Mathematics I

Basic Mathematics

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NOTICE

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I. Mathematics 1, Basic Mathematics

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II. Prerequisite Courses or Knowledge

Unit 1: (i) Sets and Functions (ii) Composite Functions

Secondary school mathematics is prerequisite.
This is a level 1 course.

Unit 2: Binary Operations

Basic Mathematics 1 is prerequisite.
This is a level 1 course.

Unit 3: Groups, Subgroups and Homomorphism

Basic Mathematics 2 is prerequisite.
This is a level 2 course.

III. Time

120 hours

IV. Material

The course materials for this module consist of:
Study materials (print, CD, on-line)
(pre-assessment materials contained within the study materials)
Two formative assessment activities per unit (always available but with specified submission date). (CD, on-line)
References and Readings from open-source sources (CD, on-line)
ICT Activity files
- Those which rely on copyright software
- Those which rely on open source software
- Those which stand alone
- Video files
- Audio files (with tape version)
- Open source software installation files

Graphical calculators and licenced software where available

V. Module Rationale

The rationale of teaching Basic mathematics is that it plays the role of filling up gaps that the student teacher could be having from secondary school mathematics. For instance, a lack of a proper grasp of the real number system and elementary functions etc. It also serves as the launching pad to University Mathematics by introducing the learner to the science of reasoning called logic and other related topics.

VI. Content

6.1 Overview

This module consists of three units which are as follows:

Unit 1: (i) Sets and Functions (ii) Composite Functions

This unit starts with the concept of a set. It then intoroduces logic which gives the learner techniques for distinguishing between correct and incorrect arguments using propositions and their connectives. A grasp of sets of real numbers on which we define elementary functions is essential. The need to have pictorial representations of a function necessitates the study of its graph. Note that the concept of a function can also be viewed as an instruction to be carried out on a set of objects. This necessitates the study of arrangements of objects in a certain order, called permutations and combinations.

Unit 2: Binary Operations

In this unit we look at the concept of binary operations. This leads to the study of elementary properties of integers such as congruence. The introduction to algebraic structures is simply what we require to pave the way for unit 3.

Unit 3: Groups, Subgroups and Homomorphism

This unit is devoted to the study of groups and rings. These are essentially sets of numbers or objects which satisfy some given axioms. The concepts of subgroup and subring are also important to study here. For the sake of looking at cases of fewer axiomatic demands we will also study the concepts of homomorphisms and isomorphisms. Here we will be reflecting on the concept of a mapping or a function from either one group to the other or from one ring to the other in order to find out what properties such a function has.

6.2 Outline

Unit 1: (i) Sets and Functions (ii) Composite Functions (50 hours)

Level 1. Priority A. No prerequisite.
Sets (4)
Elementary logic (8)
Number systems (6)
Complex numbers (4)
Relations and functions (8)
Elementary functions and their graphs (8)
Permutations (7)
Combinations (5)

Unit 2: Binary Operations (35 hours)

Level 1. Priority A. Basic Mathematics 1 is prerequisite.
Binary operations. (7)
Elementary properties of integers. (7)
Congruence. (7)
Introduction to Algebraic structures. (7)
Applications (7)

Unit 3: Groups, Subgroups and Homomorphism (35 hours)

