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The High-speed Flywheel Energy Storage System

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1. Introduction

At the present level of technology the electricity generation has already ceased to be a problem. However, years are passing by under the slogan of seeking for methods of effective energy storage. The energy storage method shall be feasible and environmentally safe. That's why the methods, once regarded as inefficient, are recently taken into consideration. The development in materials technology (carbon fibre, semiconductors, etc.) brought back the concept of a flywheel. This idea has been applied to high-speed flywheel energy storage.

2. Electromechanical energy storage using a flywheel

A flywheel energy storage system converts electrical energy supplied from DC or three-phase AC power source into kinetic energy of a spinning mass or converts kinetic energy of a spinning mass into electrical energy.
The moment of inertia of a hollow cylinder with outer radius r_z, and inner radius r_w is:

$$J = \frac{1}{2}\pi h\rho\left(r_z^4 - r_w^4\right) \tag{1}$$

Maximum amount of kinetic energy stored in a rotating mass:

$$W_{k\max} = \frac{1}{2}J\omega_{\max}^2 = \frac{\pi}{4}h\rho\left(r_z^4 - r_w^4\right)\omega_{\max}^2 \tag{2}$$

where: J – moment of inertia, ω – angular velocity.
The force acting on a segment of spinning hoop (Fig. 1) is:

$$dF_r = dm\cdot\frac{v^2}{r} = \rho\cdot h\cdot d\varphi\cdot dr\cdot v \tag{3}$$

where: ρ – density of the hoop material, h – height, r – radius, v – peripheral velocity, φ – angle, F – force, m – mass.
The net force acting in the direction of axis x, resulting from elementary forces dF_r, is:

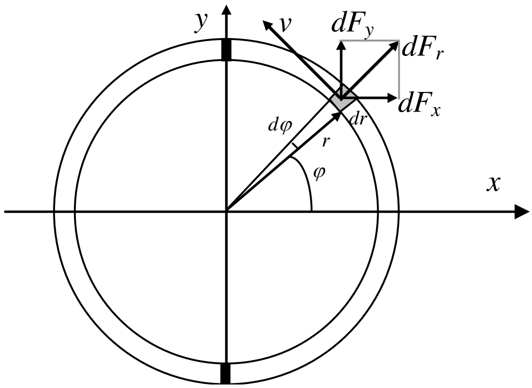

Fig. 1. Forces acting on the segment of a rotating hoop

$$F_x = 2\int_0^{\frac{\pi}{2}} dF_r \cos\varphi \cdot d\varphi = 2\rho \cdot h \cdot dr \cdot v^2 \int_0^{\frac{\pi}{2}} \cos\varphi \cdot d\varphi = 2\rho \cdot h \cdot dr \cdot v \tag{4}$$

Bursting stress (in the hoop cross sections shaded in Fig. 1):

$$\sigma_r = \frac{F_x}{2 \cdot h \cdot dr} = \frac{2\rho \cdot h \cdot dr \cdot v^2}{2 \cdot h \cdot dr} = \rho \cdot v^2 \tag{5}$$

Hence, the maximum allowable peripheral velocity for a material with the density ρ and allowable tensile stress $R_e = \sigma_{r\max}$:

$$v_{\max}^2 = \frac{R_e}{\rho} \tag{6}$$

Maximum rotational velocity of a flywheel depends on the allowable peripheral velocity at its surface (6):

$$\omega_{\max}^2 = \frac{v_{\max}^2}{r_z^2} = \frac{R_e}{r_z^2 \rho} \tag{7}$$

Substituting (7) into (2) we have:

$$W_{k\max} = \frac{\pi}{4} h\rho\left(r_z^4 - r_w^4\right)\frac{R_e}{r_z^2\rho} = \pi\left(r_z^2 - r_w^2\right)h\frac{\left(r_z^2 + r_w^2\right)R_e}{r_z^2\rho} = V\left(1 + \left(\frac{r_w}{r_z}\right)^2\right) \tag{8}$$

Hence can be found the flywheel mass:

$$m = \pi h\left(r_z^2 - r_w^2\right)\rho = \frac{4W_{k\max}}{R_e} \cdot \frac{1}{1 + \left(\frac{r_w}{r_z}\right)^2}\rho \tag{9}$$

In order to minimize the flywheel mass it shall be made in the form of a thin-walled hollow cylinder.
From relation (9) the ratio of maximum stored energy to the flywheel mass is:

$$\frac{W_{k\max}}{m} = \frac{W_{k\max}}{\rho V} = \frac{R_e}{\rho} \cdot \frac{1 + \left(\frac{r_w}{r_z}\right)^2}{4} \tag{10}$$

For $r_z \approx r_w$ relation (10) reduces to the form of:

$$\frac{W_{k\max}}{m} \approx \frac{R_e}{2\rho} = \frac{v_{\max}^2}{2} \tag{11}$$

As follows from (11), a light structure (a large amount of energy per unit of mass) can be achieved using a material with possible low density ρ and high tensile strength R_e. Materials that meet these requirements are composites (Kevlar, carbon fibre, glass fibre in combination with a filler) or composite bandage (in order to improve stiffness) on a ring of a light metal, e.g. aluminium.

	Density ρ [kg/m^3]	Strength Re [GPa]	$v_{\max}$ [m/s]	W/m [MJ/kg]
Steel	$7.8 \cdot 10^3$	1.8	480.4	0.23
Titanium	$4.5 \cdot 10^3$	1.2	516	0.27
Composite glass fibre	$2.0 \cdot 10^3$	1.6	894.4	0.80
Composite carbon fibre	$1.5 \cdot 10^3$	2.4	1256	1.60

Table 1. Parameters of typical flywheel materials

A flywheel of a larger energy per unit of mass and the given outer radius r_z, chosen for constructional reasons, has to rotate with a higher peripheral velocity (11) and, consequently, with a higher angular velocity (7).
Since in this case peripheral velocities of high-speed rotors are exceeding the speed of sound, the rotor should be enclosed in a hermetic vacuum chamber. In consequence, the energy store structure - and particularly bearings, become complicated (due to vacuum maintained in inside the enclosure should be used magnetic bearings and a system stabilizing the rotor axle position in space The flywheel, integrated with the electric machine, should rotate without a contact with motionless parts (magnetic levitation). Magnetic bearings should be made of permanent magnets (high efficiency is required) while an electromagnetic system should only assist them to a certain extent and stabilize the axle position. Due to a required very high efficiency, the flywheel shall be driven by a permanent magnet motor installed inside the enclosure. Vacuum inside the enclosure prevents exchange of heat between the FES components and causes problems with heat removal from windings of the electric machine operated as a motor or generator. An advantage of vacuum is lack of losses caused by the rotor friction in air (at peripheral velocities of 700-1000m/s) and noiseless operation.

