Mathematics 3
Calculus

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Figure 1 : Flamingo family curved out of horns of a Sebu Cow-horns

II. Prerequisite Courses or Knowledge

Unit 1: Elementary differential calculus (35 hours)

Secondary school mathematics is prerequisite. Basic Mathematics 1 is co-requisite.

This is a level 1 course.

Unit 2: Elementary integral calculus (35 hours)

Calculus 1 is prerequisite.

This is a level 1 course.

Unit 3: Sequences and Series (20 hours)

Priority A. Calculus 2 is prerequisite.

This is a level 2 course.

Unit 4: Calculus of Functions of Several Variables (30 hours)

Priority B. Calculus 3 is prerequisite.

This is a level 2 course.

III. Time

120 hours

IV. Material

The course materials for this module consist of:

Study materials (print, CD, on-line)
(pre-assessment materials contained within the study materials)
Two formative assessment activities per unit (always available but with specified submission date). (CD, on-line)
References and Readings from open-source sources (CD, on-line)

ICT Activity files

Those which rely on copyright software
Those which rely on open source software
Those which stand alone
Video files
Audio files (with tape version)
Open source software installation files
Graphical calculators and licenced software where available

(*Note*: exact details to be specified when activities completed)

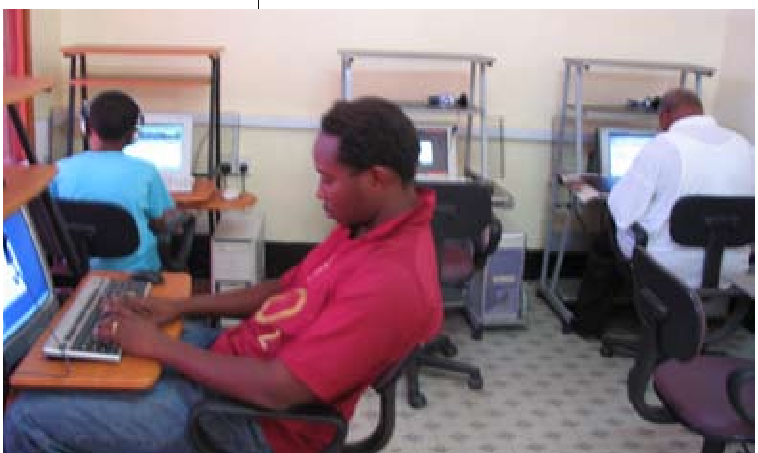


Figure 2 : A typical internet café in Dar Es Salaam

V. Module Rationale

The secondary school mathematics syllabus covers a number of topics, including differentiation and integration of functions. The module starts by introducing the concept of limits, often missed at the secondary school level, but crucial in learning these topics. It then uses limits to define continuity, differentiation and integration of a function. Also, the limit concept is used in discussing a class of special functions called sequences and the related topic of infinite series.

VI. Content

6.1 Overview

This is a four unit module. The first two units cover the basic concepts of the differential and integral calcualus of functions of a single variable. The third unit is devoted to sequences of real numbers and infinite series of both real numbers and of some special functions. The fourth unit is on the differential and integral calculus of functions of several variables.

Starting with the definitions of the basic concepts of limit and continuity of functions of a single variable the module proceeds to introduce the notions of differentiation and integration, covering both methods and applications.

Definitions of convergence for sequences and infinite series are given. Tests for convergence of infinite series are presented, including the concepts of interval and radius of convergence of a power series.

Partial derivatives of functions of several variables are introduced and used in formulating Taylor's theorem and finding relative extreme values.

6.2 Outline

Unit 1: Elementary differential calculus (35 hours)

Level 1. Priority A. No prerequisite. Basic Mathematics 1 is co-requisite.

Limits (3)
Continuity of functions. (3)
Differentiation of functions of a single variable. (6)
Parametric and implicit differentiation. (4)
Applications of differentiation. (6)
Taylor's theorem. (3)
Mean value theorems of differential calculus. (4)
Applications. (6)

Unit 2: Elementary integral calculus (35 hours)

Level 1. Priority A. Calculus 1 is prerequisite.

Anti derivatives and applications to areas. (6)
Methods of integration. (8)
Mean value theorems of integral calculus. (5)

Numerical integration. (7)
Improper integrals and their convergence. (3)
Applications of integration. (6)

Unit 3: Sequences and Series (20 hours)

Level 2. Priority A. Calculus 2 is prerequisite.

Sequences (5)
Series (5)
Power series (3)
Convergence tests (5)
Applications (2)

Unit 4: Calculus of Functions of Several Variables (30 hours)

Level 2. Priority B. Calculus 3 is prerequisite.

