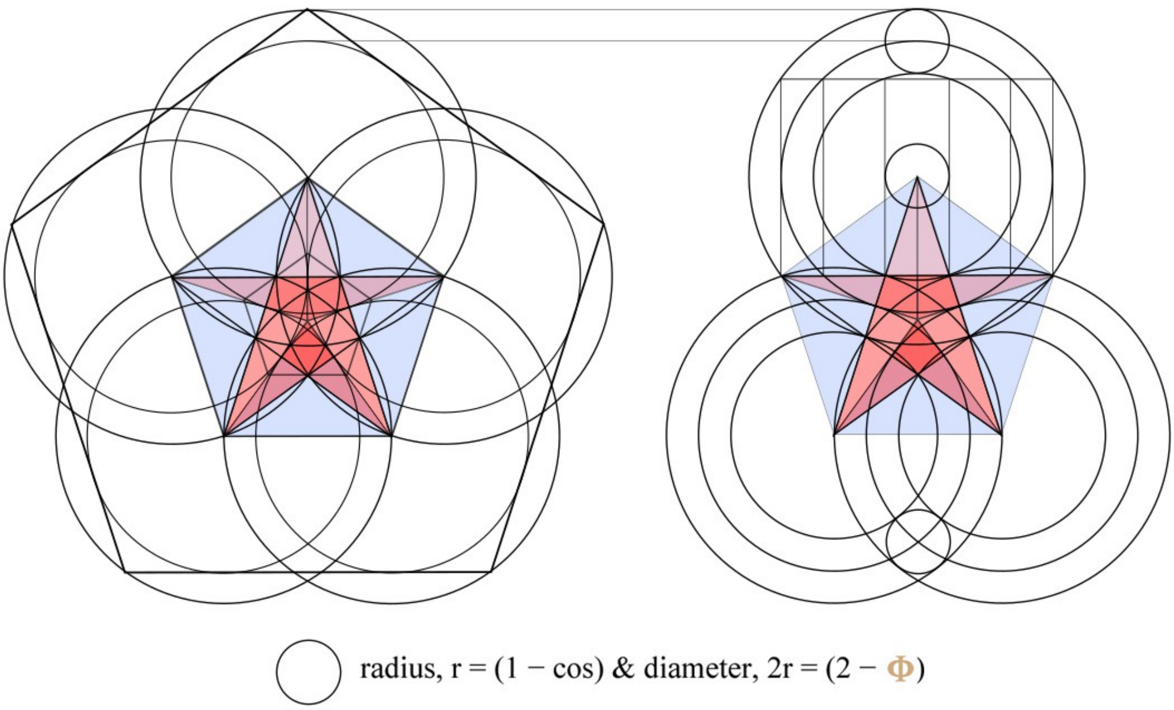
Two sets of five circles around a pentagon with perimeter=5.



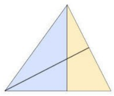
The golden ratio is unique because of the following.

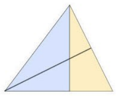
$$\Phi - 1/\Phi = 1 \qquad\qquad 2 - \Phi = 1 - 1/\Phi$$

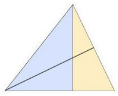
Below we can see both sets giving us five golden ratio rings.



