# Elementary Algebra

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**C O N N E X I O N S**

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# Preface[1]

Elementary Algebra is a work text that covers the traditional topics studied in a modern elementary algebra course. It is intended for students who:

1. Have no exposure to elementary algebra,
2. Have had a previously unpleasant experience with elementary algebra, or
3. Need to review algebraic concepts and techniques.

Use of this book will help the student develop the insight and intuition necessary to master algebraic techniques and manipulative skills. The text is written to promote problem-solving ability so that the student has the maximum opportunity to see that the concepts and techniques are logically based and to be comfortable enough with these concepts to know when and how to use them in subsequent sections, courses, and non-classroom situations. Intuition and understanding are some of the keys to creativity; we believe that the material presented will help make these keys available to the student.

This text can be used in standard lecture or self-paced classes. To help students meet these objectives and to make the study of algebra a pleasant and rewarding experience, Elementary Algebra is organized as follows.

## Pedagogical Features

The work text format gives the student space to practice algebraic skills with ready reference to sample problems. The chapters are divided into sections, and each section is a complete treatment of a particular topic, which includes the following features:

- **Section Overview**
- **Sample Sets**
- **Practice Sets**
- **Section Exercises**
- **Exercises for Review**

The chapters begin with **Objectives** and end with a **Summary of Key Concepts**, an **Exercise Supplement**, and a **Proficiency Exam**.

**Objectives**

Each chapter begins with a set of objectives identifying the material to be covered. Each section begins with an overview that repeats the objectives for that particular section. Sections are divided into subsections that correspond to the section objectives, which makes for easier reading.

**Sample Sets**

Elementary Algebra contains examples that are set off in boxes for easy reference. The examples are referred to as Sample Sets for two reasons:

[1] This content is available online at <http://cnx.org/content/m18873/1.4/>.

1. They serve as a representation to be imitated, which we believe will foster understanding of algebra concepts and provide experience with algebraic techniques.
2. Sample Sets also serve as a preliminary representation of problem-solving techniques that may be used to solve more general and more complicated problems. The examples have been carefully chosen to illustrate and develop concepts and techniques in the most instructive, easily remembered way. Concepts and techniques preceding the examples are introduced at a level below that normally used in similar texts and are thoroughly explained, assuming little previous knowledge.

**Practice Set**
A parallel Practice Set follows each Sample Set, which reinforces the concepts just learned. The answers to all Practice Sets are displayed with the question when viewing this content online, or at the end of the chapter in the print version.
**Section Exercises**
The exercises at the end of each section are graded in terms of difficulty, although they are not grouped into categories. There are an ample number of problems; after working through the exercises, the student will be capable of solving a variety of challenging problems.

The problems are paired so that the odd-numbered problems are equivalent in kind and difficulty to the even-numbered problems. Answers to the odd-numbered problems are provided with the exercise when viewed online, or at the back of the chapter in the print version.
**Exercises for Review**
This section consists of problems that form a cumulative review of the material covered in the preceding sections of the text and is not limited to material in that chapter. The exercises are keyed by section for easy reference.
**Summary of Key Concepts**
A summary of the important ideas and formulas used throughout the chapter is included at the end of each chapter. More than just a list of terms, the summary is a valuable tool that reinforces concepts in preparation for the Proficiency Exam at the end of the chapter, as well as future exams. The summary keys each item to the section of the text where it is discussed.
**Exercise Supplement**
In addition to numerous section exercises, each chapter includes approximately 100 supplemental problems, which are referenced by section. Answers to the odd-numbered problems are included with the problems when viewed online and in the back of the chapter in the print version.
**Proficiency Exam**
Each chapter ends with a Proficiency Exam that can serve as a chapter review or a chapter evaluation. The proficiency Exam is keyed to sections, which enables the student to refer back to the text for assistance. Answers to all Proficiency Exam problems are included with the exercises when viewed online, or in the back of the chapter in the print version.

# Content

The writing style is informal and friendly, offering a no-nonsense, straightforward approach to algebra. We have made a deliberate effort not to write another text that minimizes the use of words because we believe that students can be study algebraic concepts and understand algebraic techniques by using words **and** symbols rather than symbols alone. It has been our experience that students at the elementary level are not experienced enough with mathematics to understand symbolic explanations alone; they also need to read the explanation.

We have taken great care to present concepts and techniques so they are understandable and easily remembered. After concepts have been developed, students are warned about common pitfalls.

## Arithmetic Review

This chapter contains many examples of arithmetic techniques that are used directly or indirectly in algebra. Since the chapter is intended as a review, the problem-solving techniques are presented without being developed. Therefore, no work space is provided, nor does the chapter contain all of the pedagogical features of the text. As a review, this chapter can be assigned at the discretion of the instructor and can also be a valuable reference tool for the student.