The electric machine must be controlled by a power electronic system enabling its operation as a motor or generator and adjusting electric power parameters alternately to the needs of the accelerated spinning mass or electrical loads (or an electric network) supplied from FES. If the energy storage system is operated as an autonomous energy source (isolated operation) it must be provided with a power electronic system that prohibits propagation of load unbalance (the output voltage double-frequency ripple component) to the flywheel torque.
The amount of energy stored in FES is proportional to the square of angular velocity. It means that at the 1/3 of maximum velocity remains only ca. 10% of maximum energy. The energy store should be therefore operated within the speed range from 1/3 to maximum speed. The voltage at the electric machine winding changes with the ratio 1:3, and the power electronic system shall be designed to tolerate such changes. in order to minimize losses (conduction losses in semiconductor devices) the maximum voltage applied to electric machine should be possibly high (up to 1000V).
The design of an energy storage system that meets up-to-date requirements is an interdisciplinary and complex engineering task that requires the use of the-state-of-the art technologies and materials. The energy storage system can be applied to:

- Power quality improvement systems to compensate active power peaks and limit their impact on power supply network and reduce peak loads. Required are: a large stored energy (of the order of hundreds MJ) and large instantaneous power that enables discharging during a tens of seconds.
- Standby power supplies to backup or start other power sources (a motor-generator set or switching to another network) for particularly important and sensitive processes.
- Systems for storage and controlled release of energy produced by alternative autonomous electric power sources, like photovoltaic or wind power plants. In such systems store energy in time when there is no demand from electricity users. A flywheel energy storage system intended for supporting alternative autonomous sources shall exhibit very high energy efficiency (due to the necessity of long accumulation time) and three-phase output with possibility for unbalanced load at constant frequency (50 Hz) and constant rms voltage magnitude. The amount of stored energy is ca. 5÷10 MJ.
- Limiting wind farms power fluctuations by means of a dynamic accumulation of peak power generated during high-wind periods and release it during low-wind periods.
- Accumulation (storage) of energy recovered from regenerative braking of intermittently started and stopped (or reversing) large-power drives (e.g. rolling mills and winders) or energy recovered from discharging large electromagnets.
- Elevators in buildings with intensive traffic flow ("intelligent building"). An elevator equipped with an energy storage system will consume energy solely to compensate losses.
- Large industrial plants (large-power flywheel energy storage systems) in order to mitigate voltage fluctuations, power supply back-up during supply systems switching, and power quality improvement by means of peak loading and unloading reduction. Reduction of peak active power will result in reduced transmission losses and enable the use of more economical installations (smaller cross-sectional areas, transformer powers, etc.), smaller peak contracted power.
- Urban buses. Flywheel energy storage systems designed for mobile applications with relatively small energy stored (6÷10 MJ) and suitable for charging and discharging with large powers (100÷150 kW) can be utilized in urban buses (charged at bus stops).

- Urban and suburban electric transportation systems and hybrid vehicles (internal combustion engine, generator, electric motor), flywheel energy storage systems can absorb kinetic energy of a braking vehicle and reuse it during travel.

3. Technical requirements for flywheel energy storage systems

- High efficiency.
- Small mass and volume.
- Reliability, durability and safety.
- Capability for operation in a three-phase power network or autonomous operation with unbalanced load.
- Large short-duration power (capability for quick charging and discharging).

4. Electric machine for the flywheel energy storage purposes

Flywheel energy storage systems can utilize all types of AC three-phase machines. The choice of the machine type is determine by the energy storage application and particularly by expected duration of energy storage. In energy storage systems with expected long duration of energy storage idle losses should be radically limited. Idle losses in systems with long duration of energy storage should be radically limited. Such systems can utilize asynchronous induction machines or synchronous machines. During energy charging or discharging a small amount of energy is needed for the machine excitation (power losses in the field winding resistance in a synchronous machine or losses due to the magnetizing (reactive) component in an induction machine). In energy storage systems intended for relatively short duration storage, permanent magnet machines (synchronous or brushless) can be used. In flywheel energy storage systems with a high rotational speed and, consequently, high frequency of the fundamental component of the machine voltage, the difficulty lies in correct shaping of sinusoidal current waveform obtained by means of PWM modulation. In such a case a correct power supply of a brushless DC machine can be more easily achieved. Permanent magnet machines require no additional energy for excitation but certain small losses occur in them due to currents induced in conducting parts by variable magnetic field of rotating magnets. These losses can be reduced employing brushless coreless machines. Such machines have very small winding inductance and in order to achieve a continuous current they require additional external reactors when supplied from PWM modulated inverters.

5. Examples of flywheel energy storage applications

In an autonomous system with alternative electric energy source (Fig. 2a) the energy store supplies loads if loss of supply from a base power source occurs. The energy storage can be used in uninterruptible power supply systems (UPSs) of selected loads (Fig. 2b). Upon voltage loss or decrease in the line voltage magnitude a load and energy storage system are instantaneously disconnected (by means of thyristor switches) from the supply line and energy store turns to the generator mode, thereby powering sensitive (critical) loads Another application of an energy storage system is stabilization of supply voltage (or limitation of peak currents in a supply line) of loads characterized with fast-changing, short-duration loading far exceeding the average load.

