

Derived copy of Fundamentals of Mathematics

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C O N N E X I O N S

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Preface¹

To the next generation of explorers: Kristi, BreAnne, Lindsey, Randi, Piper, Meghan, Wyatt, Lara, Mason, and Sheanna.

Fundamentals of Mathematics is a work text that covers the traditional topics studied in a modern prealgebra course, as well as the topics of estimation, elementary analytic geometry, and introductory algebra. It is intended for students who

1. have had a previous course in prealgebra,
2. wish to meet the prerequisite of a higher level course such as elementary algebra, and
3. need to review fundamental mathematical concepts and techniques.

This text will help the student develop the insight and intuition necessary to master arithmetic techniques and manipulative skills. It was written with the following main objectives:

1. to provide the student with an understandable and usable source of information,
2. to provide the student with the maximum opportunity to see that arithmetic concepts and techniques are logically based,
3. to instill in the student the understanding and intuitive skills necessary to know how and when to use particular arithmetic concepts in subsequent material, courses, and nonclassroom situations, and
4. to give the student the ability to correctly interpret arithmetically obtained results.

We have tried to meet these objectives by presenting material dynamically, much the way an instructor might present the material visually in a classroom. (See the development of the concept of addition and subtraction of fractions in here², for example.) Intuition and understanding are some of the keys to creative thinking; we believe that the material presented in this text will help the student realize that mathematics is a creative subject.

This text can be used in standard lecture or self-paced classes. To help meet our objectives and to make the study of prealgebra a pleasant and rewarding experience, *Fundamentals of Mathematics* is organized as follows.

Pedagogical Features

The work text format gives the student space to practice mathematical skills with ready reference to sample problems. The chapters are divided into sections, and each section is a complete treatment of a particular topic, which includes the following features:

- **Section Overview**

¹This content is available online at <<http://cnx.org/content/m18884/1.4/>>.

²"Addition and Subtraction of Fractions, Comparing Fractions, and Complex Fractions: Addition and Subtraction of Fractions with Unlike Denominators" <<http://cnx.org/content/m34935/latest/>>

- **Sample Sets**
- **Practice Sets**
- **Section Exercises**
- **Exercises for Review**
- **Answers to Practice Sets**

The chapters begin with **Objectives** and end with a **Summary of Key Concepts**, an **Exercise Supplement**, and a **Proficiency Exam**.

Objectives

Each chapter begins with a set of objectives identifying the material to be covered. Each section begins with an overview that repeats the objectives for that particular section. Sections are divided into subsections that correspond to the section objectives, which makes for easier reading.

Sample Sets

Fundamentals of Mathematics contains examples that are set off in boxes for easy reference. The examples are referred to as Sample Sets for two reasons:

1. They serve as a representation to be imitated, which we believe will foster understanding of mathematical concepts and provide experience with mathematical techniques.
2. Sample Sets also serve as a preliminary representation of problem-solving techniques that may be used to solve more general and more complicated problems.

The examples have been carefully chosen to illustrate and develop concepts and techniques in the most instructive, easily remembered way. Concepts and techniques preceding the examples are introduced at a level below that normally used in similar texts and are thoroughly explained, assuming little previous knowledge.

Practice Sets

A parallel Practice Set follows each Sample Set, which reinforces the concepts just learned. There is adequate space for the student to work each problem directly on the page.

Answers to Practice Sets

The Answers to Practice Sets are given at the end of each section and can be easily located by referring to the page number, which appears after the last Practice Set in each section.

Section Exercises

The exercises at the end of each section are graded in terms of difficulty, although they are not grouped into categories. There is an ample number of problems, and after working through the exercises, the student will be capable of solving a variety of challenging problems.

The problems are paired so that the odd-numbered problems are equivalent in kind and difficulty to the even-numbered problems. Answers to the odd-numbered problems are provided at the back of the book.

Exercises for Review

This section consists of five problems that form a cumulative review of the material covered in the preceding sections of the text and is not limited to material in that chapter. The exercises are keyed by section for easy reference. Since these exercises are intended for review only, no work space is provided.

Summary of Key Concepts

A summary of the important ideas and formulas used throughout the chapter is included at the end of each chapter. More than just a list of terms, the summary is a valuable tool that reinforces concepts in preparation for the Proficiency Exam at the end of the chapter, as well as future exams. The summary keys each item to the section of the text where it is discussed.

Exercise Supplement

In addition to numerous section exercises, each chapter includes approximately 100 supplemental problems, which are referenced by section. Answers to the odd-numbered problems are included in the back of the book.

Proficiency Exam

Each chapter ends with a Proficiency Exam that can serve as a chapter review or evaluation. The Proficiency Exam is keyed to sections, which enables the student to refer back to the text for assistance. Answers to all the problems are included in the Answer Section at the end of the book.

Content

The writing style used in Fundamentals of Mathematics is informal and friendly, offering a straightforward approach to prealgebra mathematics. We have made a deliberate effort not to write another text that minimizes the use of words because we believe that students can best study arithmetic concepts and understand arithmetic techniques by using words and symbols rather than symbols alone. It has been our experience that students at the prealgebra level are not nearly experienced enough with mathematics to understand symbolic explanations alone; they need literal explanations to guide them through the symbols.

