Table of Contents

  - 1.
- Chapter 1. Background, Review, and Reference
  - 1.1. Discrete-Time Signals and Systems
    - Real- and Complex-valued Signals
    - Complex Exponentials
    - Sinusoids
    - Unit Sample
    - Unit Step
    - Symbolic Signals
    - Discrete-Time Systems
  - 1.2. Systems in the Time-Domain
  - 1.3. Discrete Time Convolution
    - Introduction
    - Convolution and Circular Convolution
      - Convolution
        - Operation Definition
        - Definition Motivation
        - Graphical Intuition
      - Circular Convolution
        - Definition Motivation
        - Graphical Intuition
    - Interactive Element
    - Convolution Summary
  - 1.4. Introduction to Fourier Analysis
    - Fourier's Daring Leap
  - 1.5. Continuous Time Fourier Transform (CTFT)
    - Introduction
    - Fourier Transform Synthesis
      - Equations
      - CTFT Definition Demonstration
    - Example Problems
    - Fourier Transform Summary
  - 1.6. Discrete-Time Fourier Transform (DTFT)
  - 1.7. DFT as a Matrix Operation
    - Matrix Review
1.8. Sampling theory

- Introduction
  - Why sample?
  - Claude E. Shannon
  - Notation
  - The Sampling Theorem

- Proof
  - Introduction
  - Proof part I - Spectral considerations
  - Proof part II - Signal reconstruction
  - Summary

- Illustrations
  - Basic examples
  - The process of sampling
    - Sampling fast enough
    - Sampling too slowly
  - Reconstruction
  - Conclusions

- Systems view of sampling and reconstruction
  - Ideal reconstruction system
  - Ideal system including anti-aliasing
  - Reconstruction with hold operation

- Sampling CT Signals: A Frequency Domain Perspective
  - Understanding Sampling in the Frequency Domain
    - Sampling
    - Relating x[n] to sampled x(t)

- The DFT: Frequency Domain with a Computer Analysis
  - Introduction
    - Sampling DTFT
      - Choosing M
      - Case 1
      - Case 2
  - Discrete Fourier Transform (DFT)
    - Interpretation
      - Remark 1
      - Remark 2
  - Periodicity of the DFT
  - A Sampling Perspective
Inverse DTFT of $S(\omega)$

Connections

Discrete-Time Processing of CT Signals

DT Processing of CT Signals

Analysis

Summary

Note

Application: 60Hz Noise Removal

DSP Solution

Sampling Period/Rate

Digital Filter

1.9. Z-Transform

Difference Equation

Introduction

General Formulas for the Difference Equation

Difference Equation

Conversion to Z-Transform

Conversion to Frequency Response

Example

Solving a LCCDE

Direct Method

Homogeneous Solution

Particular Solution

Indirect Method

The Z Transform: Definition

Basic Definition of the Z-Transform

The Complex Plane

Region of Convergence

Table of Common z-Transforms

Understanding Pole/Zero Plots on the Z-Plane

Introduction to Poles and Zeros of the Z-Transform

The Z-Plane

Examples of Pole/Zero Plots

Interactive Demonstration of Poles and Zeros

Applications for pole-zero plots

Stability and Control theory

Pole/Zero Plots and the Region of Convergence

Frequency Response and Pole/Zero Plots

Chapter 2. Digital Filter Design

2.1. Overview of Digital Filter Design

Perspective on FIR filtering

2.2. FIR Filter Design

Linear Phase Filters
- Restrictions on \( h(n) \) to get linear phase
- **Window Design Method**
  - \( L_2 \) optimization criterion
  - Window Design Method
- **Frequency Sampling Design Method for FIR filters**
  - Important Special Case
  - Important Special Case #2
    - Special Case 2a
  - Comments on frequency-sampled design
  - Extended frequency sample design
- **Parks-McClellan FIR Filter Design**
  - Formal Statement of the \( L-\infty \) (Minimax) Design Problem
  - Outline of \( L-\infty \) Filter Design
  - Conditions for \( L-\infty \) Optimality of a Linear-phase FIR Filter
    - Alternation Theorem
  - Optimality Conditions for Even-length Symmetric Linear-phase Filters
  - \( L-\infty \) Optimal Lowpass Filter Design Lemma
  - Computational Cost
- **2.3. IIR Filter Design**
  - **Overview of IIR Filter Design**
    - IIR Filter
    - IIR Filter Design Problem
    - Outline of IIR Filter Design Material
    - Comments on IIR Filter Design Methods
  - **Prototype Analog Filter Design**
    - Analog Filter Design
    - Traditional Filter Designs
      - Butterworth
      - Chebyshev
      - Inverse Chebyshev
      - Elliptic Function Filter (Cauer Filter)
  - **IIR Digital Filter Design via the Bilinear Transform**
    - Bilinear Transformation
    - Prewarping
  - Impulse-Invariant Design
  - Digital-to-Digital Frequency Transformations
  - Prony's Method
    - Shank's Method
  - Linear Prediction
    - Statistical Linear Prediction
- **Chapter 3. The DFT, FFT, and Practical Spectral Analysis**
  - **3.1. The Discrete Fourier Transform**
    - DFT Definition and Properties
DFT
IDFT
DFT and IDFT properties
- Periodicity
- Circular Shift
- Time Reversal
- Complex Conjugate
- Circular Convolution Property
- Multiplication Property
- Parseval's Theorem
- Symmetry

