Spoon Feeding Definite Integrals

Simplified Knowledge Management Classes Bangalore

My name is Subhashish Chattopadhyay. I have been teaching for IIT-JEE, Various International Exams (such as IMO [International Mathematics Olympiad], IPhO [International Physics Olympiad], IChO [International Chemistry Olympiad]), IGCSE (IB), CBSE, I.Sc, Indian State Board exams such as WB-Board, Karnataka PU-II etc since 1989. As I write this book in 2016, it is my 25 th year of teaching. I was a Visiting Professor to BARC Mankhurd, Chembur, Mumbai, Homi Bhabha Centre for Science Education (HBCSE) Physics Olympics camp BARC Campus.

I am Life Member of …

- IAPT (Indian Association of Physics Teachers)

- IPA (Indian Physics Association)

- AMTI (Association of Mathematics Teachers of India)

- National Human Rights Association

- Men's Rights Movement (India and International)

- MGTOW Movement (India and International)

And also of

IACT (Indian Association of Chemistry Teachers)

The selection for National Camp (for Official Science Olympiads - Physics, Chemistry, Biology, Astronomy) happens in the following steps ….

1) **NSEP** (National Standard Exam in Physics) and **NSEC** (National Standard Exam in Chemistry) held around 24 rth November. Approx 35,000 students appear for these exams every year. The exam fees is Rs 100 each. Since 1998 the IIT JEE toppers have been topping these exams and they get to know their rank / performance ahead of others.

2) **INPhO** (Indian National Physics Olympiad) and **INChO** (Indian National Chemistry Olympiad). Around 300 students in each subject are allowed to take these exams. Students coming from outside cities are paid fair from the Govt of India.

3) The Top 35 students of each subject are invited at HBCSE (Homi Bhabha Center for Science Education) Mankhurd, near Chembur, BARC, Mumbai. After a 2-3 weeks camp the top 5 are selected to represent India. The flight tickets and many other expenses are taken care by Govt of India.

Since last 50 years there has been no dearth of "Good Books". Those who are interested in studies have been always doing well. This e-Book does not intend to replace any standard text book. These topics are very old and already standardized.

There are 3 kinds of Text Books

- The thin Books - Good students who want more details are not happy with these. Average students who need more examples are not happy with these. Most students who want to "Cram" quickly and pass somehow find the thin books "good" as they have to read less !!

- The Thick Books - Most students do not like these, as they want to read as less as possible. Average students are "busy" with many other things and have no time to read all these.

- The Average sized Books - Good students do not get all details in any one book. Most bad students do not want to read books of "this much thickness" also !!

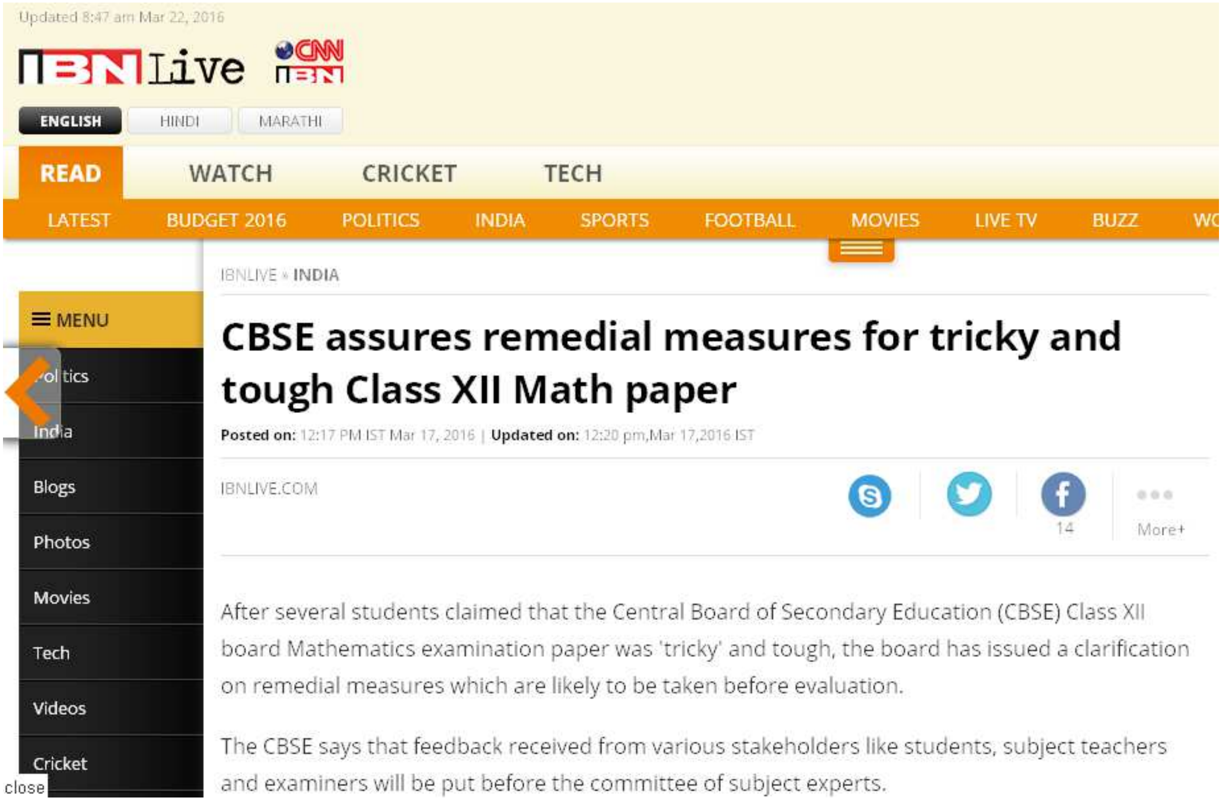
We know there can be no shoe that's fits in all.