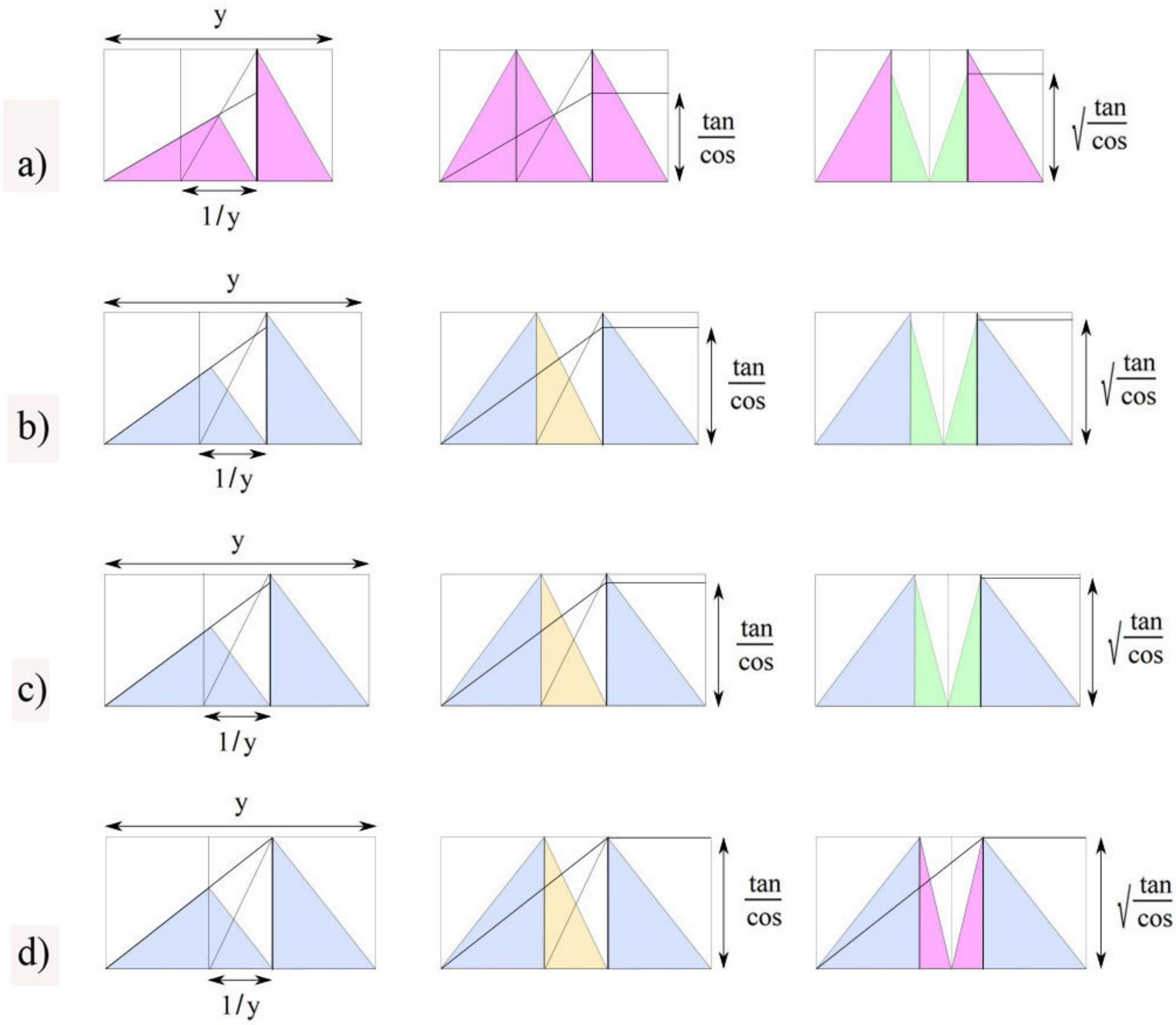
Square-root relationship between these blue and yellow triangles.

$\cos(\theta_b) = \Phi/2$  $\cos(45° - \theta_b/2) = \sqrt{\cos(\theta_e)}$

$\cos(\theta_c) = 4/5$  $\cos(45° - \theta_c/2) = \sqrt{\cos(\theta_c)}$

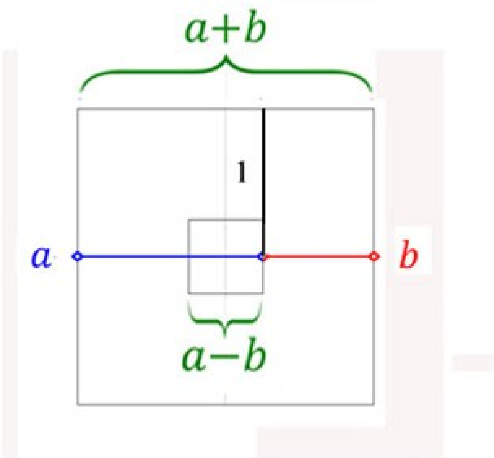
$\cos(\theta_d) = 1/\sqrt{\Phi}$  $\cos(45° - \theta_d/2) = \sqrt{\cos(\theta_b)}$

Next we detail the same angles as rectangles with (y:1) ratios.



Angle (c) gives a perfect half-square or rectangle with (2:1) ratio.