3.2. Spectrum Analysis
- Spectrum Analysis Using the Discrete Fourier Transform
  - Discrete-Time Fourier Transform
  - Discrete Fourier Transform
  - Relationships Between DFT and DTFT
    - DFT and Discrete Fourier Series
    - DFT and DTFT of finite-length data
    - DFT as a DTFT approximation
  - Relationship between continuous-time FT and DFT
  - Zero-Padding
  - Effects of Windowing
- Classical Statistical Spectral Estimation
  - Periodogram method
  - Auto-correlation-based approach
- Short Time Fourier Transform
  - Short Time Fourier Transform
    - Sampled STFT
    - Spectrogram Example
    - Effect of window length R
    - Effect of L and N
    - Effect of R and L

3.3. Fast Fourier Transform Algorithms
- Overview of Fast Fourier Transform (FFT) Algorithms
  - History of the FFT
  - Summary of FFT algorithms
- Running FFT
- Goertzel's Algorithm
  - References
- Power-of-Two FFTs
  - Power-of-two FFTs
  - Radix-2 Algorithms
    - Decimation-in-time (DIT) Radix-2 FFT
Decimation in Time

- Additional Simplification
- Radix-2 decimation-in-time FFT
- Example FFT Code

Decimation-in-Frequency (DIF) Radix-2 FFT

- Decimation in frequency
- Radix-2 decimation-in-frequency algorithm

Alternate FFT Structures

- Radix-4 FFT Algorithms
  - References
- Split-radix FFT Algorithms
  - References

Efficient FFT Algorithm and Programming Tricks

- Precompute twiddle factors
- Compiler-friendly programming
- Program in assembly language
- Special hardware
- Effective memory management
- Real-valued FFTs
- Special cases
- Higher-radix algorithms
- Fast bit-reversal
- Trade additions for multiplications
- Special butterflies
- Practical Perspective
- References

3.4. Fast Convolution

- Fast Circular Convolution
- Fast Linear Convolution
- Running Convolution
  - Overlap-Save (OLS) Method
  - Overlap-Add (OLA) Method

3.5. Chirp-z Transform

3.6. FFTs of prime length and Rader's conversion

- Rader's Conversion
  - Fact from number theory
  - Another fact from number theory
  - Rader's Conversion

- Winogrand minimum-multiply convolution and DFT algorithms
- Winogrand Fourier Transform Algorithm (WFTA)

3.7. Choosing the Best FFT Algorithm

- Choosing an FFT length
Chapter 4. Wavelets

4.1. Time Frequency Analysis and Continuous Wavelet Transform
- Why Transforms?
- Limitations of Fourier Analysis
- Time-Frequency Uncertainty Principle
- Short-time Fourier Transform
- Continuous Wavelet Transform

4.2. Hilbert Space Theory
- Hilbert Space Theory
- Vector Space
- Normed Vector Space
- Inner Product Space
- Hilbert Spaces

4.3. Discrete Wavelet Transform
- Discrete Wavelet Transform: Main Concepts
  - Main Concepts
- The Haar System as an Example of DWT
- A Hierarchy of Detail in the Haar System
- Haar Approximation at the kth Coarseness Level
- The Scaling Equation
- The Wavelet Scaling Equation
- Conditions on h[n] and g[n]
- Values of g[n] and h[n] for the Haar System
- Wavelets: A Countable Orthonormal Basis for the Space of Square-Integrable Functions
- Filterbanks Interpretation of the Discrete Wavelet Transform
- Initialization of the Wavelet Transform
- Regularity Conditions, Compact Support, and Daubechies' Wavelets
  - References
- Computing the Scaling Function: The Cascade Algorithm
- Finite-Length Sequences and the DWT Matrix
- DWT Implementation using FFTs
- DWT Applications - Choice of \( \phi(t) \)
- DWT Application - De-noising