Printed books are not e-Books! Can't be downloaded and kept in hard-disc for reading "later" ....

So if you read this book later, you will get all kinds of examples in a single place. This becomes a very good "Reference Material". I sincerely wish that all find this "very useful".

Students who do not practice lots of problems, do not do well. The rules of "doing well" had never changed .... Will never change !

After 2016 CBSE Mathematics exam, lots of students complained that the paper was tough!



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CBSE assures remedial measures for tricky and tough Class XII Math paper

Posted on: 12:17 PM IST Mar 17, 2016 | **Updated on:** 12:20 pm,Mar 17,2016 IST

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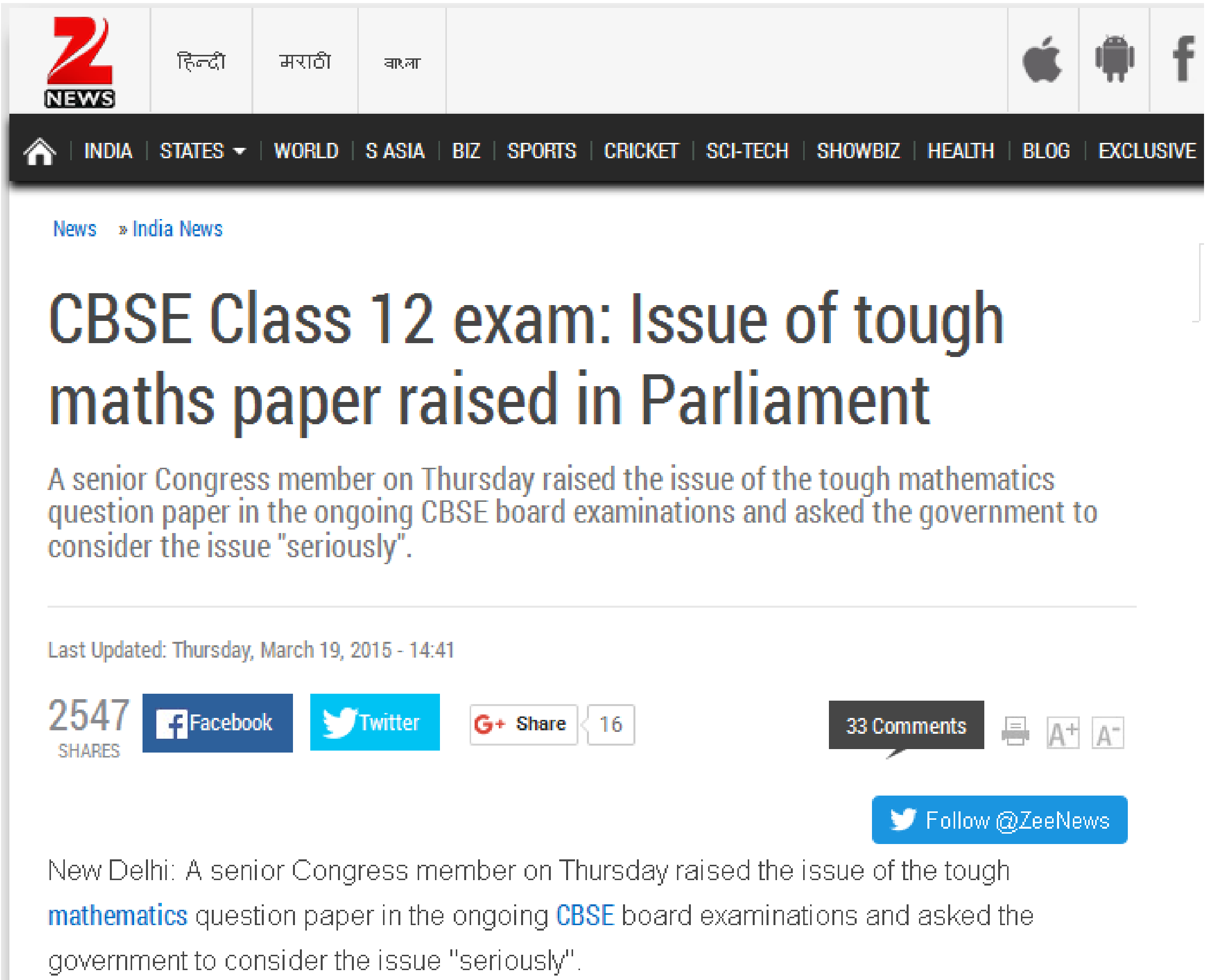
After several students claimed that the Central Board of Secondary Education (CBSE) Class XII board Mathematics examination paper was 'tricky' and tough, the board has issued a clarification on remedial measures which are likely to be taken before evaluation.