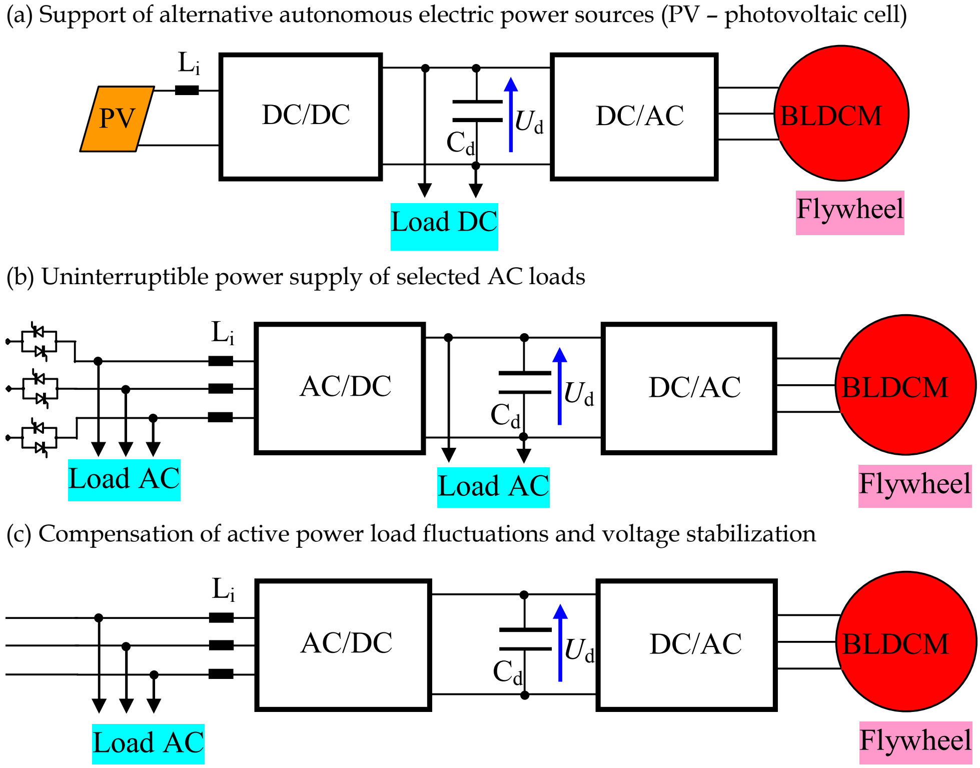

Fig. 2. Examples of spinning energy storage applications; AC/DC, DC/AC – power electronic converters, BLDCPM – electric machine (Brushless D.C. Permanent Magnet Motor)

6. Controlling energy release from a flywheel energy storage system

The amount of energy stored in a rotating mass is proportional to the angular velocity squared. It means that energy store can be effectively utilized within the range from maximum angular velocity (W_{max}) to 1/3 of angular velocity (1/9 W_{max}). There are several solutions for limiting the maximum power of energy release from (or supplied to) the energy store.

Figure 3 shows the relative energy (W/W_m) and power (P/P_m) vs. relative angular velocity (ω/ω_m). Line (1) is the characteristic of a storage system operated within the velocity range (0.5÷1)ω_{max} with limited power. The consequence of the power limitation is the necessity for limiting the current maximum value according to relation $I_{d\max} = P_{\max} / U_m$ (curve 3). Line (2) represents the power change for operation with the current maximum value determined by the straight line (4).

Another control method consists in operation with constant maximum power within the angular velocity range (0.5÷1)ω_{max}. Characteristics of the storage system controlled employing this method is shown in Fig. 4. A boundary of the control method can be set at a lower velocity; this results in limiting maximum power to a lower value.

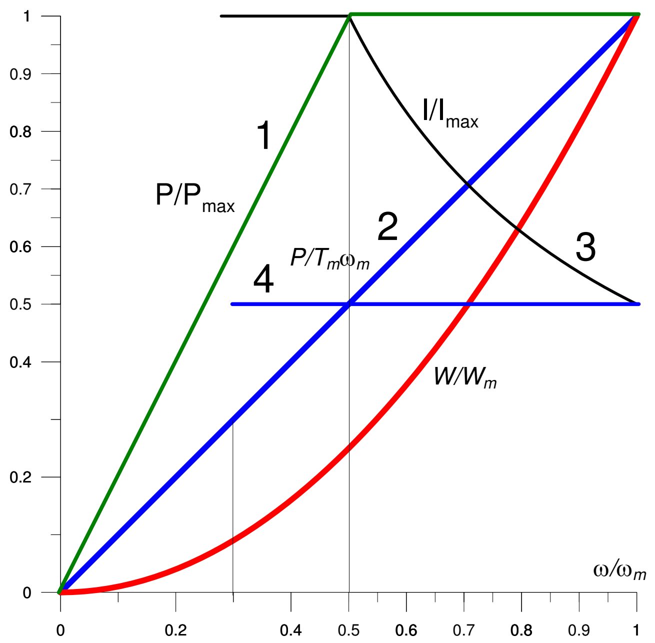

Fig. 3. Characteristic of a energy storage system with $P_{max}=f(\omega)$

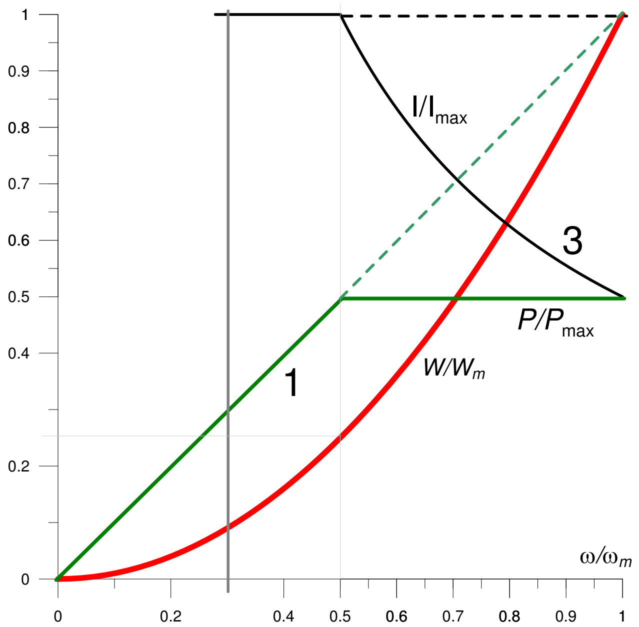

Fig. 4. Characteristic of a energy storage system with $P/P_{max}=const$ for $\omega>\omega_{max}/2$

It should be borne in mind that energy of 1kWh (3.6 MJ) is equivalent to potential energy of the mass 1000 kg at the height of 367 m, i.e. the release the amount of energy (equivalent to that consumed by a 100 W bulb during 10 hours) required to throw a 1-ton car to the height of 367 m ($3.6{\cdot}10^6$[J] =1000[kg]·9.81[m/s^2]h hence h =367[m]; air friction and the car and ground deformations are not taken into account).

7. Permanent magnet motors

Permanent magnet motors combine features of classical DC separately excited motors with advantages of an induction motor drive. They are manufactured in many structural variations with respect to both the permanent magnets arrangement and the method of their fixing, as well as the motor applications (permanent magnets in the stator or rotor). In terms of the current and the back electromotive force waveforms, permanent magnet machines can be categorized into two types:

- Permanent Magnet Synchronous Motor (PMSM),
- Brushless Permanent Magnet DC Motors (BLDCM, BLDC, BLPMDCM).