Chapter 5. Multirate Signal Processing

5.1. Overview of Multirate Signal Processing
- Applications
- Outline of Multirate DSP material
- General Rate-Changing Procedure
5.2. Interpolation, Decimation, and Rate Changing by Integer Fractions

- Interpolation: by an integer factor L
- Decimation: sampling rate reduction (by an integer factor M)
- Rate-Changing by a Rational Fraction L/M

5.3. Efficient Multirate Filter Structures

- Interpolation
- Efficient Decimation Structures
- Efficient L/M rate changers

5.4. Filter Design for Multirate Systems

- Direct polyphase filter design

5.5. Multistage Multirate Systems

- Filter design for Multi-stage Structures
- L-infinity Tolerances on the Pass and Stopbands
- Interpolation
- Efficient Narrowband Lowpass Filtering

5.6. DFT-Based Filterbanks

- Uniform DFT Filter Banks

5.7. Quadrature Mirror Filterbanks (QMF)

5.8. M-Channel Filter Banks

- Tree-structured filter banks
- Wavelet decomposition

Chapter 6. Digital Filter Structures and Quantization Error Analysis

6.1. Filter Structures

- Filter Structures
- FIR Filter Structures
  - Transpose-form FIR filter structures
    - Cascade structures
    - Lattice Structure
- IIR Filter Structures
  - Direct-form I IIR Filter Structure
  - Direct-Form II IIR Filter Structure
  - Transpose-Form IIR Filter Structure
  - IIR Cascade Form
  - Parallel form
  - Other forms
- State-Variable Representation of Discrete-Time Systems
  - State and the State-Variable Representation
  - State-Variable Transformation
  - Transfer Function and the State-Variable Description

6.2. Fixed-Point Numbers

- Fixed-Point Number Representation
  - Two's-Complement Integer Representation
6.3. Quantization Error Analysis

Finite-Precision Error Analysis

Fundamental Assumptions in finite-precision error analysis

Assumption #1
Assumption #2

Summary of Useful Statistical Facts

Input Quantization Noise Analysis

Quantization Error in FIR Filters

Data Quantization

Direct-form Structures

Transpose-form

Coefficient Quantization

Data Quantization in IIR Filters

Roundoff noise analysis in IIR filters

IIR Coefficient Quantization Analysis

Sensitivity analysis

Solution

Quantized Pole Locations

6.4. Overflow Problems and Solutions

Limit Cycles

Large-scale limit cycles

Small-scale limit cycles

Scaling

FIR Filter Scaling

IIR Filter Scaling

References

Index

Digital signal processing (DSP) has matured in the past few decades from an obscure research discipline to a large body of practical methods with very broad application. Both practicing engineers and students specializing in signal processing need a clear exposition of the ideas and methods comprising the core signal processing "toolkit" so widely used today.

This text reflects my belief that the skilled practitioner must understand the key ideas underlying the algorithms to select, apply, debug, extend, and innovate most effectively; only with real insight can the engineer make novel use of these methods in the seemingly infinite range of new problems and applications. It also reflects my belief that the needs of the typical student and the practicing engineer have converged in recent years; as the discipline of signal processing has matured, these core topics have become less a subject of active research and more a set of tools applied in the course of other research. The modern student thus has less need for exhaustive coverage of the research literature and detailed derivations and proofs as preparation for their own research on these topics, but greater need for intuition and practical guidance in their most effective use. The majority of students eventually become practicing engineers themselves and benefit from the best preparation for their future careers.

This text both explains the principles of classical signal processing methods and describes how they are used in engineering practice. It is thus much more than a recipe book; it describes the ideas behind the algorithms, gives analyses when they enhance that understanding, and includes derivations that the practitioner may need to extend when applying these methods to new situations. Analyses or derivations that are only of research interest or that do not increase intuitive understanding are left to the references. It is also much more than a theory book; it contains more description of common applications, discussion of actual implementation issues, comments on what really works in the real world, and practical "know-how" than found in the typical academic textbook. The choice of material emphasizes those methods that have found widespread practical use; techniques that have been the subject of intense research but which are rarely used in practice (for example, RLS adaptive filter algorithms) often receive only limited coverage.