We have taken great care to present concepts and techniques so they are understandable and easily remembered. After concepts have been developed, students are warned about common pitfalls. We have tried to make the text an information source accessible to prealgebra students.

Addition and Subtraction of Whole Numbers

This chapter includes the study of whole numbers, including a discussion of the Hindu-Arabic numeration and the base ten number systems. Rounding whole numbers is also presented, as are the commutative and associative properties of addition.

Multiplication and Division of Whole Numbers

The operations of multiplication and division of whole numbers are explained in this chapter. Multiplication is described as repeated addition. Viewing multiplication in this way may provide students with a visualization of the meaning of algebraic terms such as $8x$ when they start learning algebra. The chapter also includes the commutative and associative properties of multiplication.

Exponents, Roots, and Factorizations of Whole Numbers

The concept and meaning of the word root is introduced in this chapter. A method of reading root notation and a method of determining some common roots, both mentally and by calculator, is then presented. We also present grouping symbols and the order of operations, prime factorization of whole numbers, and the greatest common factor and least common multiple of a collection of whole numbers.

Introduction to Fractions and Multiplication and Division of Fractions

We recognize that fractions constitute one of the foundations of problem solving. We have, therefore, given a detailed treatment of the operations of multiplication and division of fractions and the logic behind these operations. We believe that the logical treatment and many practice exercises will help students retain the information presented in this chapter and enable them to use it as a foundation for the study of rational expressions in an algebra course.

Addition and Subtraction of Fractions, Comparing Fractions, and Complex Fractions

A detailed treatment of the operations of addition and subtraction of fractions and the logic behind these operations is given in this chapter. Again, we believe that the logical treatment and many practice exercises will help students retain the information, thus enabling them to use it in the study of rational expressions in an algebra course. We have tried to make explanations dynamic. A method for comparing fractions is introduced, which gives the student another way of understanding the relationship between the words *denominator* and *denomination*. This method serves to show the student that it is sometimes possible to compare two different types of quantities. We also study a method of simplifying complex fractions and of combining operations with fractions.

Decimals

The student is introduced to decimals in terms of the base ten number system, fractions, and digits occurring to the right of the units position. A method of converting a fraction to a decimal is discussed. The logic behind the standard methods of operating on decimals is presented and many examples of how to apply the methods are given. The word of as related to the operation of multiplication is discussed. Nonterminating divisions are examined, as are combinations of operations with decimals and fractions.

Ratios and Rates

We begin by defining and distinguishing the terms *ratio* and *rate*. The meaning of proportion and some applications of proportion problems are described. Proportion problems are solved using the "Five-Step Method." We hope that by using this method the student will discover the value of introducing a variable as a first step in problem solving and the power of organization. The chapter concludes with discussions of percent, fractions of one percent, and some applications of percent.

Techniques of Estimation

One of the most powerful problem-solving tools is a knowledge of estimation techniques. We feel that estimation is so important that we devote an entire chapter to its study. We examine three estimation techniques: estimation by rounding, estimation by clustering, and estimation by rounding fractions. We also include a section on the distributive property, an important algebraic property.

Measurement and Geometry

This chapter presents some of the techniques of measurement in both the United States system and the metric system. Conversion from one unit to another (in a system) is examined in terms of unit fractions. A discussion of the simplification of denominate numbers is also included. This discussion helps the student understand more clearly the association between pure numbers and dimensions. The chapter concludes with a study of perimeter and circumference of geometric figures and area and volume of geometric figures and objects.

Signed Numbers

A look at algebraic concepts and techniques is begun in this chapter. Basic to the study of algebra is a working knowledge of signed numbers. Definitions of variables, constants, and real numbers are introduced. We then distinguish between positive and negative numbers, learn how to read signed numbers, and examine the origin and use of the double-negative property of real numbers. The concept of absolute value is presented both geometrically (using the number line) and algebraically. The algebraic definition is followed by an interpretation of its meaning and several detailed examples of its use. Addition, subtraction, multiplication, and division of signed numbers are presented first using the number line, then with absolute value.

Algebraic Expressions and Equations

The student is introduced to some elementary algebraic concepts and techniques in this final chapter. Algebraic expressions and the process of combining like terms are discussed in here³ and here⁴. The method of combining like terms in an algebraic expression is explained by using the interpretation of multiplication as a description of repeated addition (as in here⁵).

³"Algebraic Expressions and Equations: Algebraic Expressions" <<http://cnx.org/content/m35038/latest/>>

⁴"Algebraic Expressions and Equations: Combining Like Terms Using Addition and Subtraction"
<<http://cnx.org/content/m35039/latest/>>

⁵"Multiplication and Division of Whole Numbers: Objectives" <<http://cnx.org/content/m18888/latest/>>

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⁶This content is available online at <<http://cnx.org/content/m34775/1.2/>>.

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D.B.

