High School or College or University
Calculus Practice Exam
Will INVOLVE PRECALCULUS

This exam is multiple choice which means there are several options lettered A – E usually and you pick one of the choices, by bubbling in the letter. In this exam you can use any math formula sheet to standard. fair note: different formula sheets will use different lettering to display the same variable.

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1. f(x) = $\frac{2}{3}x^3 + \frac{3}{2}x^2 + 1x$. Find f'(-1).
   A. 3
   B. 1
   C. 0
   D. -1
   E. $\frac{-1}{2}$

X (time, hr)	0	1	2	3	4	5	6
Y (speed, mph)	1	2	3	4	5	6	7

2.

The table illustrates the speed of a moving truck starting at x = 0 in function for $0 \leq x \leq 5$ is show above. Regardless, what is the value of $\int_0^4 f(x)\,dx$?

a. 10
b. 12
c. 11
d. 13
e. 9

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3. A ladder is hung from the top of a straight tree. The tree is perpendicular to the ground (90 degrees) and that the ladder is 54 degrees from the ground. The height of the ladder is 80 feet. How far is the ladder from the tree? You may use a calculator or trigonometric tables.

a. 65.72135954999
b. 47.0228201833978
c. 64.12135954999
d. 48.8228201833978
e. 63.22135954999

4. Assume $\int x^n \, dx = \frac{x^{n+1}}{n+1}$. Find $\int x + 2 \, dx$

a. $x^2 + 2x + C$

b. $\frac{x^2}{2} + x + C$

c. $\frac{-x^2}{2} + x + C$

d. $\frac{x^2}{2} + 2x + C$

e. $x^2 + x + C$

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5. $\int x^2 \sin(x) \, dx$ =

a. $x^2 \cos(x) + 2x \sin(x) - 2 \cos(x) + C$

b. $-x^2 \cos(x) + 2 \cos(x) + 2x \sin(x) + C$

c. $x^2 \cos(x) - 2x \sin(x) - 2 \cos(x) + C$

d. $-x^2 \cos(x) + 2 \cos(x) + 2 \sin(x) + C$

e. $-x^2 \cos(x) + 2 \sin(x) + 2x \cos(x) + C$

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6. Find the derivative. $\frac{dy}{dx}\left(\sqrt{x} + \sqrt[3]{x}\right) =$

a. $\frac{1}{2\sqrt{x}} + \frac{1}{3x^{2/3}}$

b. $\frac{1}{2\sqrt{x}} + \frac{1}{x^{2/3}}$

c. $\frac{1}{2\sqrt{x}} + \frac{2}{3x^{2/3}}$

d. $\frac{1}{2\sqrt{x}} + \frac{1}{x^{2/3}}$

e. $\frac{1}{2\sqrt{x}} + \frac{1}{3x^{1/3}}$

7. US Dollar Bills are being dumped from an airplane to Wolf Stadium at a rate of 28 ft³ / min. We know the height of the volume and want to find the rate that the height is increasing. What shape is this and the formula?

a. The volume of a right rectangular pyramid is $V = \frac{lwh}{3}$

b. The volume of a right cylinder is $V = \pi r^2 h$

c. The volume of a right circular cone is $V = \frac{1}{3}\pi r^2 h$.

d. None of the above

e. This situation is impossible, we need to find the surface area instead.

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8. $\frac{A}{x} + \frac{B}{x+1} =$

a. $\frac{Ax+A+Bx}{x(x+1)}$

b. $\frac{Ax+A+B}{x(x+1)}$

c. $\frac{Ax+Bx}{x(x+1)}$

d. $\frac{Ax+B+Bx}{x(x+1)}$

e. $\frac{A+B}{x(x+1)}$

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9. $\lim_{x \to 0+} \frac{\cos(x)}{x}$