The CBSE says that feedback received from various stakeholders like students, subject teachers and examiners will be put before the committee of subject experts.

In 2015 also the same complain was there by many students



CBSE Class 12 exam: Issue of tough maths paper raised in Parliament

A senior Congress member on Thursday raised the issue of the tough mathematics question paper in the ongoing CBSE board examinations and asked the government to consider the issue "seriously".

Last Updated: Thursday, March 19, 2015 - 14:41

New Delhi: A senior Congress member on Thursday raised the issue of the tough mathematics question paper in the ongoing CBSE board examinations and asked the government to consider the issue "seriously".

In March 2016, students of Karnataka PU-II also complained the same, regarding standard 12 (PU-II Mathematics Exam). Even though the Math Paper was identical to previous year, most students had not even solved the 2015 Question Paper.

Friday, March 25, 2016 - 13:28

The NEWS Minute

HOME NEWS ANDHRA KARNATAKA KERALA TAMIL NADU TELANGANA CULTURE MEDIA BLOG

Exams

Online petition for lenient evaluation of K'taka II PU math paper gets over 8000 supporters

The campaign, which was launched on Monday, has garnered over 8000 supporters

TNM Staff | Wednesday, March 16, 2016 - 09:32

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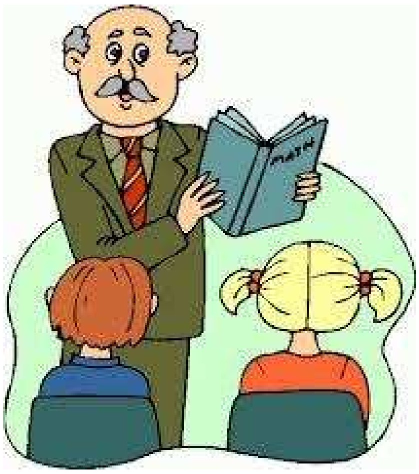
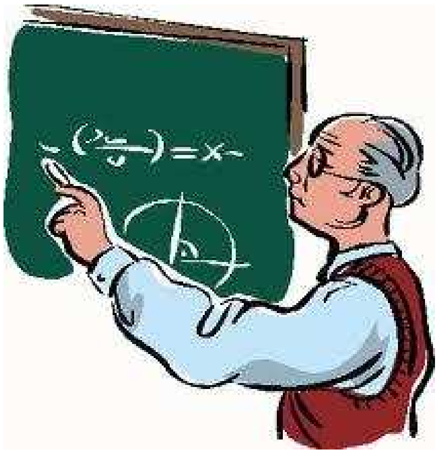
Following a “very tough” math paper that left many II PU students in tears, Saket Ravindran a student launched an online campaign demanding lenient evaluation.

These complains are not new. In fact since last 40 years, (since my childhood), I always see this; every year the same setback, same complain!

In this e-Book I am trying to solve this problem. Those students who practice can learn.

No one can help those who are not studying, or practicing.