Permanent magnet synchronous motors (*PMSM*) exhibit properties similar to those of synchronous AC machines. They are characterized by:

- sinusoidal distribution of magnetic flux in the air gap,
- sinusoidal phase currents,
- sinusoidal back electromotive force (*BEMF*).

In a brushless permanent machine the back electromotive force has a trapezoidal waveform and the required current waveform has the form of rectangular, alternating sign pulses. Idealized relations between the back electromotive force and phase currents are shown in figure 5.

In order to provide a constant torque the machine should be supplied in such a manner that the instantaneous power value remains constant (in figure 5 the instantaneous power waveform in each phase is indicated green). This requirement is met for rectangular phase currents Duration of both the positive and negative pulse is T/3, time-interval between pulses is T/6, and phase-shift between phases is T/3. During each time interval T/6 the current is conducted simultaneously only in two phases. The motor instantaneous power is the sum of powers generated in two phases. The electromagnetic torque is the quotient of the instantaneous power and the motor angular velocity. At constant angular velocity the torque is constant only if the instantaneous power is constant.

A brushless DC permanent magnet motor cannot, as a machine, be supplied without supplementary equipment, thus its integral components are:

- a power electronic converter that provides power supply of appropriate phase windings depending on the rotor position,
- a controller stabilizing the current depending on the required torque (Fig. 6).

8. Bipolar PWM of an inverter supplying a brushless DC permanent magnet motor

The pulse-width modulated voltage-source inverter, supplying a brushless DC permanent magnet motor enables shaping the required phase currents waveform by means of the supply voltage control.

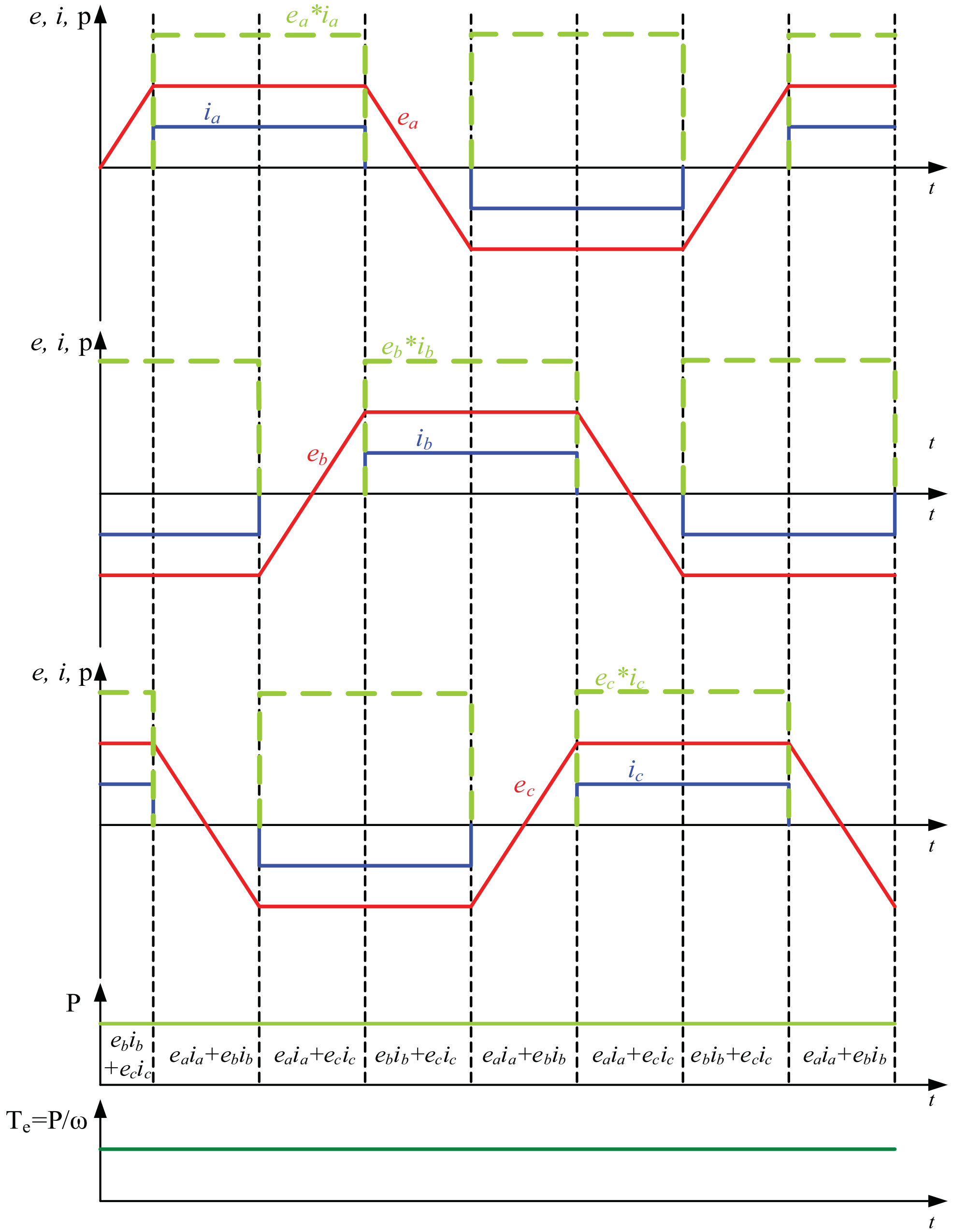

Fig. 5. Desired waveforms of electromotive force, phase currents, instantaneous power and electromagnetic torque