a. DNE

b. 1

c. ∞

d. π

e. -∞

10. $\int e^x \sqrt{e^x - 8}\, dx$

a. $\frac{3}{5}(e^x - 8)^{3/2} + C$

b. $\frac{5}{3}(e^x - 8)^{2/3} + C$

c. $\frac{3}{2}(e^x - 8)^{3/2} + C$

d. $\frac{2}{3}(e^x - 8)^{3/2} + C$

e. $\frac{3}{2}(e^x - 8)^{2/3} + C$

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11. Expand $(x + 6)(30 - x)(35 - 3x)$

a. $3x^3 - 277x^2 + 80x + 6$

b. $3x^3 - 127x^2 + 300x + 6000$

c. $3x^3 + 55x^2 - 700x + 6100$

d. $3x^3 - 99x^2 + 200x + 6300$

e. $3x^3 - 107x^2 + 300x + 6300$

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12. What applications could the derivative be useful for? Choose the most relevant one.

a. Finding the total distance traveled given the record of time and speed from speed recorders in the street.
b. Finding the average salary a person makes by reading their checking account statement.
c. Finding the angle of the ship given three coordinate points in a 2-dimensional graph.
d. Finding the average time a person plays video games given a list of data recorded from XBOX live player accounts.
e. Finding the velocity of a moving toy car after you measured the distance traveled by a stopwatch and a ruler.

13. Use L'hopital's rule to find the limit of f(x) = $\frac{g(x)}{h(x)}$. L'Hopital's Rule is finding the limit by making sure h(x) function turns to another function which the limit can be found. Like this: $\frac{6x^2}{x} \rightarrow \frac{12x}{1}$.

$$f(x) = \lim_{x \to 1} \frac{6x}{x^2 - 1}$$

a. DNE
b. diverges
c. converges
d. 3
e. ∞

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14. Simplify.

$\frac{xy^2zx^6yy^3}{y^8}$ + cos²(x) + sin²(x)

a. $\frac{x^7y^{11}z + y^8}{y^8}$
b. $\frac{x^7y^{11}z}{y^8} + 2$
c. $\frac{x^7y^{11}z + y^8}{y^8} + 1$
d. $\frac{x^7y^{11} + y^8}{y^8}$
e. $\frac{x^7y^{11}z + y^8 + 1}{y^8}$

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15. $\int_0^{\frac{\pi}{3}} \sin(x)\, dx =$

a. $\frac{1}{3}$
b. $\frac{\pi}{2}$
c. 0
d. π
e. $\frac{1}{2}$

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16. Factor $x^8 - 1$

a. $(x^2 + 1)(x + 1)(x - 1)$
b. $(x^4 + 1)(x^2 + 1)(x + 1)$
c. $(x^4 + 1)(x^2 + 1)(x + 1)(x - 1)$
d. $(x^4 + 1)(x^2 + 1)(x + 1)(x + 1)(x - 1)$
e. $(x^4 + 1)(x^2 + 1)(x^2 + 1)(x + 1)(x - 1)$

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17. Which one for these functions have a limit at x = 0?
a. y = 1/ln(x)
b. y = 1/sin(x)
c. y = $1/e^x$
d. y = 1/|x|
e. y = 1/x

Free Response

1. $\lim_{x\to\infty} \frac{|\cos(x)|}{x}$ = 0 because Squeeze/Sandwich Theorem. Explain why.

2. $\lim_{x\to\infty} \sin(x)$

3. $\frac{dy}{dx}(x^x + 2^x)$

4. Using implicit differentiation to differentiate (what is $\frac{dy}{dx}$) the following function: $6 = \frac{1}{3}x^2y$.

5. "What do you feel about Calculus" cut-out boxes are seen everywhere and many students have many opinions of how big or small they want the "Box of the Year" to be. The boxes are all cut-out so that the flaps of the "box" are easily seen and the all surfaces of this "box" can be seen. Either way the height of the rectangular box is measured by x + 6. The length of the rectangular box is measured by 30 - x. The width of the rectangular box is measured by 35 – 3x. Find the x value that would lead to the maximum volume possible using differential calculus.

6. US Dollar Bills are being dumped from an airplane to Wolf Stadium at a rate of 28 ft^3 / min. The shape formed is a right circular cone, how fast is the height of the pile increasing when the pile is 12 feet high?