And from it we can derive the geometry of a perfect **square**.



$$a = p/8 + 2/p = \sqrt{1+\tan^2} = 1/\cos$$

$$b = p/8 - 2/p = \tan$$

$$a^2 = (p^2/64 + 4/p^2) + 1/2 = 1+\tan^2$$

$$b^2 = (p^2/64 + 4/p^2) - 1/2 = \tan^2$$

Quarter Perimeter, $p/4 = a+b = y$

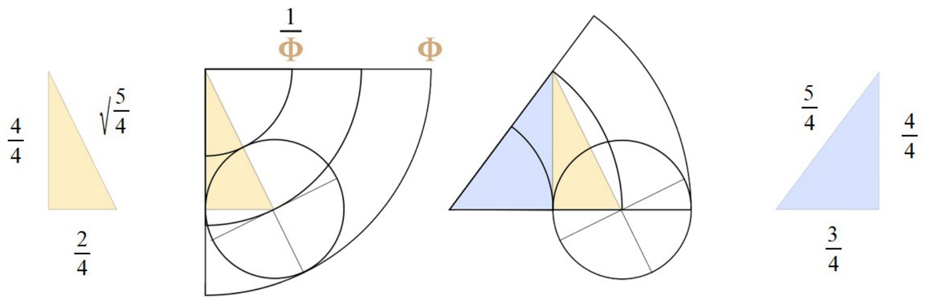
$4/p = a-b = 1/y$

And the **square numbers**, or geometric progression (1,2,4,8,16,..) and reciprocals with ratio (2:1) where y=2 and perimeter=8.

$$(a+b)^2 = a^2 + b^2 + 2ab$$

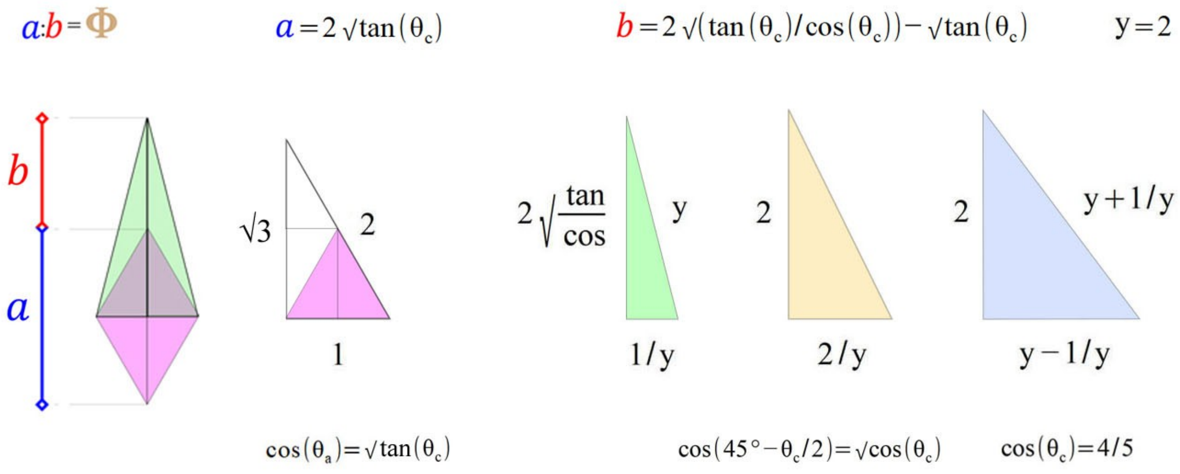
$$(a-b)^2 = a^2 + b^2 - 2ab$$

From it we get the 345 and golden (phi) triangles, shown below.