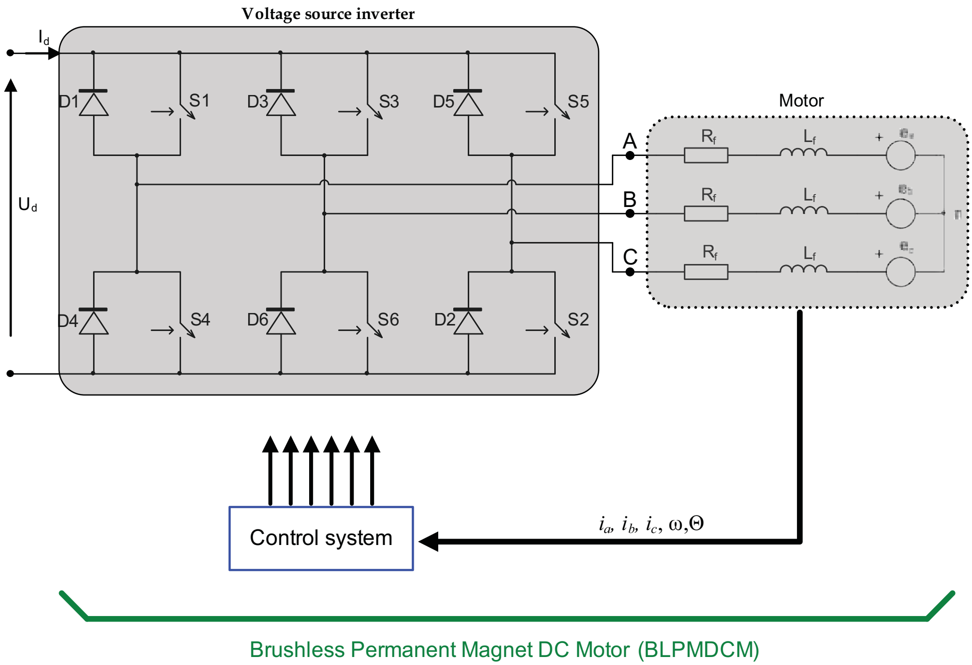

Fig. 6. A brushless permanent magnet DC motor supplied from a voltage source inverter with control system

Where this type of control is employed, only two switches are chopper controlled during the time interval of duration T/6. The sequence of switching is shown in figure 7. The inverter is controlled in the same manner as a single-phase inverter. The switches pairs, e.g. S1 and S6, are switched during the time interval equal T/6. The current flows through two phases A and B connected in series. After elapse of time equal T/6 switch S6 stops conducting and switch S2 is turned on to conduct (chopper controlled) together with the switch S1. Phase A is still connected to the DC voltage source positive terminal, phase B is being connected to its negative terminal. The current flows in phases A and C connected in series. Switch S1 is active during time period T/3 During each time interval with duration T/6 one of the phases is disconnected from both terminals of the DC voltage source, switches are switched specifically at T/6 intervals. At each time-instant the converter operates as a single-phase inverter and can be analysed as such. The inverter configurations with individual switches turned on are shown in figure 8.

9. Torque control of brushless permanent magnet DC machine

Figure 9 shows phase currents (i_a, i_b, i_c), their modules ($|i_a|$, $|i_b|$, $|i_c|$), the sum of the modules ($\Sigma|i|$) and torque (T_e). Apart from the fast-changing torque component resulting from finite time of semiconductor devices PWM switching, also torque ripple occurs due to the current commutation between the motor phase windings. Thus in each 1/6 of the period a noticeable disturbance occurs in the torque waveform.