7. Suppose you have two triangles with side lengths a, b, and c and both triangles have angles f, g, and h. a1 = 5, b1 = 6, c1 = 9. a2 = r. b2 = 10, c2 = 15. Use algebra to find a2. **Solve without calculus.**

8. Find the displacement function using integration.

$v(t) = t^3 - 6t^2 - 7t$

9. Assume there is a triangle where side A has a length of 8, side B has a length of 7, the angle N between side A and side B is 89 degrees. Find the missing length of side C using the Law of Cosines.

10. Find the derivative. In English you would find the derivative using the Chain Rule.

$f(x) = \cos(e^{\sin(x)})$

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MULTIPLE CHOICE ANSWERS

1) C 2) B 3) B 4) D 5) B 6) A 7) C 8) A 9) C 10) D 11) E 12) E 13) D 14) A 15) C 16) C 17) C

1. f(x) = $\frac{2}{3}x^3 + \frac{3}{2}x^2 + 1x$. Find f'(-1).
   A. 3
   B. 1
   C. 0
   D. -1
   E. $\frac{-1}{2}$

f(x) = $\frac{2}{3}x^3 + \frac{3}{2}x^2 + 1x$

f'(x) = $3 * \frac{2}{3}x^{3-1} + 2 * \frac{3}{2}x^{2-1} + 1 * 1x^{1-1}$

f'(x) = $2\,x^2 + 3x^1 + 1$

f(-1) = $2((-1)^2) + 3(-1) + 1$

f(-1) = 2(1) + (-3) + 1

2 – 3 + 1

-1 + 1

0

X	0	1	2	3	4	5	6
Y or f(x)	1	2	3	4	5	6	7

2. The table illustrates the speed of a moving truck starting at x = 0 in function for $0 \leq x \leq 5$ is show above. We don't know if the truck's barometer settings are miles or kilometers. Regardless, what is the value of $\int_0^4 f(x)\,dx$?
   a. 10
   b. 12
   c. 11
   d. 13
   e. 9

$$\frac{f(0)+f(1)}{2} + \frac{f(1)+f(2)}{2} + \frac{f(2)+f(3)}{2} + \frac{f(3)+f(4)}{2}$$

1.5+2.5+3.5+4.5 = 12

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3. A ladder is hung from the top of a straight tree. The tree is perpendicular to the ground (90 degrees) and that the ladder is 54 degrees from the ground. The height of the ladder is 80 feet. How far is the ladder from the tree? You may use a calculator or trigonometric tables.

a. 65.72135954999
b. 47.0228201833978
c. 64.12135954999
d. 48.8228201833978
e. 63.22135954999

cos(54° degrees) = $\frac{x}{80}$

80(cos(54°)) = x

47.0228201833978 ≈ x

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4. Assume $\int x^n\,dx = \frac{x^{n+1}}{n+1}$. Find $\int x + 2\,dx$ a. $x^2 + 2x + C$ b. $\frac{x^2}{2} + x + C$ c. $\frac{-x^2}{2} + x + C$ **d. $\frac{x^2}{2} + 2x + C$** e. $x^2 + x + C$	$\int x + 2\,dx$ $\int x^1 + 2x^0\,dx$ Remember x^0 is 1 however it is rare for anyone to write this notation $\frac{x^{1+1}}{1+1} + \frac{2x^{0+1}}{0+1} + C$ $\frac{x^2}{2} + \frac{2x^1}{1} + C$ $\frac{x^2}{2} + 2x + C$