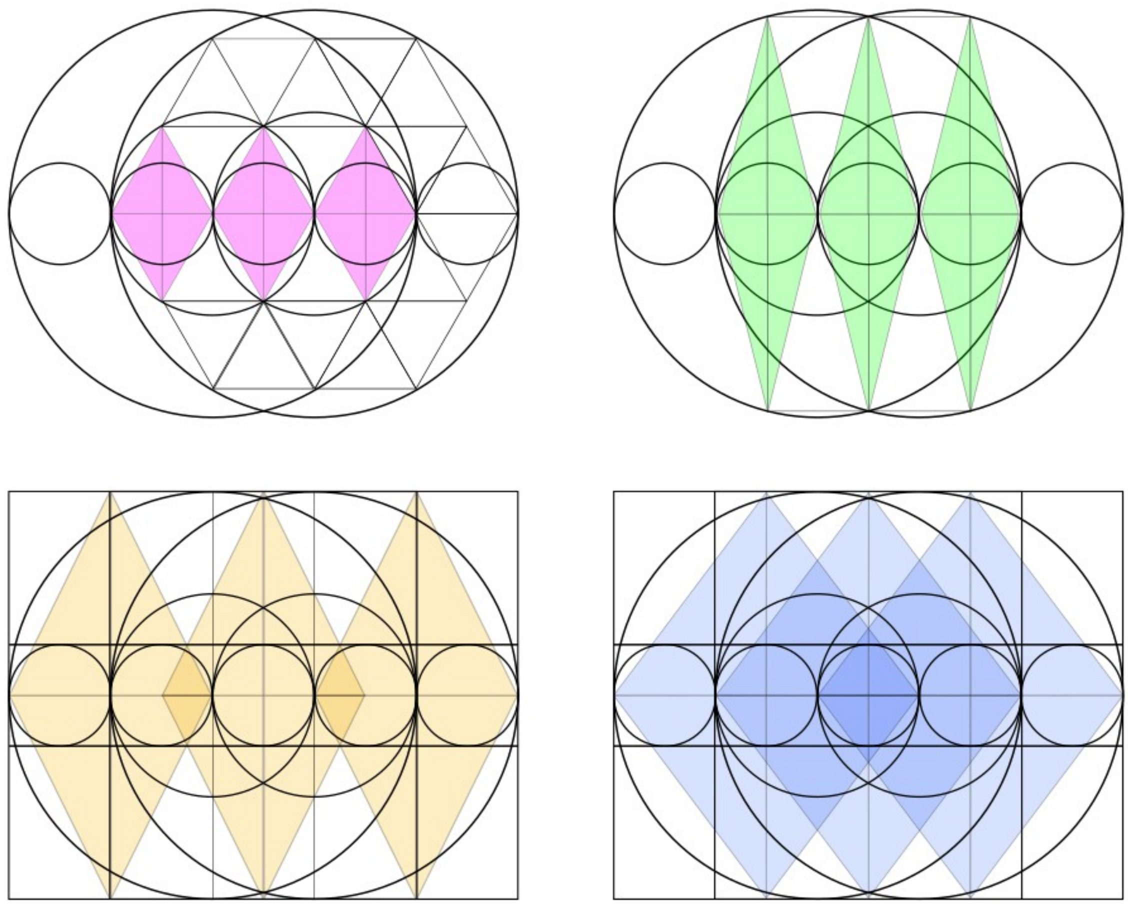


The 345 triangle is very special indeed, as shown below.

Angle (c) has a special relationship with angle (a), shown below.

