Quantitative Information Analysis III

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Chapter 1

Sampling and Data

1.1 Sampling and Data[1]

1.1.1 Student Learning Outcomes

By the end of this chapter, the student should be able to:

- Recognize and differentiate between key terms.
- Apply various types of sampling methods to data collection.
- Create and interpret frequency tables.

1.1.2 Introduction

You are probably asking yourself the question, "When and where will I use statistics?". If you read any newspaper or watch television, or use the Internet, you will see statistical information. There are statistics about crime, sports, education, politics, and real estate. Typically, when you read a newspaper article or watch a news program on television, you are given sample information. With this information, you may make a decision about the correctness of a statement, claim, or "fact." Statistical methods can help you make the "best educated guess."

Since you will undoubtedly be given statistical information at some point in your life, you need to know some techniques to analyze the information thoughtfully. Think about buying a house or managing a budget. Think about your chosen profession. The fields of economics, business, psychology, education, biology, law, computer science, police science, and early childhood development require at least one course in statistics.

Included in this chapter are the basic ideas and words of probability and statistics. You will soon understand that statistics and probability work together. You will also learn how data are gathered and what "good" data are.

1.2 Statistics[2]

The science of **statistics** deals with the collection, analysis, interpretation, and presentation of **data**. We see and use data in our everyday lives.

[1]This content is available online at <http://cnx.org/content/m16008/1.9/>.
[2]This content is available online at <http://cnx.org/content/m16020/1.16/>.

1.2.1 Optional Collaborative Classroom Exercise

In your classroom, try this exercise. Have class members write down the average time (in hours, to the nearest half-hour) they sleep per night. Your instructor will record the data. Then create a simple graph (called a **dot plot**) of the data. A dot plot consists of a number line and dots (or points) positioned above the number line. For example, consider the following data:

5; 5.5; 6; 6; 6; 6.5; 6.5; 6.5; 6.5; 7; 7; 8; 8; 9

The dot plot for this data would be as follows:

Frequency of Average Time (in Hours) Spent Sleeping per Night

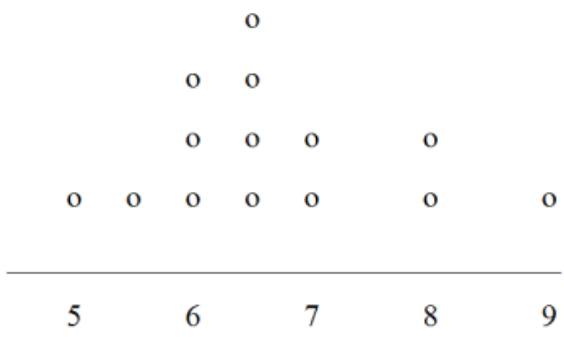

Figure 1.1

Does your dot plot look the same as or different from the example? Why? If you did the same example in an English class with the same number of students, do you think the results would be the same? Why or why not?

Where do your data appear to cluster? How could you interpret the clustering?

The questions above ask you to analyze and interpret your data. With this example, you have begun your study of statistics.

In this course, you will learn how to organize and summarize data. Organizing and summarizing data is called **descriptive statistics**. Two ways to summarize data are by graphing and by numbers (for example, finding an average). After you have studied probability and probability distributions, you will use formal methods for drawing conclusions from "good" data. The formal methods are called **inferential statistics**. Statistical inference uses probability to determine how confident we can be that the conclusions are correct.

Effective interpretation of data (inference) is based on good procedures for producing data and thoughtful examination of the data. You will encounter what will seem to be too many mathematical formulas for interpreting data. The goal of statistics is not to perform numerous calculations using the formulas, but to gain an understanding of your data. The calculations can be done using a calculator or a computer. The understanding must come from you. If you can thoroughly grasp the basics of statistics, you can be more confident in the decisions you make in life.

1.2.2 Levels of Measurement and Statistical Operations

The way a set of data is measured is called its level of measurement. Correct statistical procedures depend on a researcher being familiar with levels of measurement. Not every statistical operation can be used with every set of data. Data can be classified into four levels of measurement. They are (from lowest to highest level):

- Nominal scale level
- Ordinal scale level
- Interval scale level
- Ratio scale level

Data that is measured using a **nominal scale** is qualitative. Categories, colors, names, labels and favorite foods along with yes or no responses are examples of nominal level data. Nominal scale data are not ordered. For example, trying to classify people according to their favorite food does not make any sense. Putting pizza first and sushi second is not meaningful.