The text assumes a familiarity with basic signal processing concepts such as ideal sampling theory, continuous and discrete Fourier transforms, convolution and filtering. It evolved from a set of notes for a second signal processing course, ECE 451: Digital Signal Processing II, in Electrical and Computer Engineering at the University of Illinois at Urbana-Champaign, aimed at second-semester seniors or first-semester graduate students in signal processing. Over the years, it has been enhanced substantially to include descriptions of common applications, sometimes hard-won knowledge about what actually works and what doesn't, useful tricks, important extensions known to experienced engineers but rarely discussed in academic texts, and other relevant "know-how" to aid the real-world user. This is necessarily an ongoing process, and I continue to expand
and refine this component as my own practical knowledge and experience grows. The topics are
the core signal processing methods that are used in the majority of signal processing applications;
discrete Fourier analysis and FFTs, digital filter design, adaptive filtering, multirate signal
processing, and efficient algorithm implementation and finite-precision issues. While many of
these topics are covered at an introductory level in a first course, this text aspires to cover all of
the methods, both basic and advanced, in these areas which see widespread use in practice. I have
also attempted to make the individual modules and sections somewhat self-sufficient, so that
those who seek specific information on a single topic can quickly find what they need. Hopefully
these aspirations will eventually be achieved; in the meantime, I welcome your comments,
corrections, and feedback so that I can continue to improve this text.

As of August 2006, the majority of modules are unedited transcriptions of handwritten notes and
may contain typographical errors and insufficient descriptive text for documents unaccompanied
by an oral lecture; I hope to have all of the modules in at least presentable shape by the end of the
year.

Publication of this text in Connexions would have been impossible without the help of many
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particular to those who converted the text and equations from my original handwritten notes into
CNXML and MathML. My former and current faculty colleagues at the University of Illinois who
have taught the second DSP course over the years have had a substantial influence on the
evolution of the content, as have the students who have inspired this work and given me feedback.
I am very grateful to my teachers, mentors, colleagues, collaborators, and fellow engineers who
have taught me the art and practice of signal processing; this work is dedicated to you.
1.1. Discrete-Time Signals and Systems

Mathematically, analog signals are functions having as their independent variables continuous quantities, such as space and time. Discrete-time signals are functions defined on the integers; they are sequences. As with analog signals, we seek ways of decomposing discrete-time signals into simpler components. Because this approach leading to a better understanding of signal structure, we can exploit that structure to represent information (create ways of representing information with signals) and to extract information (retrieve the information thus represented). For symbolic-valued signals, the approach is different: We develop a common representation of all symbolic-valued signals so that we can embody the information they contain in a unified way. From an information representation perspective, the most important issue becomes, for both real-valued and symbolic-valued signals, efficiency: what is the most parsimonious and compact way to represent information so that it can be extracted later.

Real- and Complex-valued Signals

A discrete-time signal is represented symbolically as \( s(n) \), where \( n = \{ \ldots, -1, 0, 1, \ldots \} \).

![Figure 1.1. Cosine](image)

The discrete-time cosine signal is plotted as a stem plot. Can you find the formula for this signal?

We usually draw discrete-time signals as stem plots to emphasize the fact they are functions defined only on the integers. We can delay a discrete-time signal by an integer just as with analog ones. A signal delayed by \( m \) samples has the expression \( s(n-m) \).

Complex Exponentials

The most important signal is, of course, the *complex exponential sequence*.

\[ s(n) = e^{j2\pi fn} \]
Note that the frequency variable $f$ is dimensionless and that adding an integer to the frequency of the discrete-time complex exponential has no effect on the signal's value.

$$e^{i2\pi f + m\pi} = e^{i2\pi f\pi} e^{i2\pi mn} = e^{i2\pi f\pi}$$

This derivation follows because the complex exponential evaluated at an integer multiple of $2\pi$ equals one. Thus, we need only consider frequency to have a value in some unit-length interval.

**Sinusoids**

Discrete-time sinusoids have the obvious form $s(n) = A\cos(2\pi f n + \phi)$ . As opposed to analog complex exponentials and sinusoids that can have their frequencies be any real value, frequencies of their discrete-time counterparts yield unique waveforms only when $f$ lies in the interval $\left(0, \frac{1}{2}\right]$. This choice of frequency interval is arbitrary; we can also choose the frequency to lie in the interval $[0, 1)$ . How to choose a unit-length interval for a sinusoid's frequency will become evident later.

**Unit Sample**

The second-most important discrete-time signal is the **unit sample**, which is defined to be

$$\delta(n) = \begin{cases} 1 & \text{if } n = 0 \\ 0 & \text{otherwise} \end{cases}$$

![Figure 1.2. Unit sample]

Examination of a discrete-time signal's plot, like that of the cosine signal shown in Figure 1.1, reveals that all signals consist of a sequence of delayed and scaled unit samples. Because the value of a sequence at each integer $m$ is denoted by $s(m)$ and the unit sample delayed to occur at $m$ is written $\delta(n-m)$ , we can decompose any signal as a sum of unit samples delayed to the appropriate location and scaled by the signal value.

$$s(n) = \sum_{m = -\infty}^{\infty} (s(m) \delta(n - m))$$

This kind of decomposition is unique to discrete-time signals, and will prove useful subsequently.