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5. $\int x^2 \sin(x)$ =

a. $x^2\cos(x) + 2x\sin(x) - 2\cos(x) + C$

b. $-x^2\cos(x) + 2\cos(x) + 2x\sin(x) + C$

c. $x^2\cos(x) - 2x\sin(x) - 2\cos(x) + C$

d. $-x^2\cos(x) + 2\cos(x) + 2\sin(x) + C$

e. $-x^2\cos(x) + 2\sin(x) + 2x\cos(x) + C$

$u = x^2$ $du = 2x$	$\mathrm{dV} = \sin x$ $\mathrm{V} = \int dV = -\cos x$

$= -1x^2\cos(x) - \int 2x(-\cos(x))$

$u = 2x$ $du = 2dx$	$dV = -\cos x$ $V = \int -\cos(x) = -\sin x$

$= -x^2\cos(x) + 2\cos(x) + 2x\sin(x) + C$

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6. Find the derivative. **a.** $\frac{1}{2\sqrt{x}} + \frac{1}{3x^{2/3}}$ b. $\frac{1}{2\sqrt{x}} + \frac{1}{x^{2/3}}$ c. $\frac{1}{2\sqrt{x}} + \frac{2}{3x^{2/3}}$ d. $\frac{1}{2\sqrt{x}} + \frac{1}{x^{2/3}}$ e. $\frac{1}{2\sqrt{x}} + \frac{1}{3x^{1/3}}$	$\frac{dy}{dx}\left(\sqrt{x} + \sqrt[3]{x}\right) =$ $\sqrt{x} = x^{1/2}$ $\frac{dy}{dx}(x^{1/2}) = (1/2)x^{-1/2}$

$(x^{1/3}) = (1/3)x^{-2/3}$

$\frac{1}{2\sqrt{x}} + \frac{1}{3x^{2/3}}$

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7. US Dollar Bills are being dumped from an airplane to Wolf Stadium at a rate of 28 ft³ / min. We know the height of the volume and want to find the rate that the height is increasing. What shape is this and the formula?

a. The volume of a right rectangular pyramid is $V = \frac{lwh}{3}$

b. The volume of a right cylinder is $V = \pi r^2 h$

c. The volume of a right circular cone is $V = \frac{1}{3}\pi r^2 h$.

d. None of the above

e. This situation is impossible, we need to find the surface area instead.

There is no rectangular fence to trap the US Dollar Bills, so a is incorrect. Neither is there circular fence and the US Dollar bills are dropped at the center point, but there is enough of them to create a right circular cone, where more dollar bills are in the ground than at the top of the stack.

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8. $\frac{A}{x}+\frac{B}{x+1}=$

a. $\boldsymbol{\frac{Ax+A+Bx}{x(x+1)}}$ b. $\frac{Ax+A+B}{x(x+1)}$ c. $\frac{Ax+Bx}{x(x+1)}$ d. $\frac{Ax+B+Bx}{x(x+1)}$ e. $\frac{A+B}{x(x+1)}$	$\frac{A(x+1)}{x(x+1)}+\frac{B(x)}{x+1(x)}=$ $\frac{Ax+A}{x(x+1)}+\frac{Bx}{x(x+1)}=$ $\frac{Ax+A+Bx}{x(x+1)}$

9.

$\lim_{x\to 0+}\frac{\cos(x)}{x}$ a. DNE b. 1 **c. ∞** d. π e. -∞	$\lim_{x\to 0+}\frac{\cos(x)}{x}=$ $\frac{\lim_{x\to 0+}(\cos(x))}{\lim_{x\to 0+}(x)}=\frac{\lim_{x\to 0+}(\cos(0))=1}{\lim_{x\to 0+}(x)}$ $\lim_{x\to 0+}\frac{1}{x}=\infty$

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10. $\int e^x\sqrt{e^x-8}$ a. $\frac{3}{5}(e^x-8)^{3/2}+C$ b. $\frac{5}{3}(e^x-8)^{2/3}+C$ c. $\frac{3}{2}(e^x-8)^{3/2}+C$ d. $\frac{2}{3}(e^x-8)^{3/2}+C$ e. $\frac{3}{2}(e^x-8)^{2/3}+C$	$u=\mathrm{e}^x$ $du=\mathrm{e}^x\mathrm{dx}$ $\int\sqrt{\mathrm{u}}\,du$ $\frac{2}{3}u^{\frac{3}{2}}+C$ $\frac{2}{3}(e^x-8)^{3/2}+C$