Smartphone companies are another example of nominal scale data. Some examples are Sony, Motorola, Nokia, Samsung and Apple. This is just a list and there is no agreed upon order. Some people may favor Apple but that is a matter of opinion. Nominal scale data cannot be used in calculations.

Data that is measured using an **ordinal scale** is similar to nominal scale data but there is a big difference. The ordinal scale data can be ordered. An example of ordinal scale data is a list of the top five national parks in the United States. The top five national parks in the United States can be ranked from one to five but we cannot measure differences between the data.

Another example using the ordinal scale is a cruise survey where the responses to questions about the cruise are "excellent," "good," "satisfactory" and "unsatisfactory." These responses are ordered from the most desired response by the cruise lines to the least desired. But the differences between two pieces of data cannot be measured. Like the nominal scale data, ordinal scale data cannot be used in calculations.

Data that is measured using the **interval scale** is similar to ordinal level data because it has a definite ordering but there is a difference between data. The differences between interval scale data can be measured though the data does not have a starting point.

Temperature scales like Celsius (C) and Fahrenheit (F) are measured by using the interval scale. In both temperature measurements, 40 degrees is equal to 100 degrees minus 60 degrees. Differences make sense. But 0 degrees does not because, in both scales, 0 is not the absolute lowest temperature. Temperatures like -10° F and -15° C exist and are colder than 0.

Interval level data can be used in calculations but one type of comparison cannot be done. Eighty degrees C is not 4 times as hot as 20° C (nor is 80° F 4 times as hot as 20° F). There is no meaning to the ratio of 80 to 20 (or 4 to 1).

Data that is measured using the **ratio scale** takes care of the ratio problem and gives you the most information. Ratio scale data is like interval scale data but, in addition, it has a 0 point and ratios can be calculated. For example, four multiple choice statistics final exam scores are 80, 68, 20 and 92 (out of a possible 100 points). The exams were machine-graded.

The data can be put in order from lowest to highest: 20, 68, 80, 92.

The differences between the data have meaning. The score 92 is more than the score 68 by 24 points.

Ratios can be calculated. The smallest score for ratio data is 0. So 80 is 4 times 20. The score of 80 is 4 times better than the score of 20.

Exercises

What type of measure scale is being used? Nominal, Ordinal, Interval or Ratio.

1. High school men soccer players classified by their athletic ability: Superior, Average, Above average.
2. Baking temperatures for various main dishes: 350, 400, 325, 250, 300
3. The colors of crayons in a 24-crayon box.
4. Social security numbers.
5. Incomes measured in dollars
6. A satisfaction survey of a social website by number: 1 = very satisfied, 2 = somewhat satisfied, 3 = not satisfied.
7. Political outlook: extreme left, left-of-center, right-of-center, extreme right.
8. Time of day on an analog watch.
9. The distance in miles to the closest grocery store.
10. The dates 1066, 1492, 1644, 1947, 1944.
11. The heights of 21 – 65 year-old women.
12. Common letter grades A, B, C, D, F.

Answers 1. ordinal, 2. interval, 3. nominal, 4. nominal, 5. ratio, 6. ordinal, 7. nominal, 8. interval, 9. ratio, 10. interval, 11. ratio, 12. ordinal

1.3 Probability[3]

Probability is a mathematical tool used to study randomness. It deals with the chance (the likelihood) of an event occurring. For example, if you toss a **fair** coin 4 times, the outcomes may not be 2 heads and 2 tails. However, if you toss the same coin 4,000 times, the outcomes will be close to half heads and half tails. The expected theoretical probability of heads in any one toss is $\frac{1}{2}$ or 0.5. Even though the outcomes of a few repetitions are uncertain, there is a regular pattern of outcomes when there are many repetitions. After reading about the English statistician Karl Pearson who tossed a coin 24,000 times with a result of 12,012 heads, one of the authors tossed a coin 2,000 times. The results were 996 heads. The fraction $\frac{996}{2000}$ is equal to 0.498 which is very close to 0.5, the expected probability.