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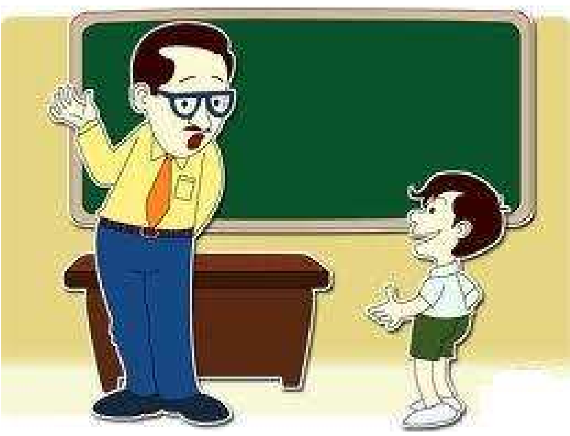

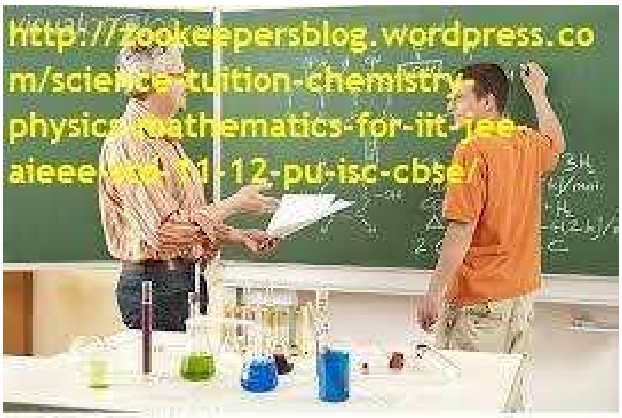

Spoon Feeding Series - Definite Integrals

In any book solution techniques of various types of Differential equations will be given. Before we proceed, Recall the various tricks, formulae, and rules of solving Indefinite Integrals

(i) $\int \frac{1}{x^2+a^2}\,dx = \frac{1}{a}\tan^{-1}\frac{x}{a} + C$

(ii) $\int \frac{1}{a^2-x^2}\,dx = \frac{1}{2a}\log\left|\frac{a+x}{a-x}\right| + C = \frac{1}{a}\tanh^{-1}\left(\frac{x}{a}\right) + C$

(iii) $\int \frac{dx}{x^2-a^2}\,dx = \frac{1}{2a}\log\left|\frac{x-a}{x+a}\right| + C = -\frac{1}{a}\coth^{-1}\left(\frac{x}{a}\right) + C$

(iv) $\int \frac{dx}{\sqrt{a^2-x^2}} = \sin^{-1}\frac{x}{a} + C$

(v) $\int \frac{dx}{\sqrt{x^2-a^2}} = \log|x+\sqrt{x^2-a^2}| + C = \cosh^{-1}\left(\frac{x}{a}\right) + C$

(vi) $\int \frac{dx}{\sqrt{x^2+a^2}} = \log|x+\sqrt{x^2+a^2}| + C = \sinh^{-1}\left(\frac{x}{a}\right) + C$

(vii) $\int \sqrt{x^2+a^2}\,dx = \frac{1}{2}\left[x\sqrt{x^2+a^2} + a^2\log|x+\sqrt{x^2+a^2}|\right] + C$

(viii) $\int \sqrt{a^2-x^2}\,dx = \frac{1}{2}\left[x\sqrt{a^2-x^2} + a^2\sin^{-1}\left(\frac{x}{a}\right)\right] + C$

(ix) $\int \sqrt{x^2-a^2}\,dx = \frac{1}{2}\left[x\sqrt{x^2-a^2} - a^2\log|x+\sqrt{x^2-a^2}|\right] + C$

(x) $\int (px+q)\sqrt{ax^2+bx+c}\,dx = \frac{p}{2a}\int (2ax+b)\sqrt{ax^2+bx+c}\,dx + \left(\frac{q-pb}{2a}\right)\int \sqrt{ax^2+bx+c}\,dx$

- $\int e^x dx = e^x$
- $\int e^{ax} dx = \frac{1}{a} e^{ax}$
- $\int e^{ax} \cos bx dx = \frac{e^{ax}}{a^2+b^2} (a \cos bx + b \sin bx)$
- $\int e^{ax} \sin bx dx = \frac{e^{ax}}{a^2+b^2} (a \sin bx - b \cos bx)$
- $\int a^x dx = \frac{a^x}{lna} + c$