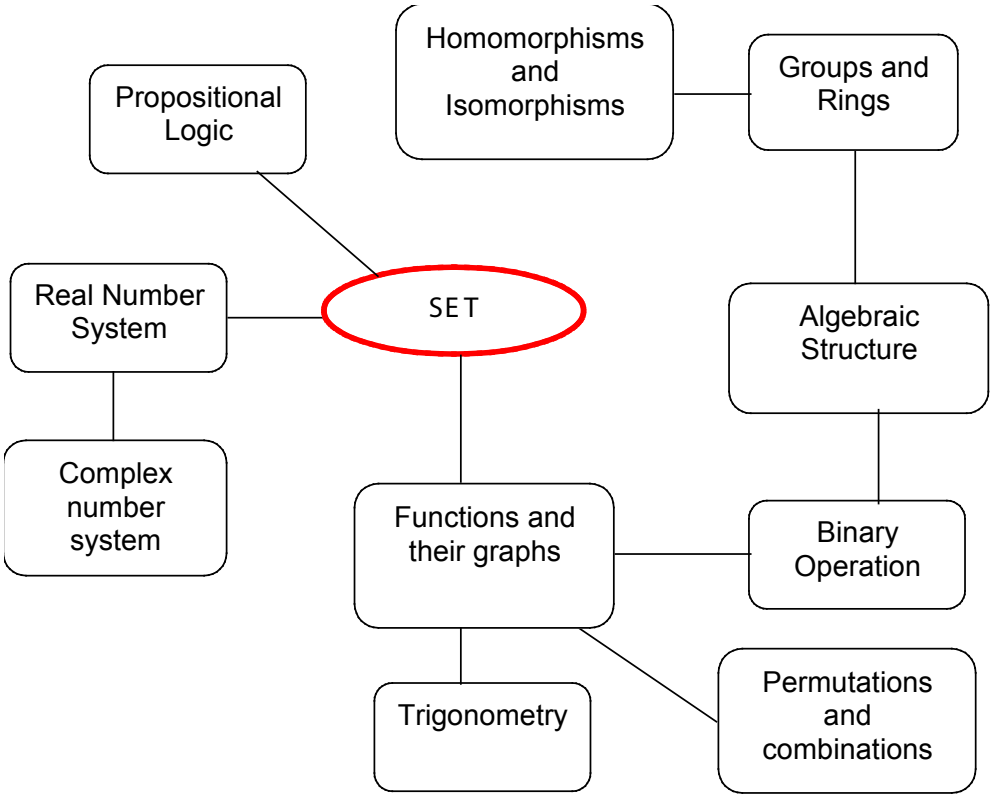
Level 2. Priority B. Basic Mathematics 2 is prerequisite.
Groups and subgroups. (7)
Cyclic groups. (2)
Permutation groups. (5)
Group homomorphisms. (4)
Factor groups. (3)
Automorphisms. (3)
Rings, sub-rings, ideals and quotient rings. (7)
Isomorphisms theorems for groups and rings. (4)

This diagram shows how the different sections of this module relate to each other.

The central or core concept is in the centre of the diagram. (Shown in red).

Concepts that depend on each other are shown by a line.

For example: Set is the central concept. The Real Number System depends on the idea of a set. The Complex Number System depend on the Real Number System.



VII. General Objective(s)

You will be equipped with knowledge of elementary mathematical logic, sets, numbers and algebraic structures required for effective teaching of mathematics in secondary schools.

VIII. Specific Learning Objectives (Instructional Objectives)

By the end of this module, the learner should be able to…"

- Construct mathematical arguments.
- make connections and communicate mathematical ideas effectively and economically.
- Examine patterns, make abstractions and generalize.
- Understand various mathematical structures and the similarities and differences among these structures.

IX. Teaching and Learning Activities

Module 1: Basic Mathematics, Pre-assessment

Unit 1: Sets and Functions

Assessments and Solutions

Pre-assessment Questions

1. Given the quadratic equation:

$$2x^2 - x - 6 = 0$$

The roots are

a. $\{-4, 3\}$

b. $\{4, -3\}$

c. $\left\{2, -\frac{3}{2}\right\}$

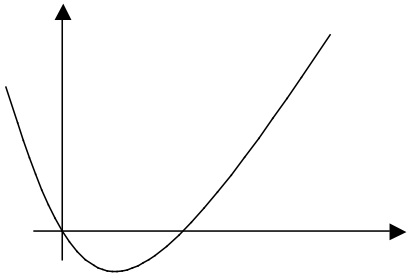
d. $\left\{-2, \frac{3}{2}\right\}$

2. The value of the function $f(x) = 2x^2 + 3x + 1$ at $x = 3$ is

a. 19
b. 28
c. 46
d. 16

3. Which of the following diagrams below represents the graph of y=3x(2-x)

a.