Chapter 1

Measurement and Geometry

1.1 Objectives¹

After completing this chapter, you should

Measurement and the United States System (Section 1.2)

- know what the word measurement means
- be familiar with United States system of measurement
- be able to convert from one unit of measure in the United States system to another unit of measure

The Metric System of Measurement (Section 1.3)

- be more familiar with some of the advantages of the base ten number system
- know the prefixes of the metric measures
- be familiar with the metric system of measurement
- be able to convert from one unit of measure in the metric system to another unit of measure

Simplification of Denominate Numbers (Section 1.4)

- be able to convert an unsimplified unit of measure to a simplified unit of measure
- be able to add and subtract denominate numbers
- be able to multiply and divide a denominate number by a whole number

Perimeter and Circumference of Geometric Figures (Section 1.5)

- know what a polygon is
- know what perimeter is and how to find it
- know what the circumference, diameter, and radius of a circle is and how to find each one
- know the meaning of the symbol π and its approximating value
- know what a formula is and four versions of the circumference formula of a circle

Area and Volume of Geometric Figures and Objects (Section 1.6)

- know the meaning and notation for area
- know the area formulas for some common geometric figures
- be able to find the areas of some common geometric figures
- know the meaning and notation for volume
- know the volume formulas for some common geometric objects
- be able to find the volume of some common geometric objects

¹This content is available online at <http://cnx.org/content/m18897/1.3/>.

1.2 Measurement and the United States System²

1.2.1 Section Overview

- Measurement
- The United States System of Measurement
- Conversions in the United States System

1.2.2 Measurement

There are two major systems of measurement in use today. They are the *United States system* and the *metric system*. Before we describe these systems, let's gain a clear understanding of the concept of measurement.

Measurement

Measurement is comparison to some standard.

Standard Unit of Measure

The concept of measurement is based on the idea of direct comparison. This means that measurement is the result of the comparison of two quantities. The quantity that is used for comparison is called the **standard unit of measure**.

Over the years, standards have changed. Quite some time in the past, the standard unit of measure was determined by a king. For example,

1 inch was the distance between the tip of the thumb and the knuckle of the king.

1 inch was also the length of 16 barley grains placed end to end.

Today, standard units of measure rarely change. Standard units of measure are the responsibility of the Bureau of Standards in Washington D.C.

Some desirable properties of a standard are the following:

1. *Accessibility*. We should have access to the standard so we can make comparisons.
2. *Invariance*. We should be confident that the standard is not subject to change.
3. *Reproducibility*. We should be able to reproduce the standard so that measurements are convenient and accessible to many people.

1.2.3 The United States System of Measurement

Some of the common units (along with their abbreviations) for the United States system of measurement are listed in the following table.

Unit Conversion Table	
Length	1 foot (ft) = 12 inches (in.) 1 yard (yd) = 3 feet (ft) 1 mile (mi) = 5,280 feet
<i>continued on next page</i>	

²This content is available online at <<http://cnx.org/content/m35018/1.2/>>.

Weight	1 pound (lb) = 16 ounces (oz) 1 ton (T) = 2,000 pounds
Liquid Volume	1 tablespoon (tbsp) = 3 teaspoons (tsp) 1 fluid ounce (fl oz) = 2 tablespoons 1 cup (c) = 8 fluid ounces 1 pint (pt) = 2 cups 1 quart (qt) = 2 pints 1 gallon (gal) = 4 quarts
Time	1 minute (min) = 60 seconds (sec) 1 hour (hr) = 60 minutes 1 day (da) = 24 hours 1 week (wk) = 7 days

Table 1.1

1.2.4 Conversions in the United States System

It is often convenient or necessary to convert from one unit of measure to another. For example, it may be convenient to convert a measurement of length that is given in feet to one that is given in inches. Such conversions can be made using *unit fractions*.

Unit Fraction

A **unit fraction** is a fraction with a value of 1.

Unit fractions are formed by using two equal measurements. One measurement is placed in the numerator of the fraction, and the other in the denominator. **Placement depends on the desired conversion.**

Placement of Units

Place the unit being converted *to* in the **numerator**.

Place the unit being converted *from* in the **denominator**.

For example,

Equal Measurements	Unit Fraction
1ft = 12in.	$\frac{1\text{ft}}{12\text{in.}}$ or $\frac{12\text{in.}}{1\text{ft}}$
1pt = 16 fl oz	$\frac{1\text{pt}}{16\text{ fl oz}}$ or $\frac{16\text{ fl oz}}{1\text{pt}}$
1wk = 7da	$\frac{7\text{da}}{1\text{wk}}$ or $\frac{1\text{wk}}{7\text{da}}$

Table 1.2

1.2.4.1 Sample Set A

Make the following conversions. If a fraction occurs, convert it to a decimal rounded to two decimal places.

Example 1.1

Convert 11 yards to feet.

Looking in the unit conversion table under *length*, we see that $1\text{ yd} = 3\text{ ft}$. There are two corresponding unit fractions, $\frac{1\text{ yd}}{3\text{ ft}}$ and $\frac{3\text{ ft}}{1\text{ yd}}$. Which one should we use? Look to see which unit we wish to convert to. Choose the unit fraction with this unit in the *numerator*. We will choose $\frac{3\text{ ft}}{1\text{ yd}}$ since

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