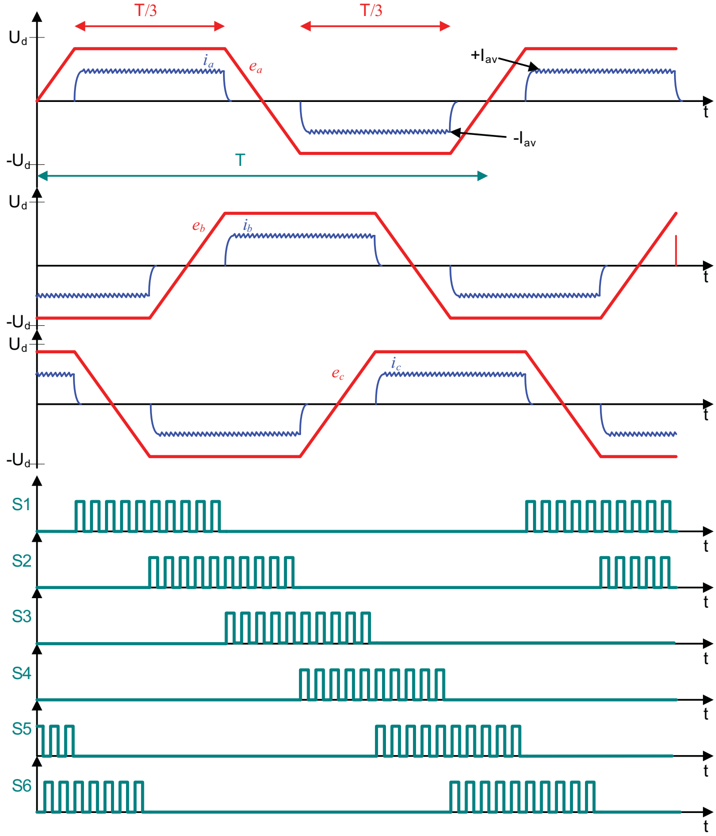

Fig. 7. Bipolar pulse width modulation: phase currents and switch control pulses

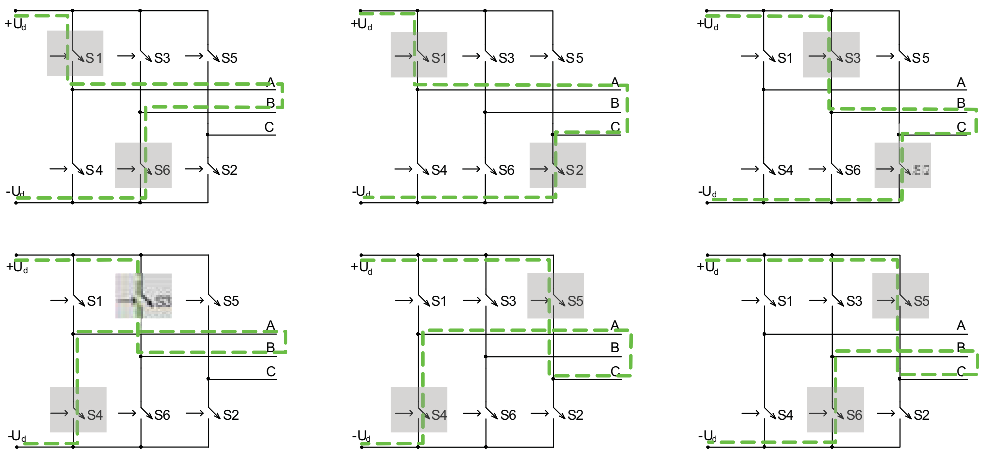

Fig. 8. Bipolar pulse width modulation: the sequence of switching

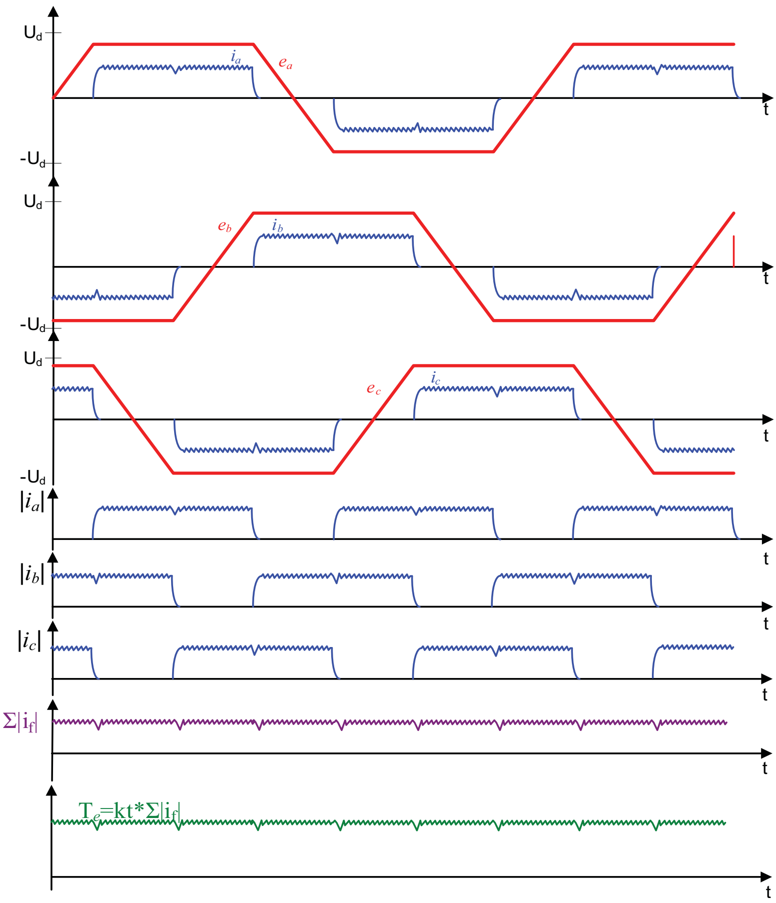

Fig. 9. Actual waveforms of phase currents, their modules and electromagnetic torque

The brushless machine torque is controlled by means of the phase currents control. The control is achieved, similarly as in a classical shunt DC machine, by modulation of fixed frequency pulses width by the output signal of a PI current controller. The feedback signal should be proportional to the actual value of the DC source current module. It can be obtained in two ways:

- measuring the module of the converter input current (DC source current) (Fig. 10), or
- measuring phase currents; the feedback signal is proportional to the sum of the load rectified phase currents (Fig. 11).

A drawback of the first solution is an additional inductance (of the sensor and its connections) connected between the capacitor and semiconductor devices. The inverter should be supplied from a voltage source and the incorporated inductance changes the source character during transient states. This inductance is the source of overvoltages occurring across semiconductor devices that require overvoltage protection in the form of RC snubber circuits to absorb overvoltage energy. These additional components increase both the system complexity and power losses in the converter.

