Stabilization of saturated switching systems

Abdellah Benzaouia
University of Cadi Ayyad, Faculty of Science Semlalia
BP 2390, Marrakech, Morocco
benzaouia@ucam.ac.ma

1. INTRODUCTION

Switched systems are a class of hybrid systems encountered in many practical situations which involve switching between several subsystems depending on various factors. Generally, a switching system consists of a family of continuous-time subsystems and a rule that supervises the switching between them. This class of systems have numerous applications in the control of mechanical systems, the automotive industry, aircraft and air traffic control, switching power converters and many other fields. Two main problems are widely studied in the literature according to the classification given in (Blanchini and Savorgnan, 2006): The first one, which is the one solved in this work, looks for testable conditions that guarantee the asymptotic stability of a switching system under arbitrary switching rules, while the second is to determine a switching sequence that renders the switched system asymptotically stable (see (Liberzon and Morse, 1999) and the reference therein). Following the first approach, (Blanchini *et al.,* 2009) investigate the problem of designing a switching compensator for a plant switching amongst a (finite) family of given configurations (A_i, B_i, C_i).

A main problem which is always inherent to all dynamical systems is the presence of actuator saturations. Even for linear systems, this problem has been an active area of research for many years. Besides approaches using anti-windup techniques (Mulder *et al.,* 2004) and model predictive controls (Camacho and Bordons, 2004), two main approaches have been developed in the literature: The first is the so-called positive invariance approach which is based on the design of controllers which work inside a region of linear behavior where saturations do not occur (see (Benzaouia and Burgat, 1988), (Benzaouia and Hmamed, 1993), (Blanchini, 1999) and the references therein). This approach has already being applied to a class of hybrid systems involving jumping parameters (Benzaouia and Boukas, 2002). It has also been used to design controllers for switching systems with constrained control under complete modelling taking into account reset functions at each switch and different system's dimension . The second approach, however allows saturations to take effect while guaranteeing asymptotic stability (see (Nguyen and Jabbari, 1999, 2000), (Tarbouriech *et al.,* 2006), (Hu *et al.,* 2002)-(Hu and Lin, 2002), (Benzaouia *et al.,* 2006) and the references therein). The main challenge in these two approaches is to obtain large domains of initial states which ensures asymptotic stability for the system despite the presence of saturations (Gilbert and Tan 1991), (Benzaouia and Baddou, 1999), (Benzaouia *et al.,* 2002), (Hu *et al.,* 2002).

The objective of this chapter is to present the available results in the literature for switching systems subject to actuator saturations. These results follow generally two ways: the first concerns the synthesis of non saturating controllers (controllers working inside a large region

of linear behavior where the saturations do not occur), while the second extends the results obtained for unsaturated switching systems by (Mignone *et al.,* 2000), (Ferrari-Trecate *et al.,* 2001) and (Daafouz *et al.,* 2001, 2002) leading to saturating controllers (controllers tolerating saturations to take effect). The second method was firstly used in (Benzaouia *et al.,* 2004) with the use of a multiple Lyapunov function. However, only the intersection of all the corresponding level sets of the local functions was considered as a region of asymptotic stability of the switching system. This drawback is improved in (Benzaouia *et al.,* 2006) and (Benzaouia *et al.,* 2009a) by considering, for the first time, a large set of asymptotic stability composed by the union of all the level sets.

In this context, two main different sufficient conditions of asymptotic stability were obtained for switching systems subject to actuator saturations. Furthermore, these conditions were presented in the form of LMIs for the state feedback control case. A particular attention was given to the output feedback case which has an additive complexity due to the output equation. It was also shown that the LMIs obtained for computing controllers working inside a large region of linear behavior are less conservative.

The obtained results are then extended to uncertain switching system subject to actuator saturations as developed in (Benzaouia *et al.,* 2009b) and (Benzaouia *et al.,* 2009c) respectively. The uncertainty types considered in these two works are the polytopic one and the structured one. This second type of uncertainty was also studied, without saturation, in (Hetel *et al.,* 2006). Thus, in this work (Benzaouia *et al.,* 2009a), two directions are explored: the first concerns the synthesis of non saturating controllers, while the second direction deals with controllers tolerating saturations to take effect under polytopic uncertainties. For structured uncertainties studies in (Benzaouia *et al.,* 2009b), the synthesis of the controller follows two different approaches, the first one deals firstly with the nominal system and then uses a test to check the asymptotic stability in presence of uncertainties while the second considers the global representation of the uncertain system.

2. Stabilization of switching systems subject to actuator saturation

2.1 PROBLEM FORMULATION

In this section, we give a more precise problem statement for the class of systems under consideration, namely, discrete-time switching linear systems with input saturation and state or output feedback. An equivalent description of such systems, based on the indicator function is also used in this work. The main results of this section are published in (Benzaouia *et al.,* 2009a).