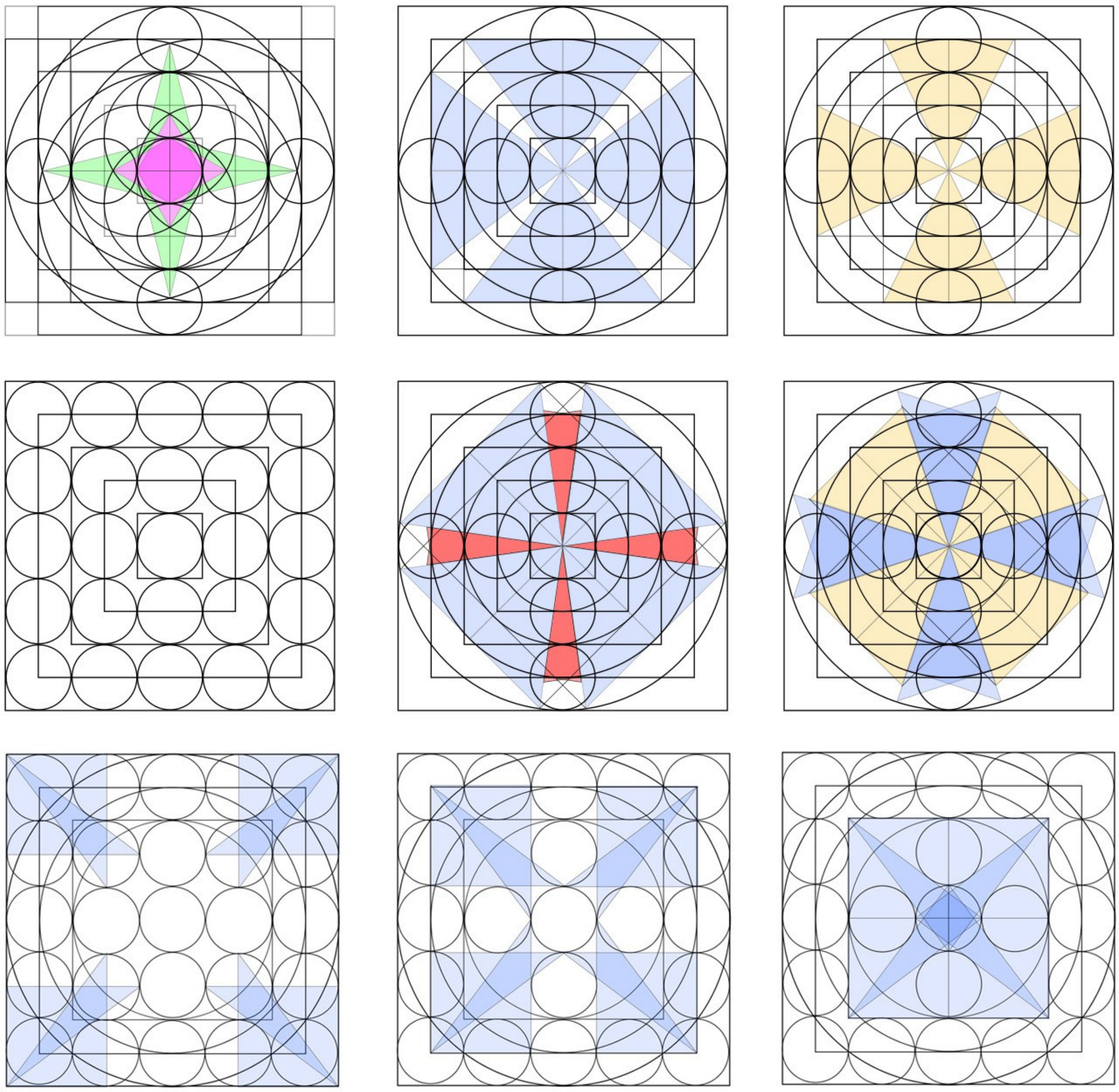


The green and pink triangles have a **phi** relationship, the pink and yellow triangles have a **square-root** relationship with the blue.

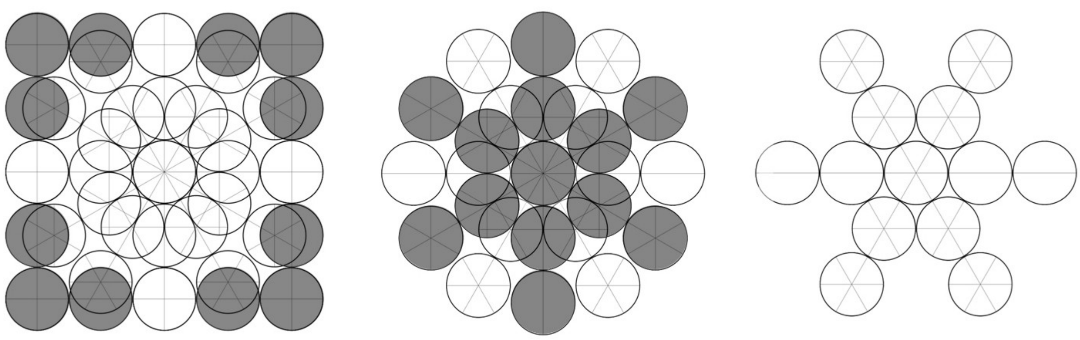


A picture speaks a thousand words. Silence, an infinity.

Nested **squares** and **circles** with triangles, as shown below.



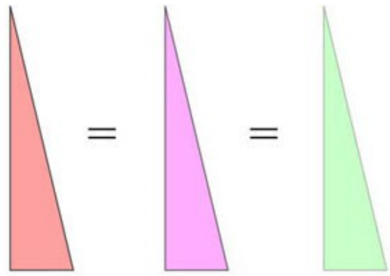
The geometric link between the **fruit of life** and the **square**.