- $\int \log x dx = x(\log x - 1) + c$
- $\int \frac{1}{x} dx = \log |x| + c$
- $\int a^x dx = a^x \log x + c$
- $\int \frac{dx}{a^2-x^2} = \frac{1}{2a} \log \frac{a+x}{a-x} + c$
- $\int \frac{dx}{x^2-a^2} = \frac{1}{2a} \log \frac{x-a}{x+a} + c$

- $\int \csc x \cot x dx = -\csc x + c$
- $\int \csc^2 x dx = -\cot x + c$
- $\int \sec x \tan x dx = \sec x + c$
- $\int \sec^2 x dx = \tan x + c$
- $\int \sin x dx = -\cos x + c$
- $\int \cos x dx = \sin x + c$

- $\int (ax+b)^n = \frac{1}{a} \frac{(ax+b)^{n+1}}{n+1} + C$, $n \neq 1$
- $\int \frac{dx}{(ax+b)} = \frac{1}{a} \log |ax+b| + C$
- $\int e^{ax+b} = \frac{1}{a} e^{ax+b} + C$
- $\int \cos(ax+b) dx = \frac{1}{a} \sin(ax+b) + C$
- $\int \sec^2(ax+b) dx = \frac{1}{a} \tan(ax+b) + C$
- $\int \csc^2(ax+b) dx = \frac{-1}{a} \cot(ax+b) + C$
- $\int \csc(ax+b) \cot(ax+b) dx = \frac{-1}{a} \csc(ax+b) + C$

For **Integrals of the form**

(i) $\int \frac{dx}{a + b \sin x}$ (ii) $\int \frac{dx}{a + b \cos x}$ (iii) $\int \frac{dx}{a \sin x + b \cos x + c}$

Put $\cos x = \frac{1 - \tan^2 x/2}{1 + \tan^2 x/2}$, $\sin x = \frac{2 \tan x/2}{1 + \tan^2 x/2}$

Some advanced procedures….

$\int \frac{x^m}{(a+bx)^p}\,dx$ m is a + ve integer	Put a + bx = z
$\int \frac{dx}{x^m (a+bx)^p}$. where either ($m$ and p positive integers) or (m and p are fractions, but $m + p$ = integers > 1)	Put a + bx = zx
$\int x^m (a+bx^n)^p\,dx$, where m, n, p are rationals.	
(i) p is a + ve integer	Apply Binomial theorem to $(a+bx^n)^p$
(ii) p is a − ve integer	Put $x = z^k$ where k = common denominator of m and n.
(iii) $\frac{m+1}{n}$ is an integer	Put $(a+bx^n) = z^k$ where k = denominator of p.
(iv) $\frac{m+1}{n} + p$ is an integer	Put $a + bx^n = x^n z^k$ where k = denominator of fraction p.

$$\int \frac{x^2\,dx}{x^4+kx^2+a^4} = \frac{1}{2}\int \frac{(x^2+a^2)\,dx}{(x^4+kx^2+x^4)} + \frac{1}{2}\int \frac{(x^2-a^2)\,dx}{(x^4+kx^2+a^4)}$$

$$\int \frac{dx}{(x^4+kx^2+a^4)} = \frac{1}{2a^2}\int \frac{(x^2+a^2)\,dx}{(x^4+kx^2+a^4)} - \frac{1}{2a^2}\int \frac{(x^2-a^2)\,dx}{(x^4+kx^2+a^4)}$$

$$\int \frac{dx}{(x^2+k)^n} = \frac{x}{k(2n-2)(x^2+k)^{n-1}} + \frac{(2n-3)}{k(2n-2)}\int \frac{dx}{(x^2+k)^{n-1}}$$

For $\int \frac{dx}{(Ax^2+Bx+C)\sqrt{(ax^2+bx+c)}}$ we need to substitute $\frac{ax^2+bx+c}{Ax^2+Bx+C} = t^2$

Every student knows that the last step is …

$$\int_a^b f(x)\,dx = [F(x) + c]_a^b = F(b) - F(a)$$

Definite Integrals have to be solved by (more than) 14 different ways, depending on the type of problem.

Type 1 - Here no property, specific to Definite Integrals is used.

The integration is solved completely as Indefinite. Finally the Upper and Lower limits are substituted.

Example - 1.1 -

If we need to solve $\int_0^{\pi} \frac{1}{1+\sin x} dx$

we should know how to integrate $\int \frac{1}{1+\sin x} dx$ (Indefinite Integral)

In the solution, notice that no special or specific property of Definite Integral is being used.