Thus, we consider systems described by:

$$
\begin{aligned}
x(t+1) &= A_\alpha x(t) + B_\alpha sat(u(t)) \\
y(t) &= C_\alpha x(t)
\end{aligned}
\tag{1}
$$

where $x \in \mathbb{R}^n$ is the state, $u \in \mathbb{R}^m$ is the control, $y \in \mathbb{R}^p$ is the output, $sat(.)$ is the standard saturation function and α a switching rule which takes its values in the finite set $\mathcal{I} := \{1, \ldots, N\}$, $t \in \mathbb{Z}_+$. The saturation function is assumed here to be normalized, *i. e.,* $(|sat(u)| = min\{1, |u|\})$. Each subsystem α is called a mode.

Definition 2.1. *(Lygeros et al., 1999) An hybrid time basis τ is an infinite or finite sequence of sets $I_s = \{t \in \mathbb{N} : t_s \leq t \leq \bar{t}_s\}$, with $\bar{t}_s = t_{s+1}$ for $s \in \mathcal{L} = \{0, \ldots, L\}$, and if $card(\tau) = L + 1 < \infty$ then $\bar{t}_L$ can be finite or infinite.*

Throughout this chapter, it is assumed that:

- The switching system is stabilizable;
- Matrices C_α are of full rank;
- $t_{s+1} \geq t_s + 1, \forall s \in \mathcal{L}$;
- the switching rule is not known *a priori* but $\alpha(t)$ is available at each t.

The third assumption ensures that at each time only one subsystem is active. The fourth assumption corresponds to practical implementations where the switched system is supervised by a discrete-event system or operator allowing for $\alpha(t)$ to be known in real time.

In this work, we are interested by the synthesis of stabilizing controllers for this class of hybrid systems subject to actuator saturation. We use a feedback control law as:

$$u(t) = F_\alpha x(t) = K_\alpha y(t), \tag{2}$$

and write the closed-loop system as

$$\begin{aligned} x(t+1) &= A_\alpha x(t) + B_\alpha sat(F_\alpha x(t)). \\ &= A_\alpha x(t) + B_\alpha sat(K_\alpha y(t)) \end{aligned} \tag{3}$$

Upon introducing the indicator function:

$$\xi(t) = [\xi_1(t), \dots, \xi_N(t)]^T \tag{4}$$

where $\xi_i(t) = 1$ if the switching system is in mode i and $\xi_i(t) = 0$ if it is in a different mode, one can write the closed-loop system (3) as follows:

$$\begin{aligned} x(t+1) &= \sum_{i=1}^{N} \xi_i(t)[A_i x(t) + B_i sat(F_i x(t))]. \\ &= \sum_{i=1}^{N} \xi_i(t)[A_i x(t) + B_i sat(K_i C_i x(t))] \end{aligned} \tag{5}$$

2.2 PRELIMINARY RESULTS

In this section, we recall two results on which our work is based. Let α be fixed. Then System (3) becomes a linear time-invariant system with input saturation given by:

$$x(t+1) = Ax(t) + Bsat(Fx(t)) \tag{6}$$

Define the following subsets of $\mathbb{R}^n$:

$$\varepsilon(P, \rho) = \{x \in \mathbb{R}^n | x^T P x \leq \rho\}, \tag{7}$$

$$\mathcal{L}(F) = \{x \in \mathbb{R}^n | |F_l x| \leq 1, l = 1, \dots, m\}, \tag{8}$$

with P a positive definite matrix, $\rho > 0$ and F_l the *lth* row of the matrix $F \in \mathbb{R}^{m \times n}$. Thus $\varepsilon(P, \rho)$ is an ellipsoid while $\mathcal{L}(F)$ is a polyhedral consisting of states for which the saturation does not occur.

Lemma 2.1. *(Hu et al., 2002) For all $u \in \mathbb{R}^m$ and $v \in \mathbb{R}^m$ such that $|v_l| < 1, l \in [1, m]$*

$$sat(u) \in co\{D_s u + D_s^- v, s \in [1, \eta]\}; \ \eta = 2^m \tag{9}$$

where co denotes the convex hull.

Consequently, there exist $\delta_1 \geq 0, \ldots, \delta_\eta \geq 0$ with $\sum_{s=1}^{\eta} \delta_s = 1$ such that,

$$sat(u) = \sum_{s=1}^{\eta} \delta_s[D_s u + D_s^- v] \tag{10}$$

Here, D_s is an m by m diagonal matrix with elements either 1 or 0 and $D_s^- = \mathbb{I}_m - D_s$. There are 2^m possible such matrices. One can also consult the work of (Benzaouia *et al.*, 2006) for more details and other extensions to linear systems with both constraints on the control and the increment or rate of the control.