## Basic Properties of Real Numbers

The symbols, notations, and properties of numbers that form the basis of algebra, as well as exponents and the rules of exponents, are introduced in Basic Properties of Real Numbers. Each property of real numbers and the rules of exponents are expressed both symbolically and literally. Literal explanations are included because symbolic explanations alone may be difficult for a student to interpret.

## Basic Operations with Real Numbers

The basic operations with real numbers are presented in this chapter. The concept of absolute value is discussed both geometrically and symbolically. The geometric presentation offers a visual understanding of the meaning of $| x |$. The symbolic presentation includes a literal explanation of how to use the definition. Negative exponents are developed, using reciprocals and the rules of exponents the student has already learned. Scientific notation is also included, using unique and real-life examples.

## Algebraic Expressions and Equations

Operations with algebraic expressions and numerical evaluations are introduced in Algebraic Expressions and Equations. Coefficients are described rather than merely defined. Special binomial products have both literal symbolic explanation and since they occur so frequently in mathematics, we have been careful to help the student remember them. In each example problem, the student is "talked" through the symbolic form.

## Solving Linear Equations and Inequalities

In this chapter, the emphasis is on the mechanics of equation solving, which clearly explains how to isolate a variable. The goal is to help the student feel more comfortable with solving applied problems. Ample opportunity is provided for the student to practice translating words to symbols, which is an important part of the "Five-Step Method" of solving applied problems (discussed in Section 5.6 and Section 5.7).

## Factoring Polynomials

Factoring is an essential skill for success in algebra and higher level mathematics courses. Therefore, we have taken great care in developing the student's understanding of the factorization process. The technique is consistently illustrated by displaying an empty set of parentheses and describing the thought process used to discover the terms that are to be placed inside the parentheses.

The factoring scheme for special products is presented with both verbal and symbolic descriptions, since not all students can interpret symbolic descriptions alone. Two techniques, the standard "trial and error" method, and the "collect and discard" method (a method similar to the "ac" method), are presented for factoring trinomials with leading coefficients different from 1.

## Graphing Linear Equations and Inequalities in One and Two Variables

In this chapter the student is shown how graphs provide information that is not always evident from the equation alone. The chapter begins by establishing the relationship between the variables in an equation, the number of coordinate axes necessary to construct the graph, and the spatial dimension of both the

coordinate system and the graph. Interpretation of graphs is also emphasized throughout the chapter, beginning with the plotting of points. The slope formula is fully developed, progressing from verbal phrases to mathematical expressions. The expressions are then formed into an equation by explicitly stating that a ratio is a comparison of two quantities of the same type (e.g., distance, weight, or money). This approach benefits students who take future courses that use graphs to display information.

The student is shown how to graph lines using the intercept method, the table method, and the slope-intercept method, as well as how to distinguish, by inspection, oblique and horizontal/vertical lines.

## Rational Expressions

A detailed study of arithmetic operations with rational expressions is presented in this chapter, beginning with the definition of a rational expression and then proceeding immediately to a discussion of the domain. The process of reducing a rational expression and illustrations of multiplying, dividing, adding, and subtracting rational expressions are also included. Since the operations of addition and subtraction can cause the most difficulty, they are given particular attention. We have tried to make the written explanation of the examples clearer by using a "freeze frame approach.

The five-step method of solving applied problems is included in this chapter to show the problem-solving approach to number problems, work problems, and geometry problems. The chapter also illustrates simplification of complex rational expressions, using the combine-divide method and the LCD-multiply-divide method.

## Roots, Radicals, and Square Root Equations

The distinction between the principal square root of the number $x$, $\sqrt{x}$, and the secondary square root of the number $x$, $\sqrt{x}$, is made by explanation and by example. The simplification of radical expressions that both involve and do not involve fractions is shown in many detailed examples; this is followed by an explanation of how and why radicals are eliminated from the denominator of a radical expression. Real-life applications of radical equations have been included, such as problems involving daily output, daily sales, electronic resonance frequency, and kinetic energy.

## Quadratic Equations

Methods of solving quadratic equations as well as the logic underlying each method are discussed. Factoring, extraction of roots, completing the square, and the quadratic formula are carefully developed. The zero-factor property of real numbers is reintroduced. The chapter also includes graphs of quadratic equations based on the standard parabola, $y = x^2$, and applied problems from the areas of manufacturing, population, physics, geometry, mathematics (number and volumes), and astronomy, which are solved using the five-step method.

## Systems of Linear Equations

Beginning with the graphical solution of systems, this chapter includes an interpretation of independent, inconsistent, and dependent systems and examples to illustrate the applications for these systems. The substitution method and the addition method of solving a system by elimination are explained, noting when to use each method. The five-step method is again used to illustrate the solutions of value and rate problems (coin and mixture problems), using drawings that correspond to the actual solution.

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[2]This content is available online at <http://cnx.org/content/m18871/1.5/>.