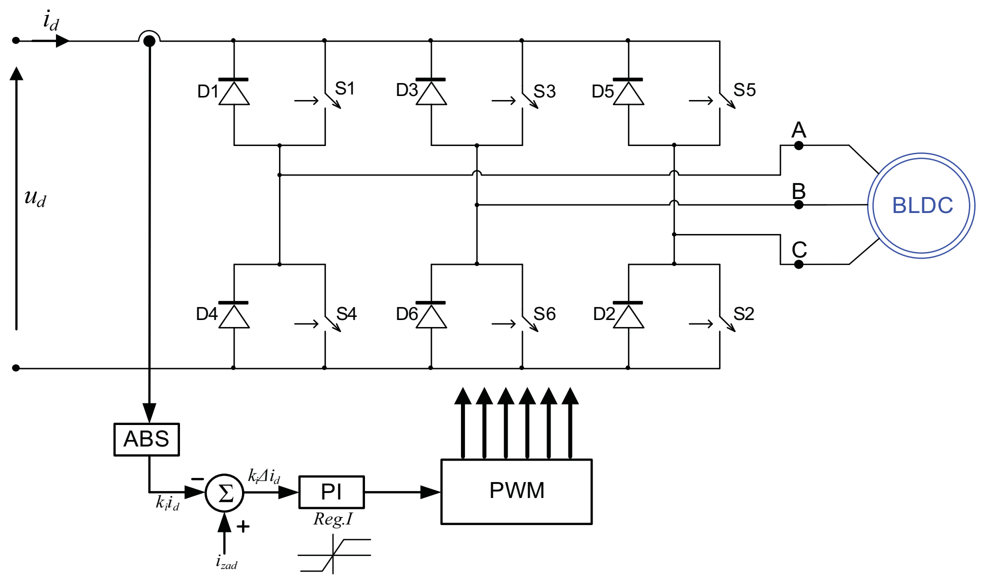

Fig. 10. Measurement of the inverter input current

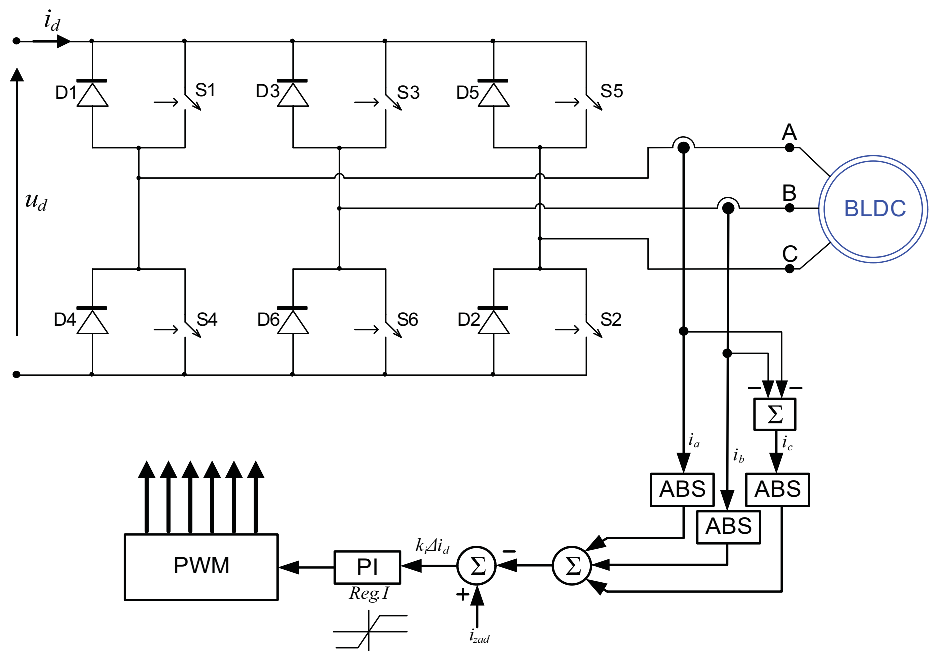

Fig. 11. The feedback signal circuit utilizing the phase currents measurement

Apart from current components from controlled switches, also the currents of backward diodes occur in the DC source current. These currents, flowing in the direction opposite to the switches current, result from the magnetic field energy stored in the machine windings and transferred back to the DC source. The phase current value depends on both these components. Therefore, in order to obtain the feedback signal, the absolute value of the signal proportional to the measured DC source current has to be taken.
The second way the feedback signal can be obtained is the measurement of phase currents. Since $i_a+i_b+i_c=0$ it is sufficient to use transducers in the load two phases. The signal proportional to the DC source current is obtained by summing the absolute values of phase currents (Fig. 11). The error signal is the difference between the DC current reference and the actual source current, reconstructed from the measured phase currents. In the pulse width modulation a high-frequency triangle carrier signal is compared with the current controller output signal. The current controller output signal limit is proportional to the phase-to-phase peak voltage value. That way are generated control pulses of fixed frequency and modulated width to control the inverter transistors switching.

10. Determining the rotor poles position relative to stator windings

Figure 12 shows the cross section of a brushless permanent magnet DC motor. The motor is assumed to have a single pole-pair rotor while the stator winding has three pole-pairs. Figure 13 shows waveforms of the current and back electromotive force in phase A depending on the mutual positions of characteristic points. The analysis starts at the instant when point K coincides with point z_1. At his time the magnet N-pole begins overlapping the stator pole denoted by a. The back electromotive force (BEMF) increases linearly until the stator pole is completely overlapped by the magnet N-pole. This takes T/6. Then, the magnetic flux increases linearly during T/3 thus the back electromotive force is constant. The rectangular waveform of the current in phase A is shaped by means of chopper control.

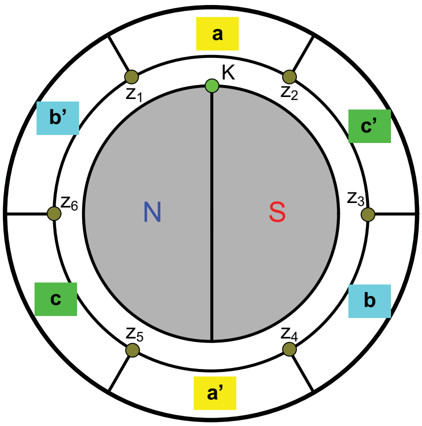

Fig. 12. The cross section of a BLDCM motor

Since point K coincides with z_4 the back electromotive force decreases linearly until point K is in the position where N-pole begins overlapping the stator pole denoted a'. Between the point z_5 and z_1 the back electromotive force is constant and negative.

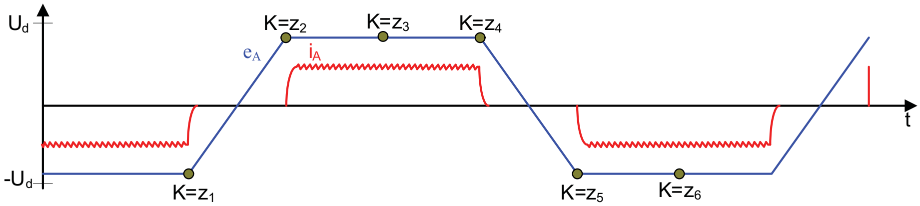

Fig. 13. Waveforms of the current and back electromotive force in one phase depending on the permanent magnet poles position

In motors with trapezoidal BEMF it is essential that voltage switching on or off to a given winding is synchronized with the rotor position relative to this winding axis.

11. AC/DC converter

A unity input power factor control of a three-phase step-up converter is feasible in the rotating co-ordinate frame because in this system the source frequency quantities are represented by constant values. The diagram of the rectifier connection to a supply network is shown in figure 1. Since X_L>>R, the resistances of reactors are disregarded in the diagram.

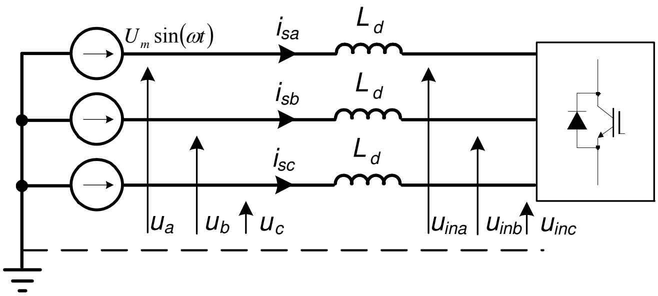

Fig. 14. Diagram of the rectifier connection to a supply network

The following designations are used the diagram of figure 1: i_{sn}– phase currents, u_{sn}– the supply line phase-to-neutral voltages, u_{inn}– the converter output voltage (where n= a, b, c). The phase currents, according to the diagram, are described by equation (12).

$$u_{sn} - u_{inn} = L\frac{di_{sn}}{dt} \tag{12}$$

Converting the equation (12) into the rotating reference frame dq we obtain equation (13).

$$\boldsymbol{u}_{sdq} - \boldsymbol{u}_{indq} = \Delta\boldsymbol{u}_{dq} = L_d\frac{d\boldsymbol{i}_{sdq}}{dt} + j\omega L_d\boldsymbol{i}_{sdq} \tag{13}$$

Decomposing the equation (12) into dq components we obtain (14).

$$u_{ind} = u_{sd} - \Delta u_d = u_{sd} - (L_d\frac{di_{sd}}{dt} - \omega L_d i_{sq}) \tag{14}$$

$$u_{inq} = u_{sq} - \Delta u_q = u_{sq} - (L_d \frac{di_{sq}}{dt} - \omega L_d i_{sd}) \tag{15}$$

Equations (14) and (15) describe the converter input voltages. Inserting the required line current values into these equations we can determine the output voltage waveforms forcing the required current. The components $L_d(di_{sdq}/dt)$ represent the converter dynamic states (load switching or changes in the load parameters). Assuming the control system comprises only proportional terms we obtain from equations (14) and (15) relationships describing the control system (16) and (17).

$$u_{ind} = u_{sd} - (K_R \Delta i_{sd} - K_d \Delta i_{sq}) = u_{sd} - [K_R(i_{sdr} - i_{sd}) - K_d(i_{sqr} - i_{sq})] \tag{16}$$

$$u_{inq} = u_{sq} - (K_R \Delta i_{sq} - K_q \Delta i_{sd}) = u_{sq} - [K_R(i_{sqr} - i_{sq}) - K_q(i_{sdr} - i_{sd})] \tag{17}$$