The theory of probability began with the study of games of chance such as poker. Predictions take the form of probabilities. To predict the likelihood of an earthquake, of rain, or whether you will get an A in this course, we use probabilities. Doctors use probability to determine the chance of a vaccination causing the disease the vaccination is supposed to prevent. A stockbroker uses probability to determine the rate of return on a client's investments. You might use probability to decide to buy a lottery ticket or not. In your study of statistics, you will use the power of mathematics through probability calculations to analyze and interpret your data.

1.4 Key Terms[4]

In statistics, we generally want to study a **population**. You can think of a population as an entire collection of persons, things, or objects under study. To study the larger population, we select a **sample**. The idea of **sampling** is to select a portion (or subset) of the larger population and study that portion (the sample) to gain information about the population. Data are the result of sampling from a population.

[3]This content is available online at <http://cnx.org/content/m16015/1.11/>.
[4]This content is available online at <http://cnx.org/content/m16007/1.17/>.

Because it takes a lot of time and money to examine an entire population, sampling is a very practical technique. If you wished to compute the overall grade point average at your school, it would make sense to select a sample of students who attend the school. The data collected from the sample would be the students' grade point averages. In presidential elections, opinion poll samples of 1,000 to 2,000 people are taken. The opinion poll is supposed to represent the views of the people in the entire country. Manufacturers of canned carbonated drinks take samples to determine if a 16 ounce can contains 16 ounces of carbonated drink.

From the sample data, we can calculate a statistic. A **statistic** is a number that is a property of the sample. For example, if we consider one math class to be a sample of the population of all math classes, then the average number of points earned by students in that one math class at the end of the term is an example of a statistic. The statistic is an estimate of a population parameter. A **parameter** is a number that is a property of the population. Since we considered all math classes to be the population, then the average number of points earned per student over all the math classes is an example of a parameter.

One of the main concerns in the field of statistics is how accurately a statistic estimates a parameter. The accuracy really depends on how well the sample represents the population. The sample must contain the characteristics of the population in order to be a **representative sample**. We are interested in both the sample statistic and the population parameter in inferential statistics. In a later chapter, we will use the sample statistic to test the validity of the established population parameter.

A **variable**, notated by capital letters like X and Y, is a characteristic of interest for each person or thing in a population. Variables may be **numerical** or **categorical**. **Numerical variables** take on values with equal units such as weight in pounds and time in hours. **Categorical variables** place the person or thing into a category. If we let X equal the number of points earned by one math student at the end of a term, then X is a numerical variable. If we let Y be a person's party affiliation, then examples of Y include Republican, Democrat, and Independent. Y is a categorical variable. We could do some math with values of X (calculate the average number of points earned, for example), but it makes no sense to do math with values of Y (calculating an average party affiliation makes no sense).

Data are the actual values of the variable. They may be numbers or they may be words. Datum is a single value.

Two words that come up often in statistics are **mean** and **proportion**. If you were to take three exams in your math classes and obtained scores of 86, 75, and 92, you calculate your mean score by adding the three exam scores and dividing by three (your mean score would be 84.3 to one decimal place). If, in your math class, there are 40 students and 22 are men and 18 are women, then the proportion of men students is $\frac{22}{40}$ and the proportion of women students is $\frac{18}{40}$. Mean and proportion are discussed in more detail in later chapters.

NOTE: The words "mean" and "average" are often used interchangeably. The substitution of one word for the other is common practice. The technical term is "arithmetic mean" and "average" is technically a center location. However, in practice among non-statisticians, "average" is commonly accepted for "arithmetic mean."

Example 1.1

Define the key terms from the following study: We want to know the average (mean) amount of money first year college students spend at ABC College on school supplies that do not include books. We randomly survey 100 first year students at the college. Three of those students spent $150, $200, and $225, respectively.

Solution

The **population** is all first year students attending ABC College this term.

The **sample** could be all students enrolled in one section of a beginning statistics course at ABC College (although this sample may not represent the entire population).

The **parameter** is the average (mean) amount of money spent (excluding books) by first year college students at ABC College this term.

The **statistic** is the average (mean) amount of money spent (excluding books) by first year college students in the sample.

The **variable** could be the amount of money spent (excluding books) by one first year student. Let X = the amount of money spent (excluding books) by one first year student attending ABC College.