Consider the following autonomous switching system:

$$x(t+1) = \sum_{i=1}^{N} \xi_i(t) A_i x(t) \tag{11}$$

The use of the Lyapunov functions revealed two ways:

- The existence of a common Lyapunov function to the various subsystems guarantees the asymptotic stability of the switching system. In general, the search for such function is not always obvious (Shorten and Narenda, 1997, 1998).

- The multiple Lyapunov functions were introduced in (Branicky, 1998). They are considered as a strong tool in the analysis of the stability of the hybrid systems and in particular the switching systems.

Proposition 2.1. *(Branicky, 1998) If there exists a multiple Lyapunov function $V(t, x), t \in I_s$, with $V : \mathbb{N} \times \mathbb{R}^n \longrightarrow \mathbb{R}_+$, such that the following hold:*

- $\Delta V(t, x(t)) < 0$, *for any time $t \in I_s$*
- $V(t_k, x(t_k)) < V(t_{k-1}, x(t_{k-1})), \forall k \in \mathcal{L}$

then, the switching autonomous system (11) is asymptotically stable.

We now recall a useful stability result for switching systems with no input saturations presented by many authors (see (Mignone *et al.*, 2000), (Ferrari-Trecate *et al.*, 2001) and (Daafouz *et al.*, 2001, 2002)) firstly used for linear time varying systems (Daafouz and Bernussou, 2001).

Theorem 2.1. *The closed-loop switching system (11) is asymptotically stable at the origin if there exist N symmetric and positive definite matrices $P_1, \ldots, P_N$ satisfying,*

$$A_i^T P_j A_i - P_i < 0, \forall (i, j) \in \mathcal{I} \times \mathcal{I} \tag{12}$$

A corresponding Lyapunov function for the system is then given by:

$$V(t, x) = x^T(t)(\sum_{i=1}^{N} \xi_i(t) P_i) x(t) \tag{13}$$

It is worth to note that function $V(t, x)$, which is a multiple Lyapunov function candidate involving matrices P_i, can be seen as a standard Lyapunov function candidate. It was the way followed by (Mignone *et al.*, 2000) and (Daafouz *et al.*, 2001, 2002). Further, condition (12) is equivalent, by using Schur complement to,

$$\begin{bmatrix} P_i & A_i^T P_j \\ * & P_j \end{bmatrix} > 0 , \ \forall (i, j) \in \mathcal{I} \times \mathcal{I} \tag{14}$$

where $*$ denotes the transpose of the off diagonal element of the LMI. Subsequently, we will need the following equivalent LMI representation of (14):

$$\left[\begin{array}{cc} X_i & X_i A_i^T \\ * & X_j \end{array}\right] > 0 \ , \ \forall (i,j) \in \mathcal{I} \times \mathcal{I} \tag{15}$$

where $X_i = P_i^{-1}$. By noting that inequality (12) is equivalent to:

$$G_i A_i^T P_j A_i G_i^T - G_i P_i G_i^T < 0, \forall (i,j) \in \mathcal{I} \times \mathcal{I} \tag{16}$$

for any nonsingular matrix G_i. By using the fact that $(G_i - X_i)^T X_i^{-1}(G_i - X_i) \geq 0$, implies $G_i X_i^{-1} G_i^T \geq G_i + G_i^T - X_i$. The LMI (15) is also in turn equivalent to the following LMI, generally used to relax the previous one (Daafouz $et\ al.$, 2001, 2002):

$$\left[\begin{array}{cc} G_i + G_i^T - X_i & G_i^T A_i^T \\ * & X_j \end{array}\right] > 0, \forall (i,j) \in \mathcal{I} \times \mathcal{I} \tag{17}$$

where matrices G_i called slack variables are nonsingular matrices and X_i the positive definite matrices.

It is worth nothing that condition (12) has to be satisfied $\forall (i,j) \in \mathcal{I} \times \mathcal{I}$, in particular for $i = j$. This means that each mode is necessarily asymptotically stable and $V_i(t,x) = x^T(t)P_i x(t)$ is the associated Lyapunov function. Recall that a level set of the Lyapunov function $V(t,x)$ given by (13) and associated to the switching system (11) is given by the set $\varepsilon(P,\rho)$ defined by (7) with $P = \sum_{i=1}^{N} \xi_i(t)P_i$. This region of asymptotic stability is very difficult to construct. Nevertheless, an estimate can be obtained as large as possible as presented in the next section based on the union of the sub-level ellipsoid sets $\varepsilon(P_i, 1)$. Other type of level sets obtained with different Lyapunov functions for switched systems can be found in (Hu $et\ al.$, 2006). A useful lemma is also recalled.