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11. Expand $(x+6)(30-x)(35-3x)$ a. $3x^3-277x^2+80x+6$ b. $3x^3-127x^2+300x+6000$ c. $3x^3+55x^2-700x+6100$ d. $3x^3-99x^2+200x+6300$ **e. $3x^3-107x^2+300x+6300$**	$(30x-x^2+180-6x)(35-3x)$ $(24x-x^2+180)(35-3x)$ 840x-35x²+6300-72x²+3x³-540x 3x³-35x²-72x²+840x-540x+6300 $3x^3-107x^2+300x+6300$

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12. What applications could the derivative be useful for? Choose the most relevant one.

a. Finding the total distance traveled given the record of time and speed from speed recorders in the street.
b. Finding the average salary a person makes by reading their checking account statement.
c. Finding the angle of the ship given three coordinate points in a 2-dimensional graph.
d. Finding the average time a person plays video games given a list of data recorded from XBOX live player accounts.
e. Finding the velocity of a moving toy car after you measured the distance traveled by a stopwatch and a ruler.

You integrate the velocity function (which is essentially speed but could be negative) to get a displacement function. Other answer choices are better solved using algebra or trigonometry.

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13. Use L'hopital's rule to find the limit of f(x) = $\frac{g(x)}{h(x)}$. L'Hopital's Rule is finding the limit by making sure h(x) function turns to another function which the limit can be found. Like this: $\frac{6x^2}{x} \rightarrow \frac{12x}{1}$.

$$f(x) = \lim_{x\to 1} \frac{6x}{x^2 - 1}$$

a. DNE
b. diverges
c. converges
d. 3
e. ∞

$$\lim_{x\to 1} \frac{6x}{x^2-1} \rightarrow \lim_{x\to 1} \frac{6}{2x} = \frac{6}{2(1)} = \frac{6}{2} = 3.$$

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14. Simplify.

$\frac{xy^2zx^6yy^3}{y^8}$ + cos²(x) + sin²(x)

a. $\frac{x^7y^{11}z+ y^8}{y^8}$
b. $\frac{x^7y^{11}z}{y^8} + 2$
c. $\frac{x^7y^{11}z+ y^8}{y^8} + 1$
d. $\frac{x^7y^{11}+ y^8}{y^8}$
e. $\frac{x^7y^{11}z+ y^8+1}{y^8}$

$\frac{xy^2zx^6yy^3}{y^8}$ + cos²(x) + sin²(x) => $\frac{xx^6yy^3y^2z}{y^8}$ + 1 => $\frac{x^7y^{11}z}{y^8} + \frac{y^8}{y^8}$

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15. *Find the integral of* $\int_0^{\frac{\pi}{3}} \sin(x)\,dx =$

a. $\frac{1}{3}$
b. $\frac{\pi}{2}$
c. 0
d. π
e. $\frac{1}{2}$

$\int \sin(x) = -\cos(x) + C$

$\int_0^{\pi/3} \sin(x) = -\cos(x)\Big]_0^{\pi/3}$

$-\cos\left(\frac{\pi}{3}\right) - (-\cos(0)) = 0$

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16. Factor $x^8 - 1$
a. $(x^2+1)(x+1)(x-1)$
b. $(x^4+1)(x^2+1)(x+1)$
c. $(x^4+1)(x^2+1)(x+1)(x-1)$
d. $(x^4+1)(x^2+1)(x+1)(x+1)(x-1)$
e. $(x^4+1)(x^2+1)(x^2+1)(x+1)(x-1)$

$x^8 - 1 = (x^4+1)(x^4-1)$
$(x^4+1)(x^2+1)(x^2-1)$
$(x^4+1)(x^2+1)(x+1)(x-1)$

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17. Which one for these functions have a limit at x = 0?
a. y = 1/ln(x)
b. y = 1/sin(x)
c. y = $1/e^x$
d. y = 1/|x|
e. y = 1/x