The **data** are the dollar amounts spent by the first year students. Examples of the data are \$150, \$200, and \$225.

1.4.1 Optional Collaborative Classroom Exercise

Do the following exercise collaboratively with up to four people per group. Find a population, a sample, the parameter, the statistic, a variable, and data for the following study: You want to determine the average (mean) number of glasses of milk college students drink per day. Suppose yesterday, in your English class, you asked five students how many glasses of milk they drank the day before. The answers were 1, 0, 1, 3, and 4 glasses of milk.

1.5 Data[5]

Data may come from a population or from a sample. Small letters like x or y generally are used to represent data values. Most data can be put into the following categories:

- Qualitative
- Quantitative

Qualitative data are the result of categorizing or describing attributes of a population. Hair color, blood type, ethnic group, the car a person drives, and the street a person lives on are examples of qualitative data. Qualitative data are generally described by words or letters. For instance, hair color might be black, dark brown, light brown, blonde, gray, or red. Blood type might be AB+, O-, or B+. Researchers often prefer to use quantitative data over qualitative data because it lends itself more easily to mathematical analysis. For example, it does not make sense to find an average hair color or blood type.

Quantitative data are always numbers. Quantitative data are the result of **counting** or **measuring** attributes of a population. Amount of money, pulse rate, weight, number of people living in your town, and the number of students who take statistics are examples of quantitative data. Quantitative data may be either **discrete** or **continuous**.

All data that are the result of counting are called **quantitative discrete data**. These data take on only certain numerical values. If you count the number of phone calls you receive for each day of the week, you might get 0, 1, 2, 3, etc.

[5]This content is available online at <http://cnx.org/content/m16005/1.18/>.

All data that are the result of measuring are **quantitative continuous data** assuming that we can measure accurately. Measuring angles in radians might result in the numbers $\frac{\pi}{6}$, $\frac{\pi}{3}$,$\frac{\pi}{2}$, π , $\frac{3\pi}{4}$, etc. If you and your friends carry backpacks with books in them to school, the numbers of books in the backpacks are discrete data and the weights of the backpacks are continuous data.

NOTE: In this course, the data used is mainly quantitative. It is easy to calculate statistics (like the mean or proportion) from numbers. In the chapter **Descriptive Statistics**, you will be introduced to stem plots, histograms and box plots all of which display quantitative data. Qualitative data is discussed at the end of this section through graphs.

Example 1.2: Data Sample of Quantitative Discrete Data
The data are the number of books students carry in their backpacks. You sample five students. Two students carry 3 books, one student carries 4 books, one student carries 2 books, and one student carries 1 book. The numbers of books (3, 4, 2, and 1) are the quantitative discrete data.

Example 1.3: Data Sample of Quantitative Continuous Data
The data are the weights of the backpacks with the books in it. You sample the same five students. The weights (in pounds) of their backpacks are 6.2, 7, 6.8, 9.1, 4.3. Notice that backpacks carrying three books can have different weights. Weights are quantitative continuous data because weights are measured.

Example 1.4: Data Sample of Qualitative Data
The data are the colors of backpacks. Again, you sample the same five students. One student has a red backpack, two students have black backpacks, one student has a green backpack, and one student has a gray backpack. The colors red, black, black, green, and gray are qualitative data.

NOTE: You may collect data as numbers and report it categorically. For example, the quiz scores for each student are recorded throughout the term. At the end of the term, the quiz scores are reported as A, B, C, D, or F.

Example 1.5
Work collaboratively to determine the correct data type (quantitative or qualitative). Indicate whether quantitative data are continuous or discrete. Hint: Data that are discrete often start with the words "the number of."

1. The number of pairs of shoes you own.
2. The type of car you drive.
3. Where you go on vacation.
4. The distance it is from your home to the nearest grocery store.
5. The number of classes you take per school year.
6. The tuition for your classes
7. The type of calculator you use.
8. Movie ratings.
9. Political party preferences.
10. Weight of sumo wrestlers.
11. Amount of money won playing poker.
12. Number of correct answers on a quiz.
13. Peoples' attitudes toward the government.
14. IQ scores. (This may cause some discussion.)