Multiplying Numerator and Denominator by $(1 - \sin x)$

$$
\begin{aligned}
I &= \int_0^{\pi} \frac{1}{1+\sin x} \times \frac{1-\sin x}{1-\sin x} dx \\
&= \int_0^{\pi} \frac{(1-\sin x)}{(1^2-\sin^2 x)} dx \\
&= \int_0^{\pi} \frac{1-\sin x}{(\cos^2 x)} dx \\
&= \int_0^{\pi} \frac{1}{\cos^2 x} dx - \int_0^{\pi} \frac{\sin x}{\cos^2 x} dx \\
&= \int_0^{\pi} \sec^2 x dx - \int_0^{\pi} \tan x . \sec x dx \\
&= \left[\tan x\right]_0^{\pi} - \left[\sec x\right]_0^{\pi} \\
&= \left[\tan \pi - \tan 0\right] - \left[\sec \pi - \sec 0\right] \\
&= \left[0-0\right] - \left[-1-1\right] \\
&= 2
\end{aligned}
$$

Similarly **example - 1.2 -**

$$\int_1^2 \log x dx = \left[x \log x - x\right]_1^2$$

As we know from indefinite integrals that Integration of Ln |x| is x Ln | x | - x

If we substitute the upper limit we get 2 ln 2 - 2

And substituting the lower limit we get 1 ln 1 - 1 = -1

So final result is 2 ln 2 - 2 - (-1) = 2 ln 2 - 1

Example - 1.3 -

If we need to integrate by parts then do not apply the limits at intermediate steps.

Solve the whole problem as indefinite and then finally apply the limits

Recall $\int uv \ dx = u \int v \ dx - \int \left(u' \int v \ dx\right) \ dx.$

So to solve $\int_0^1 (x^2+1)\, e^{-x}\, dx$ we proceed as above equation

Let $u = x^2 + 1$ and $dv = e^{-x}\, dx$. Then $du = 2x\, dx$ and $v = -e^{-x}$

I = $\int_0^1 (x^2+1)\, e^{-x}\, dx = \left[-(x^2+1)e^{-x}\right]_0^1 + 2\int_0^1 xe^{-x}\, dx$

$$\int_0^1 xe^{-x}\, dx = \left[-xe^{-x}\right]_0^1 + \int_0^1 e^{-x}\, dx = \left[-e^{-x}(x+1)\right]_0^1$$

Thus finally the required Solution is $\int_0^1 (x^2+1)\, e^{-x}\, dx = \left[-e^{-x}\left(x^2+2x+3\right)\right]_0^1 = -6e^{-1} + 3$

Example - 1.4 -

Show that $\int_0^1 x \tan^{-1} x dx = \frac{\pi}{4} - \frac{1}{2}$

$$\int_0^1 x \tan^{-1} x\, dx = \tan^{-1} x \int_0^1 x dx - \int_0^1 \left(\int x dx\right) \frac{d}{dx}\left(\tan^{-1} x\right) dx$$

$$= \left[\frac{x^2}{2} \tan^{-1} x\right]_0^1 - \frac{1}{2}\int_0^1 \frac{x^2}{1+x^2} dx$$

$$= \left[\frac{x^2}{2} \tan^{-1} x\right]_0^1 - \frac{1}{2}\int_0^1 \frac{1+x^2-1}{1+x^2} dx$$

$$= \frac{1}{2}\left(\frac{\pi}{4}\right) - \frac{1}{2}\left[\int_0^1 dx - \int_0^1 \frac{dx}{1+x^2}\right]$$

$$= \frac{\pi}{8} - \frac{1}{2}\left[x - \tan^{-1} x\right]_0^1$$

$$= \frac{\pi}{8} - \frac{1}{2}\left[1 - \frac{\pi}{4}\right]$$

$$= \frac{\pi}{8} - \frac{1}{2} + \frac{\pi}{8}$$

$$= \frac{\pi}{4} - \frac{1}{2}$$

Example - 1.5 -

Solve $\int_0^2 x\sqrt{x+2}$ **dx** Put x + 2 = t^2 so dx = 2t dt at x = 0 t = √2 at x = 2 x + 2 = 4 = t^2 => t = 2