Functions of several variables and their applications (4)
Partial differentiation (4)
Center of masses and moments of inertia (4)
Differential and integral calculus of functions of several variables:
Taylors theorem (3)
Minimum and Maximum points (2)
Lagrange's Multipliers (2)
Multiple integrals (8)
Vector fields (2)

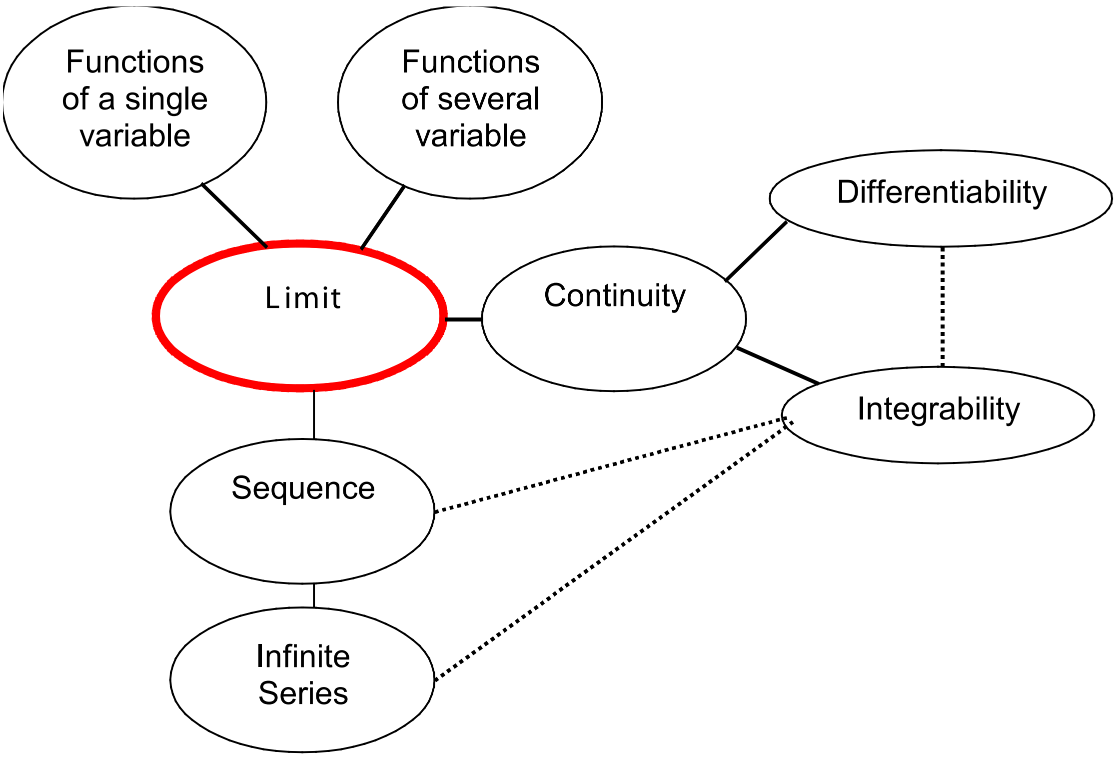
6.3 Graphic Organizer

This diagram shows how the different sections of this module relate to each other.

The central or core concept is in the centre of the diagram. (Shown in red).

Concepts that depend on each other are shown by a line.

For example: Limit is the central concept. Continuity depends on the idea of Limit. The Differentiability depend on Continuity.



VII. General Objective(s)

You will be equipped with knowledge and understanding of the properties of elementary functions and their various applications necessary to confidently teach these subjects at the secondary school level.

You will have a secure knowledge of the content of school mathematics to confidently teach these subjects at the secondary school level

You will acquire knowledge of and the ability to apply available ICT to improve the teaching and learning of school mathematics

VIII. Specific Learning Objectives (Instructional Objectives)

You should be able to demonstrate an understanding of

- The concepts of limits and the necessary skills to find limits.
- The concept of continuity of elementary functions.
- … and skills in differentiation of elementary functions of both single and several variables, and the various applications of differentiation.
- … and skills in integration of elementary functions and the various applications of integration.
- Sequences and series, including convergence properties.

You should **secure your knowledge** of the following school mathematics:

- Graphs of real value functions.
- Idea of limits, continuity, gradients and areas under curves using graphs of functions.
- Differentiation and integration a wide of range of functions.
- Sequences and series (including A.P., G.P. and $\sum$ notation).
- Appropriate notation, symbols and language.