Lemma 2.2. *Let R, S and Γ be matrices with appropriate dimension. Suppose $\Gamma^T \Gamma \leq \mathbb{I}$, then for any scalar $\lambda > 0$, we have:*

$$R\Gamma S + S^T \Gamma^T R^T \leq \lambda R R^T + \lambda^{-1} S S^T \tag{18}$$

2.3 Analysis and synthesis of stabilizability

In this section, the region of local asymptotic stability associated to the saturated switching system is firstly studied. The design of a stabilizing controller for the class of switching system with actuator saturation is then presented by following two ways, the first concerns controllers working inside the region of linear behavior where the saturations do not occur while the second is based on Lemma 2.1 tolerating saturations to occur.

2.3.1 Region of asymptotic stability

The link between the result of Theorem 2.1 and the level set (7) is not mentioned in (Mignone $et\ al.$, 2000) nor in (Daafouz $et\ al.$, 2001, 2002). The absence of saturation on the control of the switching systems in these works does not necessitate to take care with the level sets since the asymptotic stability is global. We present here after an interesting result on this subject inspired from the general class studied in (Benzaouia $et\ al.$, 2007).

Theorem 2.2. *If there exist N symmetric positive definite matrices $P_1, \ldots, P_N$ such that the inequalities (12) are satisfied then the set given by,*

$$\Omega = \bigcup_{i=1}^{N} \varepsilon(P_i, 1) \tag{19}$$

is a level set of the multiple Lyapunov function $V(t, x)$ given by (13) with respect to the trajectories of the autonomous switching system (11).

Proof: Note that condition (12) for $i = j$ ensures that $\Delta V(t, x) < 0$, for $t \in I_s, s \in \mathcal{L}$ (Daafouz *et al.*, 2001, 2002). Now let $x(\bar{t}_{k-1}) \in \varepsilon(P_i, 1)$. What happens at the switching time $\bar{t}_{k-1}$?
For this, compute $x(t_k)$. With an identity reset function we have,

$$x(t_k) = x(\bar{t}_{k-1}) \tag{20}$$

The switch to the next region $\varepsilon(P_j, 1)$ will be seen an unit time after,

$$\begin{aligned} x(t_k + 1) &= x(\bar{t}_{k-1} + 1) \\ &= A_i x(\bar{t}_{k-1}) \end{aligned} \tag{21}$$

We have,

$$x^T(t_k + 1)P_j x(t_k + 1) = x^T(\bar{t}_{k-1}) \left(A_i^T P_j A_i \right) x(\bar{t}_{k-1}) \tag{22}$$

According to condition (12), one can obtain,

$$x^T(t_k + 1)P_j x(t_k + 1) < x^T(\bar{t}_{k-1})P_i x(\bar{t}_{k-1}) \tag{23}$$

Since $x(\bar{t}_{k-1}) \in \varepsilon(P_i, 1)$, then, $x(t_k + 1) \in \varepsilon(P_j, 1), \forall i, j \in \mathcal{I} \times \mathcal{I}$. This proves that $V(t_k, x(t_k)) < V(t_{k-1}, x(t_{k-1})), \forall k \in \mathcal{L}$. Consequently, function $V(t, x)$ is a multiple Lyapunov function of the switching system according to Proposition 2.1 and the set Ω is a level set associated to this function. It is worth to note that the same reasoning holds for $\varepsilon(P_j, \rho)$ for any positive scalar ρ. $\square$
Note that an important remark is to be done at this level: since the studied system is a switching one composed of N subsystems (modes), the switching system can be initialized inside any level set $\varepsilon(P_i, 1)$, however, the initial mode to be selected is the corresponding mode i.

2.3.2 State feedback control
We assume that the state is available, so a state feedback control can be performed. The first result we present is a simple extension of known result given by (Gutman and Hagandar, 1985) for linear systems and concerns the synthesis of non saturating controllers ensuring that a large region of linear behavior is a region of asymptotic stability.

Theorem 2.3. *If there exists N symmetric matrices $X_1, \ldots, X_N$ and N matrices $Y_1, \ldots, Y_N$ solutions of the following LMIs:*

$$\begin{bmatrix} X_i & (A_i X_i + B_i Y_i)^T \\ * & X_j \end{bmatrix} > 0, \tag{24}$$

$$\begin{bmatrix} 1 & Y_{li} \\ * & X_i \end{bmatrix} > 0, \tag{25}$$

$$\forall (i, j) \in \mathcal{I} \times \mathcal{I}, \forall l \in [1, m]$$

where Y_{li} is the lth row of matrix Y_i; then the switching system with saturations in closed-loop (3), with,

$$F_i = Y_i X_i^{-1}, P_i = X_i^{-1} \tag{26}$$

is asymptotically stable at the origin $\forall x_0 \in \Omega$ and for any sequences of switching $\alpha(t)$.