Qualitative Data Discussion
Below are tables of part-time vs full-time students at De Anza College in Cupertino, CA and Foothill College in Los Altos, CA for the Spring 2010 quarter. The tables display counts (frequencies) and percentages or proportions (relative frequencies). The percent columns make comparing the same categories in the colleges easier. Displaying percentages along with the numbers is often helpful, but it is particularly important when comparing sets of data that do not have the same totals, such as the total enrollments for both colleges in this example. Notice how much larger the percentage for part-time students at Foothill College is compared to De Anza College.

De Anza College

	Number	Percent
Full-time	9,200	40.9%
Part-time	13,296	59.1%
Total	22,496	100%

Table 1.1

Foothill College

	Number	Percent
Full-time	4,059	28.6%
Part-time	10,124	71.4%
Total	14,183	100%

Table 1.2

Tables are a good way of organizing and displaying data. But graphs can be even more helpful in understanding the data. There are no strict rules concerning what graphs to use. Below are pie charts and bar graphs, two graphs that are used to display qualitative data.

In a **pie chart**, categories of data are represented by wedges in the circle and are proportional in size to the percent of individuals in each category.

In a **bar graph**, the length of the bar for each category is proportional to the number or percent of individuals in each category. Bars may be vertical or horizontal.

A **Pareto chart** consists of bars that are sorted into order by category size (largest to smallest).

Look at the graphs and determine which graph (pie or bar) you think displays the comparisons better. This is a matter of preference.

It is a good idea to look at a variety of graphs to see which is the most helpful in displaying the data. We might make different choices of what we think is the "best" graph depending on the data and the context. Our choice also depends on what we are using the data for.

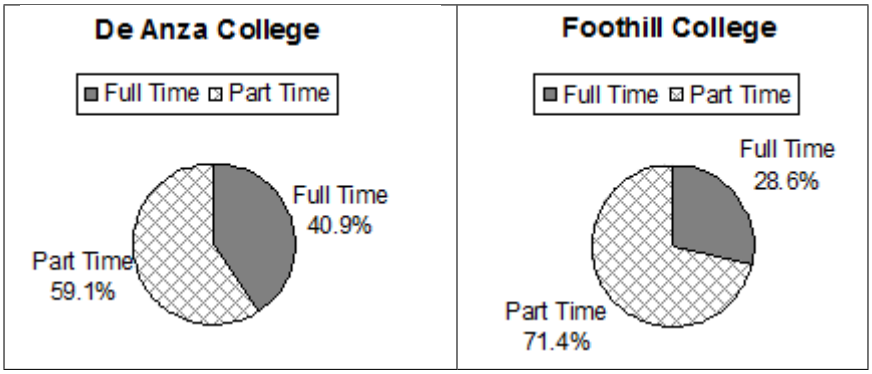

Table 1.3

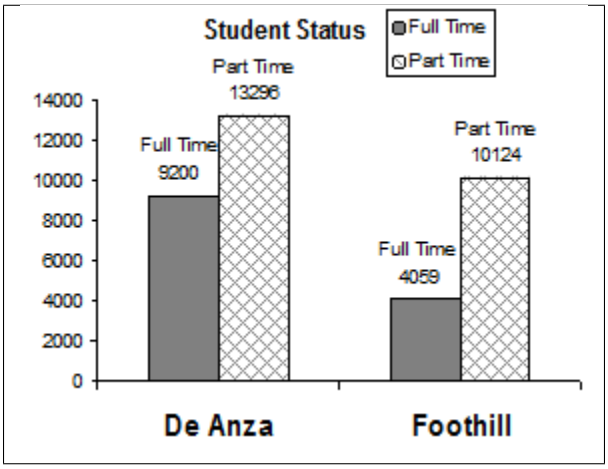

Table 1.4

Percentages That Add to More (or Less) Than 100%
Sometimes percentages add up to be more than 100% (or less than 100%). In the graph, the percentages add to more than 100% because students can be in more than one category. A bar graph is appropriate to compare the relative size of the categories. A pie chart cannot be used. It also could not be used if the percentages added to less than 100%.

De Anza College Spring 2010

Characteristic/Category	Percent
Full-time Students	40.9%
Students who intend to transfer to a 4-year educational institution	48.6%
Students under age 25	61.0%
TOTAL	150.5%

Table 1.5