IX. Teaching And Learning Activities

9.1 Pre-assessment

Module 3: Calculus

Unit 1: Elementary Differential Calculus

1. Which of the following sets of ordered pairs (x, y) represents a function?

   (a) (1,1), (1,2), (1,2), (1,4)

   (b) (1,1), (2,1), (3,1), (4,1)

   (c) (1,1), (2,2), (3,1), (3,2)

   (d) (1,1), (1,2), (2,1), (2,2)

The set which represents a function is $\begin{cases} (a) \\ (b) \\ (c) \\ (d) \end{cases}$

2. The sum of the first n terms of a GP, whose first term is a and common ratio is r, is $S_n = a\left[\dfrac{1-r^{n+1}}{1-r}\right]$. For what values of r will the GP converge?

The GP will converge if $\begin{cases} (a) & r < -1 \\ (b) & |r| = 1 \\ (c) & r > 1 \\ (d) & |r| < 1 \end{cases}$

3. Find the equation of the tangent to the curve $y = 2x^2 - 3x + 2$ at $(2, 4)$.

The equation of the tangent at $(2,4)$ is
$$\begin{cases} (a) & y = 5x + 6 \\ (b) & y = 6x - 5 \\ (c) & y = 5x - 6 \\ (d) & y = 6x + 5 \end{cases}$$

4. Given the function $y = \sin(x) + \cos(x)$, find $\dfrac{d^2y}{dx^2} + 2\dfrac{dy}{dx} - y$.

The value of the expression is
$$\begin{cases} (a) & -4\sin(x) \\ (b) & 4\cos(x) \\ (c) & 4\sin(x) \\ (d) & -4\cos(x) \end{cases}$$

5. Using Maclaurin's series expansion, give a cubic approximation of $y = \tan(x)$.

The required cubic is
$$\begin{cases} (a) & 1 + x + \frac{1}{3}x^3 \\ (b) & x - \frac{1}{3}x^3 \\ (c) & 1 - x + \frac{1}{3}x^3 \\ (d) & x + \frac{1}{3}x^3 \end{cases}$$

Unit 2: Elementary Integral Calculus

6. If $f(x) = \begin{cases} x^2 - 2 & x \geq 2 \\ x - 4 & x < 2 \end{cases}$ then, $\lim_{x \to 2} f(x) = \begin{cases} (a) & \text{Non-existent} \\ (b) & -2 \\ (c) & 2 \\ (d) & 0 \end{cases}$

7. If $f(x) = x^x$ Then $\frac{df}{dx} = \begin{cases} (a) & x^x \\ (b) & x^{x-1} \\ (c) & x^x[1 + \ln(x)] \\ (d) & \ln(x)x^x \end{cases}$

8. The anti-derivative of a function $f(x)$ is defined as any function whose derivative is $f(x)$. Therefore, the anti-derivative of $f(x) = \sin(x) + e^{-x}$

is $F(x) = \begin{cases} (a) & -\cos(x) + e^{-x} \\ (b) & \cos(x) - e^{-x} \\ (c) & \cos(x) + e^{-x} \\ (d) & -[\cos(x) + e^{-x}] \end{cases}$

9. A trapezium (also known as a trapezoid) is any quadrilateral with a pair of opposite sides being parallel. If the lengths of the sides of a trapezium are f_0 and f_1, and if the distance between the pair of parallel sides is h, then the

area of the trapezium is Area $= \begin{cases} (a) & h(f_0 + f_1) \\ (b) & \frac{h}{2}(f_0 - f_1) \\ (c) & h(f_0 - f_1) \\ (d) & \frac{h}{2}(f_0 + f_1) \end{cases}$

10. If the points $A(-h, f_{-1})$, $B(0, f_0)$, $C(h, f_1)$, lie on a parabola $y = ax^2 + bx + c$, then, it can be shown that:

$$A = \frac{f_{-1} - 2f_0 + f_1}{2h^2}, \quad B = \frac{f_1 - f_0}{2h}, \text{ and } C = f_0.$$

Then, the area under the parabola that lies between the ordinates at $x = -h$

and $x = h$ is given by $\int_{-h}^{h} y\,dx = \begin{cases} \text{(a)} & \frac{h}{2}(f_{-1} + 4f_0 + f_1) \\ \text{(b)} & \frac{h}{3}(f_{-1} - 4f_0 + f_1) \\ \text{(c)} & \frac{h}{3}(f_{-1} + 4f_0 + f_1) \\ \text{(d)} & \frac{h}{2}(f_{-1} + 4f_0 + f_1) \end{cases}$

Unit 3: Sequences and Series

11. The first four terms of a sequence $\{a_n\}$ are $\frac{2}{3}$, $\frac{3}{5}$, $\frac{4}{7}$, $\frac{5}{9}$.

Therefore, $a_{27} = \begin{cases} \text{(a)} & 28/47 \\ \text{(b)} & 28/57 \\ \text{(c)} & 28/49 \\ \text{(d)} & 28/55 \end{cases}$