**Unit Step**
The **unit sample** in discrete-time is well-defined at the origin, as opposed to the situation with analog signals.

\[ u(n) = \begin{cases} 1 & \text{if } n \geq 0 \\ 0 & \text{if } n < 0 \end{cases} \]

### Symbolic Signals

An interesting aspect of discrete-time signals is that their values do not need to be real numbers. We do have real-valued discrete-time signals like the sinusoid, but we also have signals that denote the sequence of characters typed on the keyboard. Such characters certainly aren't real numbers, and as a collection of possible signal values, they have little mathematical structure other than that they are members of a set. More formally, each element of the **symbolic-valued** signal \( s(n) \) takes on one of the values \( \{a_1, \ldots, a_K\} \) which comprise the **alphabet** \( A \). This technical terminology does not mean we restrict symbols to being members of the English or Greek alphabet. They could represent keyboard characters, bytes (8-bit quantities), integers that convey daily temperature. Whether controlled by software or not, discrete-time systems are ultimately constructed from digital circuits, which consist **entirely** of analog circuit elements. Furthermore, the transmission and reception of discrete-time signals, like e-mail, is accomplished with analog signals and systems. Understanding how discrete-time and analog signals and systems intertwine is perhaps the main goal of this course.

### Discrete-Time Systems

Discrete-time systems can act on discrete-time signals in ways similar to those found in analog signals and systems. Because of the role of software in discrete-time systems, many more different systems can be envisioned and "constructed" with programs than can be with analog signals. In fact, a special class of analog signals can be converted into discrete-time signals, processed with software, and converted back into an analog signal, all without the incursion of error. For such signals, systems can be easily produced in software, with equivalent analog realizations difficult, if not impossible, to design.

### 1.2. Systems in the Time-Domain

A discrete-time signal \( s(n) \) is **delayed** by \( n_0 \) samples when we write \( s(n-n_0) \), with \( n_0 > 0 \). Choosing \( n_0 \) to be negative advances the signal along the integers. As opposed to **analog delays**, discrete-time delays can **only** be integer valued. In the frequency domain, delaying a signal
corresponds to a linear phase shift of the signal's discrete-time Fourier transform:
\[ s(n-n_0) \leftrightarrow e^{-j2\pi fn_0}S(e^{j2\pi f}) \]

**Linear discrete-time systems** have the superposition property.

\[ S(a_1x_1(n)+a_2x_2(n))=a_1S(x_1(n))+a_2S(x_2(n)) \]

A discrete-time system is called **shift-invariant** (analogous to time-invariant analog systems) if delaying the input delays the corresponding output.

\[ S(x(n))=y(n) , \text{Then} \ S(x(n-n_0))=y(n-n_0) \]

We use the term shift-invariant to emphasize that delays can only have integer values in discrete-time, while in analog signals, delays can be arbitrarily valued.

We want to concentrate on systems that are both linear and shift-invariant. It will be these that allow us the full power of frequency-domain analysis and implementations. Because we have no physical constraints in "constructing" such systems, we need only a mathematical specification. In analog systems, the differential equation specifies the input-output relationship in the time-domain. The corresponding discrete-time specification is the **difference equation**.

\[ y(n)=a_1y(n-1)+\ldots+a_py(n-p)+b_0x(n)+b_1x(n-1)+\ldots+b_qx(n-q) \]

Here, the output signal \( y(n) \) is related to its past values \( y(n-l) , l\in\{1,\ldots,p\} \), and to the current and past values of the input signal \( x(n) \). The system's characteristics are determined by the choices for the number of coefficients \( p \) and \( q \) and the coefficients' values \( \{a_1,\ldots,a_p\} \) and \( \{b_0, b_1, \ldots, b_q\} \).

There is an asymmetry in the coefficients: where is \( a_0 \)? This coefficient would multiply the \( y(n) \) term in the difference equation. We have essentially divided the equation by it, which does not change the input-output relationship. We have thus created the convention that \( a_0 \) is always one.

As opposed to differential equations, which only provide an implicit description of a system (we must somehow solve the differential equation), difference equations provide an explicit way of computing the output for any input. We simply express the difference equation by a program that calculates each output from the previous output values, and the current and previous inputs.
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