$$I = \int_{\sqrt{2}}^{2}\left(t^2-2\right)\sqrt{t^2}\,2t\,dt$$

$$= 2\int_{\sqrt{2}}^{2}\left(t^2-2\right)t^2\,dt$$

$$= 2\int_{\sqrt{2}}^{2}\left(t^4-2t^2\right)dt$$

$$= 2\left[\frac{t^5}{5}-\frac{2t^3}{3}\right]_{\sqrt{2}}^{2}$$

$$= 2\left[\frac{32}{5}-\frac{16}{3}-\frac{4\sqrt{2}}{5}+\frac{4\sqrt{2}}{3}\right]$$

$$= 2\left[\frac{96-80-12\sqrt{2}+20\sqrt{2}}{15}\right]$$

$$= 2\left[\frac{16+8\sqrt{2}}{15}\right]$$

$$= \frac{16\left(2+\sqrt{2}\right)}{15}$$

$$= \frac{16\sqrt{2}\left(\sqrt{2}+1\right)}{15}$$

Example - 1.6 -

Solve $\int_{\frac{1}{3}}^{1}\frac{\left(x-x^3\right)^{\frac{1}{3}}}{x^4}dx$

let $x = \sin\theta \Rightarrow dx = \cos\theta\, d\theta$

When $x = \frac{1}{3}$, $\theta = \sin^{-1}\left(\frac{1}{3}\right)$ and when $x = 1$, $\theta = \frac{\pi}{2}$

$$\Rightarrow I = \int_{\sin^{-1}\left(\frac{1}{3}\right)}^{\frac{\pi}{2}}\frac{\left(\sin\theta-\sin^3\theta\right)^{\frac{1}{3}}}{\sin^4\theta}\cos\theta\, d\theta$$

$$= \int_{\sin^{-1}\left(\frac{1}{3}\right)}^{\frac{\pi}{2}}\frac{\left(\sin\theta\right)^{\frac{1}{3}}\left(1-\sin^2\theta\right)^{\frac{1}{3}}}{\sin^4\theta}\cos\theta\, d\theta$$

$$= \int_{\sin^{-1}\left(\frac{1}{3}\right)}^{\frac{\pi}{2}}\frac{\left(\sin\theta\right)^{\frac{1}{3}}\left(\cos\theta\right)^{\frac{2}{3}}}{\sin^4\theta}\cos\theta\, d\theta$$

$$= \int_{\sin^{-1}\left(\frac{1}{3}\right)}^{\frac{\pi}{2}}\frac{\left(\sin\theta\right)^{\frac{1}{3}}\left(\cos\theta\right)^{\frac{2}{3}}}{\sin^2\theta\sin^2\theta}\cos\theta\, d\theta$$

$$= \int_{\sin^{-1}\left(\frac{1}{3}\right)}^{\frac{\pi}{2}}\frac{\left(\cos\theta\right)^{\frac{5}{3}}}{\left(\sin\theta\right)^{\frac{5}{3}}}\operatorname{cosec}^2\theta\, d\theta$$

$$= \int_{\sin^{-1}\left(\frac{1}{3}\right)}^{\frac{\pi}{2}}\left(\cot\theta\right)^{\frac{5}{3}}\operatorname{cosec}^2\theta\, d\theta$$

Let $\cot\theta = t \Rightarrow -\operatorname{cosec}2\theta\, d\theta = dt$

When $\theta = \sin^{-1}\left(\frac{1}{3}\right)$, $t = 2\sqrt{2}$ and when $\theta = \frac{\pi}{2}$, $t = 0$

$$\therefore I = -\int_{2\sqrt{2}}^{0}\left(t\right)^{\frac{5}{3}}dt$$

$$= -\left[\frac{3}{8}\left(t\right)^{\frac{8}{3}}\right]_{2\sqrt{2}}^{0}$$

$$= -\frac{3}{8}\left[\left(t\right)^{\frac{8}{3}}\right]_{2\sqrt{2}}^{0}$$

$$= -\frac{3}{8}\left[-\left(2\sqrt{2}\right)^{\frac{8}{3}}\right]$$

$$= \frac{3}{8}\left[\left(\sqrt{8}\right)^{\frac{8}{3}}\right]$$
$$= \frac{3}{8}\left[(8)^{\frac{4}{3}}\right]$$
$$= \frac{3}{8}[16]$$
$$= 3 \times 2$$
$$= 6$$

AIEEE (now known as IIT-JEE main) - 2004

Solve

The value of $I = \int_0^{\pi/2} \frac{(\sin x + \cos x)^2}{\sqrt{1+\sin 2x}}\,dx$ is

(a) 2 (b) 1 (c) 0 (d) 3

(a) : $\int_0^{\pi/2} \frac{(\sin x + \cos x)^2}{\sqrt{(\sin x + \cos x)^2}}\,dx$