12. The limit L of a sequence with $a_n = \frac{1}{n(n+1}$ is $L = \begin{cases} (a) & -1 \\ (b) & 1 \\ (c) & 0 \\ (d) & 1/2 \end{cases}$

13. The sequence whose $n-th$ term is given by $a_n = (-1)^{n+1}$ is:

$\begin{cases} (a) & \text{convergent} \\ (b) & \text{increa sin g} \\ (c) & \text{divergent} \\ (d) & \text{decreasin g} \end{cases}$

14. If $\{a_n\}$ is a sequence of real numbers, and if $\lim_{n\to\infty} a_n = L$, then the infinite

series $\sum_{k=1}^{\infty} a_k$ converges only if $\begin{cases} (a) & L < \infty \\ (b) & L =< 1 \\ (c) & L = 0 \\ (d) & |L| < 1 \end{cases}$

15. If $S_n = \sum_{k=1}^{n} \left(\frac{1}{k} - \frac{1}{n+1} \right)$ then $S_n = \begin{cases} (a) & 1 - \frac{1}{n} \\ (b) & 1 - \frac{1}{n+1} \\ (c) & \frac{1}{n} - \frac{1}{n+1} \\ (d) & \frac{1}{n+1} - \frac{1}{n} \end{cases}$

Unit 4: Calculus of Functions of Several Variables

16. The area enclosed by the curve $y = 4$ and $y = 1 + x^2$ is

(a) $= \frac{3}{20}$

(b) $= 7$

(c) $= 0$

(d) $= \frac{20}{3}$

17. If $f(x, y) = \ln\left(x^3 - x^2 y^2 + y^3\right)$ and $g(t) = e^{-t}\sin(t)$, then $g\left(f(x, y)\right)$ is given by:

(a) $= \dfrac{\sin\left(x^3 - x^2 y^2 + y^3\right)}{\ln\left(x^3 - x^2 y^2 + y^3\right)}$

(b) $= \dfrac{\sin\left[\ \ln\left(x^3 - x^2 y^2 + y^3\right)\right]}{\ln\left(x^3 - x^2 y^2 + y^3\right)}$

(c) $= \dfrac{\sin\left[\ln\left(x^3 - x^2 y^2 + y^3\right)\right]}{\left(x^3 - x^2 y^2 + y^3\right)}$

(d) $= \dfrac{\left(x^3 - x^2 y^2 + y^3\right)}{\sin\left[\ \ln\left(x^3 - x^2 y^2 + y^3\right)\right]}$

18. The volume **V** of an ideal gas depends on (**is a function of**) two independent variables, namely, temperature **T** and pressure **P.** Specifically, **V** is directly proportional to **T** but inversely proportional to **P.** Assume that for some unspecified temperature and pressure, the volume **V** is **100** units. If one then doubled the pressure and halves the temperature, then **V** becomes:

(a) 200 units

(b) 100 units

(c) 50 units

(d) 25 units

19. The domain D and range R of the function

$f(x, y) = \sin\sqrt{9 - x^2 - y^2}$ are:

$$\begin{cases} (a) \Rightarrow D = \{(x, y) | 9 < x^2 + y^2\} R = (-1,1) \\ (b) \Rightarrow D = \{(x, y) | 9 \le x^2 + y^2\} R = [-1,1] \\ (c) \Rightarrow D = \{(x, y) | 9 \ge x^2 + y\} R = [-1,1] \\ (d) \Rightarrow D = \{(x, y) | 9 > x^2 + y^2\} R = (-1,1) \end{cases}$$

20. If $f(x, y) = 2xy^3 + x^2y^2 - 3yx^3 + 4y$ and one denotes the limit $\lim_{h\to 0}\left[\frac{f(x+h, y) - f(x, y)}{h}\right]$ with $\frac{\partial f}{\partial x}$, then

$$\frac{\partial f}{\partial x} = \begin{cases} (a) & 6xy^2\frac{dy}{dx} + 2x^2y\frac{dy}{dx} - 3x^3\frac{dy}{dx} + 4\frac{dy}{dx} \\ (b) & 2y^3 + 6xy^2\frac{dy}{dx} + 2xy^2 + 2x^2y\frac{dy}{dx} - 9x^2y - 3x^3\frac{dy}{dx} + 4\frac{dy}{dx} \\ (c) & 2y^3 + 2xy^2 - 9x^2y \\ (d) & 6xy^2 - 3x^3 + 4 \end{cases}$$

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