Proof: In order to guarantee that the control by state feedback is always admissible, each sub-level set of the Lyapunov function has to be contained inside the polyhedral set where the saturations do not occur $\varepsilon(P_i, 1) \subset \mathcal{L}(F_i), \forall i \in \mathcal{I}$ (Gutman and Hagandar, 1985). Using (Boyd *et al.*, 1994), this inclusion condition can also be transformed to the equivalent LMI (25) by letting $X_i = P_i^{-1}$ and $F_i X_i = Y_i$. Let $x(t) \in \Omega, \forall t \in I_s$. According to condition (25), the system in closed-loop (5) can be equivalently written as,

$$x(t+1) \quad = \quad \sum_{i=1}^{N} \xi_i(t) A_{c_i} x(t). \tag{27}$$

with, $A_{c_i} = A_i + B_i F_i$. The proof follows from the asymptotic stability conditions of the switching system given by Theorem 3.2, which is expressed equivalently by (15), with matrix A_{c_i} instead of A_i. Further, Theorem 2.2 ensures that the set Ω is a set of asymptotic stability of the switching system with saturations in closed-loop (3). $\qquad\square$

To achieve a domain of attraction as large as possible, we can solve the following optimization problem:

$$(Pb.1) : \begin{cases} \sup_{(X_i, Y_i)} Trace(X_i) \\ s.t. \quad (24), (25), \\ \quad i = 1, \ldots, N \end{cases}$$

When this optimization problem is feasible, the obtained ellipsoid volumes are maximum with respect to the data of the system.

The obtained LMIs (24) of Theorem 2.1 are similar to those obtained in (Mignone *et al.*, 2000) and (Ferrari-Trecate *et al.*, 2001) for non saturated switching systems. The presence of saturation on the control in our problem leads to the additional LMIs (25), which will obviously restrict the set of solutions. Nevertheless, the associated large region of asymptotic stability Ω for the saturated switching system enables one to conclude that these LMIs are not conservative.

The second result we present concerns the synthesis of saturating controllers tolerating saturations to take effect inside a large region of asymptotic stability.

Theorem 2.4. *If there exist symmetric positive definite matrices $P_1, \ldots, P_N \in \mathbb{R}^{n \times n}$ and matrices $H_1, \ldots, H_N \in \mathbb{R}^{m \times n}$ such that,*

$$\begin{bmatrix} P_i & [A_i + B_i(D_{is}F_i + D_{is}^- H_i)]^T P_j \\ * & P_j \end{bmatrix} > 0 , \tag{28}$$
$$\forall (i,j) \in \mathcal{I} \times \mathcal{I}, \forall s \in [1, \eta]$$

and,

$$\varepsilon(P_i, 1) \subset \mathcal{L}(H_i), \forall i \in \mathcal{I} \tag{29}$$

then, the closed-loop switching system (3) is asymptotically stable at the origin $\forall x_0 \in \Omega$ and for all sequences of switching $\alpha(t)$.

Proof: Assume that there exist N matrices $H_1, \ldots, H_N$ and N symmetric matrices $P_1, \ldots, P_N$ such that condition (28) and (29) are satisfied. Using the expression in (10) and rewriting System (3) as in (5) yields that:

$$sat(F_i x(t)) = \sum_{s=1}^{\eta} \delta_{is}(t)[D_{is}F_i + D_{is}^- H_i]x(t) ; \tag{30}$$

$$\delta_{si}(t) \geq 0, \ \sum_{s=1}^{\eta} \delta_{si}(t) = 1 \tag{31}$$

and, subsequently :

$$x(t+1) = \sum_{s=1}^{\eta} \sum_{i=1}^{N} \xi_i(t)\delta_{is}(t)Ac_{is}x(t); \tag{32}$$

$$Ac_{is} := A_i + B_i(D_{is}F_i + D_{is}^- H_i), \ s \in [1, \eta]$$

The rate of change of the Lyapunov function candidate (13) along the trajectories of (32) is given by:

$$\Delta V(t, x(t)) = x^T(t+1)(\sum_{j=1}^{N} \xi_j(t+1)P_j)x(t+1) - x^T(t)(\sum_{i=1}^{N} \xi_i(t)P_i)x(t) = x^T(t) \tag{33}$$

$$\left[\left(\sum_{s=1}^{\eta} \sum_{i=1}^{N} \xi_i(t)\delta_{is}(t)Ac_{is} \right)^T \left(\sum_{j=1}^{N} \xi_j(t+1)P_j \right) \left(\sum_{s=1}^{\eta} \sum_{i=1}^{N} \xi_i(t)\delta_{is}(t)Ac_{is} \right) - \sum_{i=1}^{N} \xi_i(t)P_i \right] x(t)$$