Figure 15 shows block diagram of the control system and the power circuit. The following designations are used in the diagram: *TP* – switch-on delay units (blanking time),

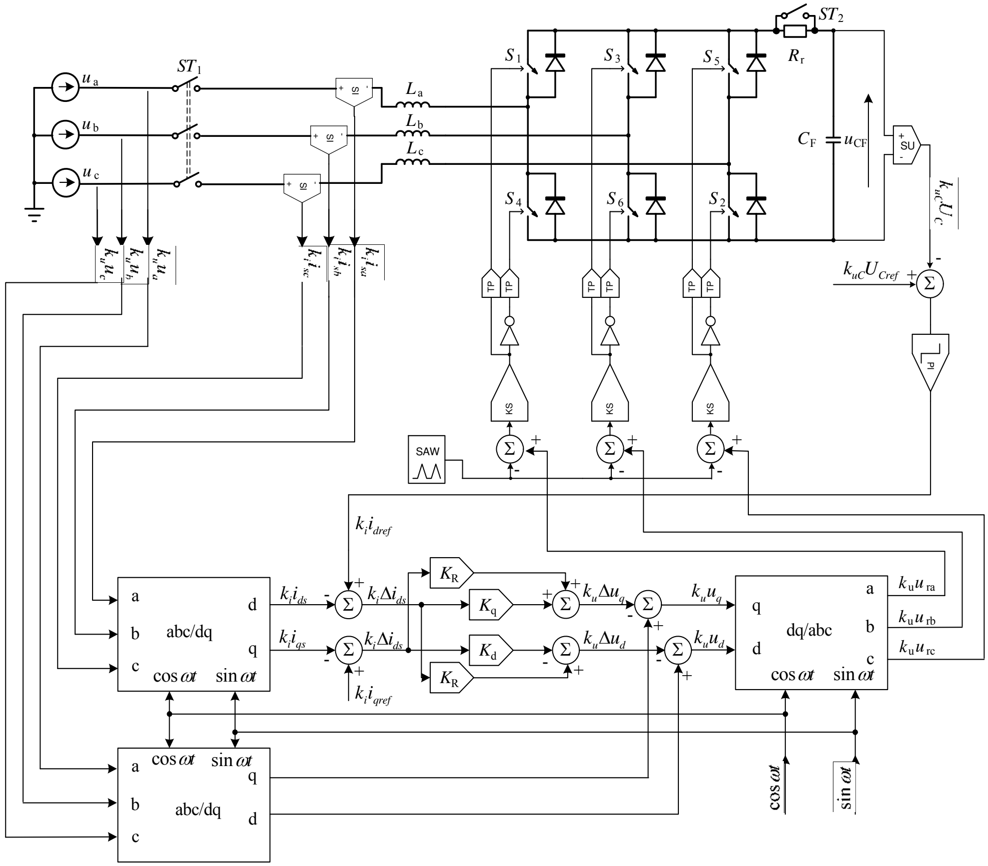

Fig. 15. Block diagram of the control system and the power circuit

PI – proportional-integral controller, *KS*- sign comparator, *SAW*- triangle wave generator, K_R, K_d, K_q- proportional terms, *ST*- contactors, R_a, R_b, R_c - resistors limiting the capacitor charging current, *Σ*- adder.
The control circuit of diagram 15 employs transformation from the thee-phase system to the rotating co-ordinate system (abc→dq), described by equation (12).

$$\begin{bmatrix} v_d \\ v_q \end{bmatrix} = \begin{bmatrix} \cos\omega t & \sin\omega t \\ -\sin\omega t & \cos\omega t \end{bmatrix} \begin{bmatrix} v_a \\ \frac{1}{\sqrt{3}}(v_b - v_c) \end{bmatrix} \tag{18}$$

Where:

$$\begin{cases} v_a = V_m \cos\omega t \\ v_b = V_m \cos(\omega t - \frac{2}{3}\pi) \\ v_c = V_m \cos(\omega t + \frac{2}{3}\pi) \end{cases} \tag{19}$$

11.1 Synchronization circuit

In order to determine the transformation abc→dq it is necessary to generate functions $\cos\omega t$ and $\sin\omega t$, as follows from equation (18), such that the function $\cos\omega t$ will correspond (i.e. be cophasal) to $v_a = V_m \cos\omega t$. In practical solutions various methods for generating the $\cos\omega t$ and $\sin\omega t$ functions are employed, e.g. synchronization with a single, selected phase (normally *a*) employing a single-phase PLL loop. The advantage of this method is an easy implementation in digital technique. Microprocessor systems employ an external, specialized device performing the functions of a phase-locked loop, connected with a microprocessor port dedicated for counting external events. Therefore the CPU workload due to generating the $\cos\omega t$ and $\sin\omega t$ functions is reduced to minimum. A drawback of this method is the generated function is related to only one phase of the synchronizing signal and the system does not control the other phases. In the event of a disturbance starting in phase *c* (a phase jump in the synchronizing voltage caused by switching a large active power load) the control system will respond with large delay. In order to protect the converter from effects of a phase jump the synchronization circuit should control all phases of the synchronizing voltage. Substituting equations (19) describing the three-phase synchronizing voltage into equation (18), the transformation abc→dq takes the form (20).

$$\begin{bmatrix} v_d \\ v_q \end{bmatrix} = \begin{bmatrix} V_m(\cos^2\omega t + \sin^2\omega t) \\ V_m(-\sin\omega t\cos\omega t + \sin\omega t\cos\omega t) \end{bmatrix} = \begin{bmatrix} V_m \\ 0 \end{bmatrix} \tag{20}$$

It follows from equation (20) that if the functions $\cos\omega t$ and $\sin\omega t$ are generated correctly ($\cos\omega t$ is cophasal with voltage in phase *a*), the component in axis *d* equals the amplitude of the synchronizing voltage, whereas the component *q* is zero. This property of the abc→dq transformation is employed in the design of the three-phase synchronization circuit depicted in figure 16.
The following designations are used in figure 16: *PI*- proportional-integral controller, *VCO*- voltage controlled square-wave generator. The *PI* controller input signal is the instantaneous value of the *q*-axis component of abc→dq transformation. The controller tunes the *VCO* oscillator, whose output signal controls the $\cos\omega t$ and $\sin\omega t$ generation circuit. The controller