$1/e^0 = 1$

SELECTED FREE RESPONSE SOLUTIONS/ANSWERS

3) $\frac{\mathrm{dy}}{\mathrm{dx}}(x^x + 2^x)$

$y = x^x$

$\ln(y) = \ln(x^x)$

$\ln(y) = x\ln(x)$

$\frac{d}{dx}(\ln(y)) = \frac{d}{dx}\big(xln(x)\big) + \frac{d}{dx}(2^x)$

$\frac{\mathrm{dy}}{\mathrm{dx}}(x^x + 2^x) = x^x(\ln(x) + 1) + \ln(2)\, 2^x$

6). US Dollar Bills are being dumped from an airplane to Wolf Stadium at a rate of 28 ft³ / min. It is recorded from a drone. How fast is the height of the pile increasing when the pile is 12 feet high?

Answer: $\frac{28}{3\pi}\ ft/min$

10) $\frac{d}{dx}\cos(e^{\sin(x)})$ =

$\frac{d}{dx}\cos(\mathrm{x}) = -\sin(x)$

$\frac{d}{dx}\mathrm{e}^x = e^x$

$\frac{d}{dx}\sin(x) = \cos(x)$

$= \frac{d}{dx}\big(e^{\sin(x)}\big)(-1)\sin\big(\mathrm{e}^{\sin(x)}\big)\frac{d}{dx}\sin(x)$

$-e^{\sin(x)}\sin(e^{\sin(x)})\cos(x)$

Partial List Formulas Equations PreCalculus

$a^{b+c} = a^b a^c$

Example: $x^{2+3} = x^2 x^3$

Example: $2^{2+3} = 2^2 2^3$

$\ln(1) = 0$

$\ln(2 * 7) = \ln(2) + \ln(7)$

$\ln(a * b) = \ln(a) + \ln(b)$

$\ln(2/7) = \ln(2) - \ln(7)$

$\ln(a/b) = \ln(a) - \ln(b)$

$\ln(2^7) = 7 * \ln(2)$

$\ln(a^b) = b * \ln(a)$

Order of operations: g = 2 * 3 + (3 + 5) – 6^2 / 3

g = 2

X = radians	Sin(x)	Cos(x)	Tan(x) = $\sin(x)/\cos(x)$
0	0	1	0
$\pi/6$	$1/2$	$\sqrt{3}/2$	$\sqrt{3}/3$
$\pi/4$	$\sqrt{2}/2$	$\sqrt{2}/2$	1
$\pi/3$	$\sqrt{3}/2$	$1/2$	$\sqrt{3}$
$\pi/2$	1	0	1/0 = undefined
$\pi/1$	0	-1	0

Law of Cosines:

$a^2 = b^2 + c^2 - 2 * b * c * \cos A$

$b^2 = a^2 + c^2 - 2 * a * c * \cos B$

$c^2 = a^2 + b^2 - 2 * a * b * \cos C$

Partial List Formulas Equations Calculus 1 differentiation

$(a^b)' = b * a^{b-1}$

$(a * b)' = a'b + ab'$

$\left(a/b\right)' = \frac{a'b - ab'}{b^2}$

$(\ln(x))' = \frac{1}{x}$

$\frac{d}{dx}\mathrm{e}^x = e^x$

$\frac{d}{dx}\sin(\mathrm{x}) = \cos(x)$

$\frac{d}{dx}\cos(\mathrm{x}) = -\sin(x)$

$\frac{d}{dx}\tan(\mathrm{x}) = \sec^2(x)$

$\frac{d}{dx} log_a x = \frac{1}{x \ln(a)}$

$\frac{d}{dx}\mathrm{a}^x = \mathrm{a}^x \ln(a)$

Partial List Formulas Equations Calculus 2 integration

$\int f'(x) = f(x)$

$\int f(x) = g(x) + C$

$\int u * dv = uv - \int v\, du$

$\int_b^c f(x) = g(c) - g(b)$

$\int sin(x) = -\cos(x) + C$

$\int cos(x) = \sin(x) + C$