Let condition (28) be satisfied. At this level, for each i, multiply the $j = 1, \ldots, N$ inequalities (28) by $\xi_j(t+1)$ and sum. Multiply the resulting $i = 1, \ldots, N$ inequalities by $\xi_i(t)$ and sum. Multiply again the resulting $s = 1, \ldots, \eta$ inequalities by $\delta_{is}(t)$ and sum. As $\sum_{i=1}^{N} \xi_i(t) = \sum_{j=1}^{N} \xi_j(t+1) = \sum_{s=1}^{\eta} \delta_{is}(t) = 1$, one gets:

$$\begin{bmatrix} \sum_{i=1}^{N} \xi_i(t)P_i & \Gamma \sum_{j=1}^{N} \xi_j(t+1)P_j \\ * & \sum_{j=1}^{N} \xi_j(t+1)P_j \end{bmatrix} > 0, \tag{34}$$

with, $\Gamma = \left[\sum_{s=1}^{\eta} \sum_{i=1}^{N} \xi_i(t)\delta_{is}(t)Ac_{is} \right]^T$. The use of Schur complement allows us to write condition (34) under the equivalent form,

$$\left(\sum_{s=1}^{\eta} \sum_{i=1}^{N} \xi_i(t)\delta_{is}(t)Ac_{is} \right)^T \left(\sum_{j=1}^{N} \xi_j(t+1)P_j \right) \left(\sum_{s=1}^{\eta} \sum_{i=1}^{N} \xi_i(t)\delta_{is}(t)Ac_{is} \right) - \sum_{i=1}^{N} \xi_i(t)P_i < 0, \tag{35}$$

$$\forall (i, j) \in \mathcal{I} \times \mathcal{I}, \forall s \in [1, \eta]$$

Letting λ be the largest eigenvalue among all the above matrices, we obtain that

$$\Delta V(t, x(t)) \leq \lambda x^T(t)x(t) < 0. \tag{36}$$

which ensures the desired result. Noting that condition (28) is also satisfied for $i = j$, this implies that each set $\varepsilon(P_i, 1)$ is a contractively invariant set for the corresponding subsystem.

Further, by taking account of condition (29), one can guarantee that for every $x_o \in \varepsilon(P_i, 1) \subset \mathcal{L}(H_i)$, each subsystem is asymptotically stable at the origin. Besides, following Theorem 2.2, conditions (28)-(29) also allow for a state belonging, before the switch, to a set $\varepsilon(P_i, 1) \subset \mathcal{L}(H_i)$, if a switch occurs at any time t_k, that the switch will handle the state to the desired set $\varepsilon(P_j, 1) \subset \mathcal{L}(H_j)$. That means that the set Ω is a set of asymptotic stability of the switching system. □

This stability result can be used for control synthesis as follows.

Theorem 2.5. *If there exists N symmetric matrices $X_1, \ldots, X_N$ and $2N$ matrices $Y_1, \ldots, Y_N$ and $Z_1, \ldots, Z_N$ solutions of the following LMIs:*

$$\left[\begin{array}{cc} X_i & (A_i X_i + B_i D_{is} Y_i + B_i D_{is}^- Z_i)^T \\ * & X_j \end{array} \right] > 0, \tag{37}$$

$$\left[\begin{array}{cc} 1 & Z_{li} \\ * & X_i \end{array} \right] > 0, \tag{38}$$

$$\forall (i, j) \in \mathcal{I} \times \mathcal{I}, \forall s \in [1, \eta], \forall l \in [1, m]$$

then the switching system with saturations in closed-loop (3), with,

$$F_i = Y_i X_i^{-1}, H_i = Z_i X_i^{-1}, P_i = X_i^{-1} \tag{39}$$

is asymptotically stable at the origin $\forall x_0 \in \Omega$ and for any sequences of switching $\alpha(t)$.

Proof: The Inequalities (28) can be transformed equivalently by Schur complement to the following:

$$P_j \left[A_i + B_i (D_{is} F_i + D_{is}^- H_i) \right] P_i^{-1} \left[A_i + B_i (D_{is} F_i + D_{is}^- H_i) \right]^T P_j - P_j < 0 \tag{40}$$

By pre and post-multiplying the latter by P_j^{-1}, it follows:

$$\left[A_i + B_i (D_{is} F_i + D_{is}^- H_i) \right] P_i^{-1} \left[A_i + B_i (D_{is} F_i + D_{is}^- H_i) \right]^T - P_j^{-1} < 0 \tag{41}$$

Pose $X_i = P_i^{-1}$, inequality (41) can then be rewritten as,

$$\left[A_i + B_i (D_{is} F_i + D_{is}^- H_i) \right] X_i \left[A_i + B_i (D_{is} F_i + D_{is}^- H_i) \right]^T - X_j < 0,$$

that is

$$\left[A_i X_i + B_i (D_{is} F_i X_i + D_{is}^- H_i X_i) \right] X_i^{-1} \left[A_i X_i + B_i (D_{is} F_i X_i + D_{is}^- H_i X_i) \right]^T - X_j < 0.$$

The use of the Schur complement a second time leads to:

$$\left[\begin{array}{cc} X_i & (A_i X_i + B_i D_{is} F_i X_i + B_i D_{is}^- H_i X_i)^T \\ * & X_j \end{array} \right] > 0 \tag{42}$$