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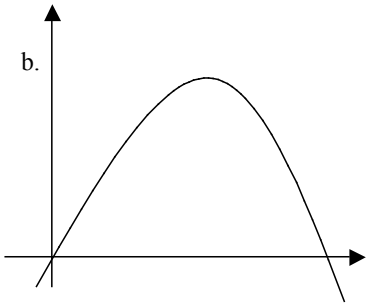
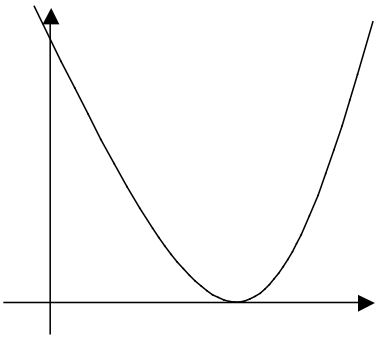
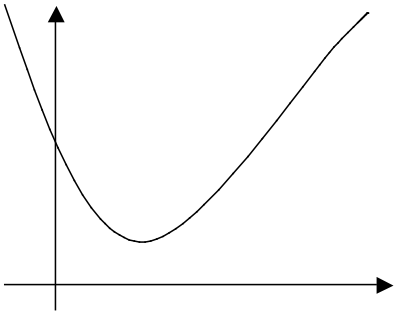


c.



d



4. The solution of the equation

$\sin x = -\frac{1}{2}$ in the range $0 \le x^\circ \le 360$ is:

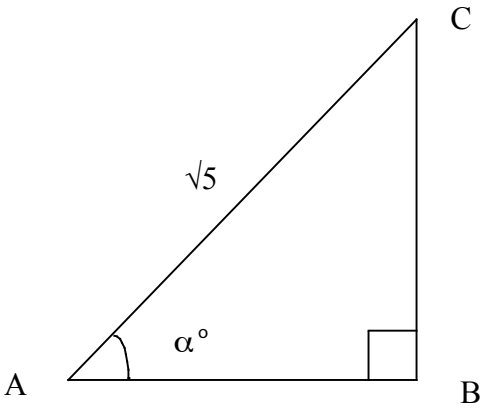
a. $\{150^\circ, 210^\circ\}$

b. $\{30^\circ, 150^\circ\}$

c. $\{210^\circ, 330^\circ\}$

d. $\{30^\circ, 330^\circ\}$

5. Given the triangle ABC below



Which of the following statements is correct?

a) $\text{Cos}\ \alpha = \frac{2}{\sqrt{15}}$

b) $\text{Sin}\ \alpha = \frac{\sqrt{5}}{2}$

c) $\text{Tan}\ \alpha = 2$

d) $\text{Sec}\ \alpha = \frac{1}{\sqrt{5}}$

Unit 1: Pre-assessment Solutions

The following are the answers to the multiple choice questions.

Q 1 c Q 2 b Q 3 b Q 4 c Q 5 c

Unit 2: Binary Operations

1. The inverse of the function

$f(x) = \frac{1}{x-1}$ is

(a) $f^{-1}(x) = x - 1$

(b) $f^{-1}(x) = \frac{1-x}{x}$

(c) $f^{-1}(x) = \frac{x+1}{x}$

(d) $f^{-1}(x) = \frac{1}{x} - 1$

2. If $\sin\frac{x}{2} = \frac{a}{2}$ then

   $\sin x$ in terms a is:

   (a) $\frac{a}{\sqrt{4-a^2}}$

   (b) $a\sqrt{4-a^2}$

   (c) a

   (d) $\frac{\sqrt{4-a^2}}{2}$

3. A girl has 3 skirts, 5 blouses and 4 scarves. The number of different outfits consisting of skirt, blouse and scarf that she can make out of these is:

   a. 220
   b. 60
   c. 12
   d. 150

4. Given the complex number

   $z = 1 - i$ we have that Arg z is:

   (a) 45^0
   (b) 135^0
   (c) 225^0
   (d) 315^0

5. If $a * b = a^2 + ab - 1$, then

   $5 * 3$ is

   (a) 39
   (b) 41
   (c) 23
   (d) 25

Unit 2: Pre-assessment Solutions

Q1. c Q2. b Q3. b Q4. b Q5. a

Unit 3: Groups, Subgroups and Homomorphism

1. Which of the following is a binary operation?

   (a) Squaring a number.
   (b) Taking the predecessor of a natural number.
   (c) Taking the successor of a natural number.
   (d) Finding the sum of two natural numbers

2. Recall the definition of a homormorphism and state which one of the following is a homomorphism on a group G of real numbers under either multiplication or addition?

   (a) $\phi(x) = 2^x$

   (b) $\phi(x) = 6x$

   (c) $\phi(x) = x^2$

   (d) $\phi(x) = x + 5$

3. For a group G if $a x a = b$ in G, then x is

   (a) b

   (b) ba^{-1}

   (c) $a^{-1}b$

   (d) $a^{-1}ba^{-1}$

4. If an element a is a ring $\mathfrak{R}$ is such that $a^2 = a$ then a is called

   (a) nilpotent
   (b) characteristic
   (c) idempotent
   (d) identity

5. Let $\mathfrak{R}$ be a ring and $x \in R$ if there exists a unique element $a \in R$ such that $xa = x$, then ax is:

   (a) e
   (b) a
   (c) $-x$
   (d) x

Unit 3: Pre-assessment Solutions

1. d 2. c 3. d 4. c 5. d

Title of Pre-assessment : Pedagogical comment for learners

The questions in this pre-assessment are designed to test your readiness for studying the module.

The 5 questions preparing you for unit 1 require high school mathematics. If you make any errors, this should suggest the need to re-visit the high school mathematical topic referred to in the question.

The questions for unit 2 and unit 3 test your readiness **after** having completed the learning activities for unit 1 and unit 2.

If you make errors in the unit 2 pre-assessment, you should check through your work on unit 1 in this module. Likewise, If you make errors in the unit 3 pre-assessment, you should check through your work on unit 2 in this module.

X. Key concepts (glossary)

1. **Abelian group**: This is a group $\langle G, * \rangle$ in which $a * b = b * a$ for $a,b, \in G$.

2. **Algebraic structure**: This is the collection of a given set G together with a binary operation * that satisfies a given set of axioms.

3. **Binary operation**: This is a mapping which assigns to each ordered pair of elements of a set G, exactly one element of G.

4. **Composite Function**: This is a function obtained by combing two or more other simple functions in a given order.

5. **Function**: This is a special type of mapping where an object is mapped to a unique image.

6. **Group**: This is a non-empty set say G with a binary operation $*$ such that:

   (i) $a * b \in G$ for all $a, b \in G$.

   (ii) $a * (b * c) = (a * b) * c$ for all $a, b, c \in G$.

   (iii) There exists an element e in G such that $e * a = a = a * e$ for all $a \in G$ where e is called identity.

   (iv) For every $a \in G$ there exists

   $a^{-1} \in G$ such that $a * a^{-1} = e = a^{-1} * a$

   Where a^{-1} is called the inverse of a

7. **Homomorphism**: This is a mapping ϕ from a group G into another group H such that for any pair $a,b, \in G$. We have $\phi\ (ab) = \phi\ (a)\ \phi\ (b)$.

8. **Isomorphism:** This is a homorphism which is also a bijection.

9. **Mapping**: This is simply a relationship between any two given sets.