Angle (d) is fascinating for many reasons, the red and green triangles are <u>equal</u>, hence the different colour.

$$x + 1/x = y = z + 1/z$$

$$x - 1/x = 1/y = z - 1/z$$



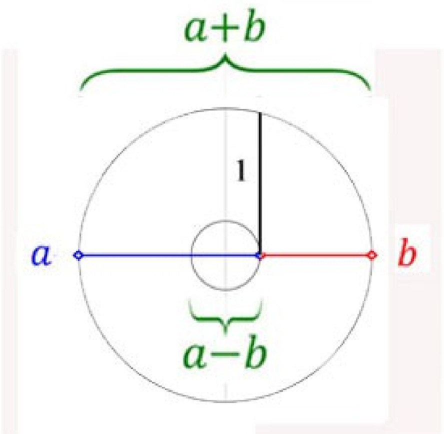
This is because cos and tan are <u>equal</u>, as shown below.

$$\cos = \tan \quad \therefore \quad 1 = \sqrt{\frac{\tan}{\cos}} = \frac{\tan}{\cos}$$

And so uniquely a <u>and</u> b are reciprocals, therefore.

$$(a+b)^2 = a^2 + b^2 + 2$$
$$(a-b)^2 = a^2 + b^2 - 2$$

This is very special indeed and means that using this method, <u>only</u> this angle gives us the geometry of a **circle**, as shown below.



$$x = a = r + 1/4r = \sqrt{1+\tan^2} = 1/\cos$$
$$1/x = b = r - 1/4r = \tan$$
$$a^2 = (r^2 + 1/16r^2) + 1/2 = 1 + \tan^2$$
$$b^2 = (r^2 + 1/16r^2) - 1/2 = \tan^2$$

$$\text{Diameter, d} = a+b = y$$
$$1/d = a-b = 1/y$$

Angle (d) has a very special unique relationship with angle (c).

	Angle (c)	Angle (d)
$1/\cos(\theta) = a$	$\frac{5}{4} = 1.25$	$\frac{\Phi^{1/2}}{1} = 1.2720..$
$\tan(\theta) = b$	$\frac{3}{4} = 0.75$	$\frac{1}{\Phi^{1/2}} = 0.7861..$
a^2	$(\frac{5}{4})^2 = 1.5625$	$\frac{\Phi}{1} = 1.6180..$
b^2	$(\frac{3}{4})^2 = 0.5625$	$\frac{1}{\Phi} = 0.6180..$
$\sqrt{a}$	$(\frac{5}{4})^{1/2} = 1.1180..$	$\frac{\Phi^{1/4}}{1} = 1.1278..$
$\sqrt{b}$	$(\frac{3}{4})^{1/2} = 0.8660..$	$\frac{1}{\Phi^{1/4}} = 0.8866..$
$a-b$	$\frac{1}{2} = 0.5$	$\frac{1}{\Phi^{3/2}} = 0.4858..$
$a+b$	$\frac{2}{1} = 2$	$\frac{\Phi^{3/2}}{1} = 2.0581..$
$(a-b)^2$	$\frac{1}{2^2} = 0.25$	$\frac{1}{\Phi^3} = 0.2360..$
$(a+b)^2$	$\frac{2^2}{1} = 4$	$\frac{\Phi^3}{1} = 4.2360..$
$\sqrt{(a-b)}$	$\frac{1}{2^{1/2}} = 0.7071..$	$\frac{1}{\Phi^{3/4}} = 0.6970..$
$\sqrt{(a+b)}$	$\frac{2^{1/2}}{1} = 1.4142..$	$\frac{\Phi^{3/4}}{1} = 1.4346..$

<u>All</u> decimals used in this book can be written as fractions.

$4 \times (a-b)^2$	$\frac{1}{1} = 1$	$\frac{4}{\Phi^3} = 0.9442..$
$1/4 \times (a+b)^2$	$\frac{1}{1} = 1$	$\frac{\Phi^3}{4} = 1.0590..$
$2 \times (a-b)$	$\frac{1}{1} = 1$	$\frac{2}{\Phi^{3/2}} = 0.9717..$
$1/2 \times (a+b)$	$\frac{1}{1} = 1$	$\frac{\Phi^{3/2}}{2} = 1.0290..$
$\sqrt{2} \times \sqrt{(a-b)}$	$\frac{1}{1} = 1$	$\frac{2^{1/2}}{\Phi^{3/4}} = 0.9857..$
$1/\sqrt{2} \times \sqrt{(a+b)}$	$\frac{1}{1} = 1$	$\frac{\Phi^{3/4}}{2^{1/2}} = 1.0144..$