By letting $F_i X_i = Y_i$ and $H_i X_i = Z_i$, the LMI (37) follows readily. Using (Boyd *et al.*, 1994), the inclusion condition (29) can also be transformed to the equivalent LMI (38). □

To obtain larger ellipsoid domains $\varepsilon(P_i, 1)$, we can apply the following optimization problem:

$$(Pb.2): \begin{cases} \sup_{(X_i, Y_i, Z_i)} Trace(X_i) \\ s.t. \quad (37), (38), \\ \quad i = 1, \dots, N \end{cases}$$

When this optimization problem is feasible, the obtained ellipsoid volumes are maximum with respect to the data of the system.

Remark 2.1. *The results presented in this section can be extended easily to level sets $\varepsilon(P_i, \rho_i)$ where the scalars $\rho_i, i = 1, \dots, N$ can be a priori fixed, by using the variables $X_i = P_i / \rho_i$ in the LMIs (24)-(25) and (37)-(38).*

Commennt 2.1. *The resolution of the LMIs (37) for $s \in [1, \eta]$, may be very restrictive. One can relax this resolution by accepting controllers working in a region of linear behavior where the saturations do not occur. The obtained results in this work, where the switching system is with saturated control, are a set of $(2^m \times N^2 + Nm)$ LMIs for computing stabilizing controllers working inside the saturated regions $\mathcal{L}(H_i)$ or only a set of $(N^2 + Nm)$ LMIs for computing stabilizing controllers working inside regions of linear behavior $\mathcal{L}(F_i)$. Note also that the LMIs of Theorem 2.3 can be obtained from LMIs of Theorem 2.5 by letting $D_{is} = \mathbb{I}$ and $D_{is}^- = 0$.*

Example 2.1. *Consider a numerical switching discrete-time system with saturation specified by the two subsystems:*

$$A_1 \quad = \quad \begin{bmatrix} -0.7 & 1 \\ -0.5 & -1.5 \end{bmatrix}; B_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}; A_2 = \begin{bmatrix} 0.9 & -1 \\ 1.7 & -1.5 \end{bmatrix}; B_2 = \begin{bmatrix} 0 \\ -1 \end{bmatrix}.$$

We have to solve 5 LMIs with 4 variables to compute controllers working inside a region of linear behavior. The use of the Matlab LMI Toolbox to check our conditions leads to the following results without using the optimization problem $(Pb.1)$.

$$P_1 = \begin{bmatrix} 7.3328 & 18.7125 \\ 18.7125 & 55.9711 \end{bmatrix}; F_1 = \begin{bmatrix} 1.2244 & 0.7535 \end{bmatrix}; Ac_1 = \begin{bmatrix} 0.5244 & 1.7535 \\ -0.5000 & -1.5000 \end{bmatrix};$$

$$P_2 = \begin{bmatrix} 4.8237 & -5.1001 \\ -5.1001 & 5.6715 \end{bmatrix}; F_2 = \begin{bmatrix} 1.7559 & -1.5699 \end{bmatrix}; Ac_2 = \begin{bmatrix} 0.9000 & -1.0000 \\ -0.0559 & 0.0699 \end{bmatrix};$$

Figure 1 presents the two ellipsoid sets of invariance and contractivity $\varepsilon(P_i, 1)$ together with the sets of linear behavior $\mathcal{L}(F_i)$ computed with LMIs (24)-(25).

For the same example, the use of the Matlab LMI Toolbox to check conditions (37)-(38) leads to the following results by using the optimization problem $(Pb.2)$ with 10 LMIs and 6 variables.

$$P_1 \quad = \quad \begin{bmatrix} 7.4032 & 18.1841 \\ 18.1841 & 56.6345 \end{bmatrix}; H_1 = \begin{bmatrix} 1.2534 & 0.8569 \end{bmatrix}; F_1 = \begin{bmatrix} 1.2641 & 0.8526 \end{bmatrix};$$

$$Ac_1 \quad = \quad \begin{bmatrix} 0.5641 & 1.8526 \\ -0.5000 & -1.5000 \end{bmatrix};$$

$$P_2 \quad = \quad \begin{bmatrix} 4.5202 & -4.3908 \\ -4.3908 & 6.5909 \end{bmatrix}; H_2 = \begin{bmatrix} 1.7693 & -1.5845 \end{bmatrix}; F2 = \begin{bmatrix} 1.7697 & -1.5836 \end{bmatrix};$$

$$Ac_2 \quad = \quad \begin{bmatrix} 0.9000 & -1.0000 \\ -0.0697 & 0.0836 \end{bmatrix};$$
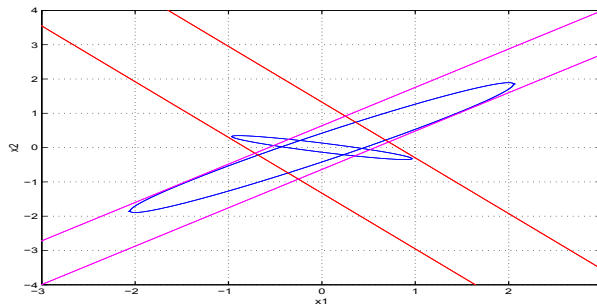
Fig. 1. The ellipsoids sets of invariance and contractivity for the switching discrete-time linear system computed with LMIs (24)-(25).