$= \int_0^{\pi/2} (\sin x + \cos x)\,dx$

$= \left(\frac{\cos x}{-1} + \sin x\right)_0^{\frac{\pi}{2}}$

$= 1 - (-1) = 2$

AIEEE (now known as IIT-JEE main) - 2007

The solution for x of the equation $\int_{\sqrt{2}}^{x} \frac{dt}{t\sqrt{t^2-1}} = \frac{\pi}{2}$ is

(a) $\frac{\sqrt{3}}{2}$ (b) $2\sqrt{2}$ (c) 2 (d) π

Solution :

$$\left[\sec^{-1} t\right]_{\sqrt{2}}^{x} = \frac{\pi}{2}$$

$$\sec^{-1} x - \sec^{-1}\sqrt{2} = \frac{\pi}{2} \Rightarrow \sec^{-1} x = \frac{\pi}{2} + \frac{\pi}{4} = \frac{3\pi}{4}$$

$x = -\sqrt{2}$ There is no correct option.

Example - 1.7 -

If $I = \int_2^3 \frac{2x^5 + x^4 - 2x^3 + 2x^2 + 1}{(x^2+1)(x^4-1)}\, dx$, then

I equals

(a) $\frac{1}{2} \log 6 + \frac{1}{10}$ (b) $\frac{1}{2} \log 6 - \frac{1}{10}$

(c) $\frac{1}{2} \log 3 - \frac{1}{10}$ (d) $\frac{1}{2} \log 2 + \frac{1}{10}$

Solution

$$2x^5 + x^4 - 2x^3 + 2x^2 + 1$$
$$= 2x^3(x^2 - 1) + (x^2 + 1)^2$$

$$\therefore\ I = \int_2^3 \frac{2x^3(x^2-1) + (x^2+1)^2}{(x^2+1)^2 + (x^2-1)}\, dx$$

$$= \int_2^3 \frac{2x^3\, dx}{(x^2+1)^2} + \int_2^3 \frac{dx}{x^2-1}$$

$$= I_1 + \frac{1}{2} \log \left|\frac{x-1}{x+1}\right| \Bigg]_2^3$$

$$= I_1 + \frac{1}{2}\left(\log\frac{1}{2} - \log\frac{1}{3}\right)$$

where $I_1 = \int_2^3 \frac{x^2}{(x^2+1)^2}\, (2x)\, dx$

Put $x^2 + 1 = t$, $2x\, dx = dt$

$$\therefore\ I_1 = \int_5^{10} \frac{t-1}{t^2}\, dt = \left(\log|t| + \frac{1}{t}\right)\Bigg]_5^{10}$$

$$= \log 2 - \frac{1}{10}$$

Thus, $I = \frac{1}{2} \log 6 - \frac{1}{10}$

Type 2 - Here special properties of Definite Integrals are used

Let us see the list of properties

$$\int_a^b f(x)\,dx = \int_a^b f(t)\,dt$$

$$\int_a^b f(x)\,dx = -\int_b^a f(x)\,dx \text{. In particular, } \int_a^a f(x)\,dx = 0$$

$$\int_a^b f(x)\,dx = \int_a^c f(x)\,dx + \int_c^b f(x)\,dx$$

$$\int_a^b f(x)\,dx = \int_a^b f(a+b-x)\,dx$$

$$\int_0^a f(x)\,dx = \int_0^a f(a-x)\,dx$$

$$\int_0^{2a} f(x)\,dx = \int_0^a f(x)\,dx + \int_0^a f(2a-x)\,dx$$

$$\int_0^{2a} f(x)\,dx = 2\int_0^a f(x)\,dx, \text{ if } f(2a-x) = f(x) \text{ and}$$
$$0 \text{ if } f(2a-x) = -f(x)$$

(i) $\int_{-a}^a f(x)\,dx = 2\int_0^a f(x)\,dx$, if f is an even function, i.e., if $f(-x) = f(x)$.

(ii) $\int_{-a}^a f(x)\,dx = 0$, if f is an odd function, i.e., if $f(-x) = -f(x)$.

The property of Modulus

$$\left| \int_a^b f(x)\,dx \right| < \int_a^b |f(x)|\,dx$$

An Example to start the discussion

The absolute value of $\int_{10}^{19} \frac{\sin x}{1+x^8}\,dx$ is

(a) less than 10^{-7} (b) more than 10^{-7}

(c) less than 10^{-6} (d) more than 10^{-6}