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**D.B.**

*À Sandi*
*C'est pour toi, l'étoile au centre de mon univers.*

# Chapter 1

# Arithmetic Review

## 1.1 Objectives[1]

This chapter contains many examples of arithmetic techniques that are used directly or indirectly in algebra. Since the chapter is intended as a review, the problem-solving techniques are presented without being developed. If you would like a quick review of arithmetic before attempting the study of algebra, this chapter is recommended reading. If you feel your arithmetic skills are pretty good, then move on to Basic Properties of Real Numbers (Section 2.1). However you feel, do not hesitate to use this chapter as a **quick reference of arithmetic techniques**.

The other chapters include Practice Sets paired with Sample Sets with sufficient space for the student to work out the problems. In addition, these chapters include a Summary of Key Concepts, Exercise Supplements, and Proficiency Exams.

## 1.2 Factors, Products, and Exponents [2]

### 1.2.1 Overview

- Factors
- Exponential Notation

### 1.2.2 Factors

Let's begin our review of arithmetic by recalling the meaning of multiplication for whole numbers (the counting numbers and zero).

**Multiplication**

**Multiplication** is a description of repeated addition.

In the addition

$7+7+7+7$

the number 7 is repeated as an **addend*** 4 **times.** Therefore, we say we have **four times seven** and describe it by writing

$4 \cdot 7$

[1]This content is available online at <http://cnx.org/content/m22784/1.4/>.

[2]This content is available online at <http://cnx.org/content/m18882/1.5/>.

The raised dot between the numbers 4 and 7 indicates multiplication. The dot directs us to multiply the two numbers that it separates. In algebra, the dot is preferred over the symbol $\times$ to denote multiplication because the letter $x$ is often used to represent a number. Thus,

$4 \cdot 7 = 7 + 7 + 7 + 7$

**Factors and Products**

In a multiplication, the numbers being multiplied are called **factors.** The result of a multiplication is called the **product.** For example, in the multiplication

$4 \cdot 7 = 28$

the numbers 4 and 7 are factors, and the number 28 is the product. We say that 4 and 7 are factors of 28. (They are not the only factors of 28. Can you think of others?)

Now we know that

$$(\text{factor}) \cdot (\text{factor}) = \text{product}$$

This indicates that a first number is a factor of a second number if the first number divides into the second number with no remainder. For example, since

$4 \cdot 7 = 28$

both 4 and 7 are factors of 28 since both 4 and 7 divide into 28 with no remainder.

### 1.2.3 Exponential Notation

Quite often, a particular number will be repeated as a factor in a multiplication. For example, in the multiplication

$7 \cdot 7 \cdot 7 \cdot 7$

the number 7 is repeated as a factor 4 times. We describe this by writing $7^4$. Thus,

$7 \cdot 7 \cdot 7 \cdot 7 = 7^4$

The repeated factor is the lower number (the base), and the number recording how many times the factor is repeated is the higher number (the superscript). The superscript number is called an **exponent.**

**Exponent**

An **exponent** is a number that records how many times the number to which it is attached occurs as a factor in a multiplication.

### 1.2.4 Sample Set A

For Examples 1, 2, and 3, express each product using exponents.

**Example 1.1**

$3 \cdot 3 \cdot 3 \cdot 3 \cdot 3 \cdot 3$. Since 3 occurs as a factor 6 times,

$3 \cdot 3 \cdot 3 \cdot 3 \cdot 3 \cdot 3 = 3^6$

**Example 1.2**
$8 \cdot 8$. Since 8 occurs as a factor 2 times,

$$8 \cdot 8 = 8^2$$

**Example 1.3**
$5 \cdot 5 \cdot 5 \cdot 9 \cdot 9$. Since 5 occurs as a factor 3 times, we have $5^3$. Since 9 occurs as a factor 2 times, we have $9^2$. We should see the following replacements.

$$\underbrace{5 \cdot 5 \cdot 5}_{5^3} \cdot \underbrace{9 \cdot 9}_{9^2}$$

Then we have

$$5 \cdot 5 \cdot 5 \cdot 9 \cdot 9 = 5^3 \cdot 9^2$$

**Example 1.4**
Expand $3^5$. The base is 3 so it is the repeated factor. The exponent is 5 and it records the number of times the base 3 is repeated. Thus, 3 is to be repeated as a factor 5 times.

$$3^5 = 3 \cdot 3 \cdot 3 \cdot 3 \cdot 3$$

**Example 1.5**
Expand $6^2 \cdot 10^4$. The notation $6^2 \cdot 10^4$ records the following two facts: 6 is to be repeated as a factor 2 times and 10 is to be repeated as a factor 4 times. Thus,

$$6^2 \cdot 10^4 = 6 \cdot 6 \cdot 10 \cdot 10 \cdot 10 \cdot 10$$