Note that the optimal values of the optimization problem (Pb.2) are given by $Trace(P_1^{-1}) = 0.7227$ and $Trace(P_2^{-1}) = 1.0568$. Figure 2 presents the two ellipsoid sets of invariance and contractivity
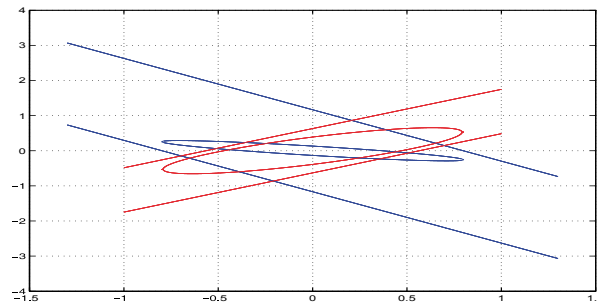

Fig. 2. The ellipsoids sets of invariance and contractivity for the switching discrete-time linear system computed with LMIs (37)-(38) and (*Pb.*2).

$\varepsilon(P_i, 1)$ *together with the set of saturations $\mathcal{L}(H_i)$ computed with (Pb.2).*

Comparing the results obtained by LMIs (37)-(38) and LMIs (24)-(25), one can note that the matrices H_i are very closed up to matrices F_i obtained with LMIs (37)-(38), that is, $\mathcal{L}(H_i) \cong \mathcal{L}(F_i)$. This means that in the case of this example, the saturations allowed by this technique are not very important. Recall that inside the sets $\mathcal{L}(F_i)$ no saturations occur. Furthermore, the results obtained with LMIs (24)-(25) are less conservative. This is due to the fact that among the number of LMIs to be solved in (37)-(38), one find the LMIs (24)-(25).

2.3.3 Output feedback control

Static output-feedback control plays a very important role in control applications: The purpose is to design controllers such that the resulting closed-loop system is asymptotically stable without using any reconstruction method of the unavailable states. In this section, we begin by the synthesis of the saturating controllers since the non saturating ones can be derived as a particular case. The synthesis of the stabilizing controller by output feedback control for the class of switching system with actuator saturation is presented by applying

the results of Theorem 2.4.

Theorem 2.6. *If there exist symmetric matrices X_i, matrices V_i, Y_i and Z_i solutions of the following LMIs:*

$$\begin{bmatrix} X_i & (A_iX_i + B_iD_{is}Y_iC_i + B_iD_{is}^-Z_i)^T \\ * & X_j \end{bmatrix} \quad > \quad 0, \tag{43}$$

$$\begin{bmatrix} 1 & Z_{il} \\ * & X_i \end{bmatrix} \quad > \quad 0, \tag{44}$$

$$V_iC_i = C_iX_i \tag{45}$$

$$\forall(i,j) \in \mathcal{I}^2, \ \forall s \in [1,\eta], \ \forall l \in [1,m]$$

then the closed-loop saturated switching system (3) with

$$K_i \quad = \quad Y_iV_i^{-1}, H_i = Z_iX_i^{-1}, P_i = X_i^{-1} \tag{46}$$

is asymptotically stable $\forall x_0 \in \Omega$ and for all switching sequences $\alpha(k)$.

Proof: The same development as (40)-(42) while replacing F_i by K_iC_i, can be followed to obtain,

$$\begin{bmatrix} X_i & (A_iX_i + B_iD_{is}K_iC_iX_i + B_iD_{is}^-H_iX_i)^T \\ * & X_j \end{bmatrix} > 0.$$

According to Equation (45), we have $K_iC_iX_i = K_iV_iC_i$. By letting $K_iV_i = Y_i$ and $H_iX_i = Z_i$, the LMI (43) follows together with relations (46). Similarly, the inclusion condition (29) can also be transformed to the equivalent LMI (44). Finally, the inequality (43) ensures that the obtained solutions X_i are positive definite while equalities (45) guarantee that matrices V_i are nonsingular.   □

It is worth noting that the state feedback follows readily from Theorem 2.6 by letting $C_i = \mathbb{I}_m$. In this case, $V_i = X_i$.

The LMI (43), relating matrices C_i to matrix X_i by means of Equation (45), can be relaxed by using the LMI (17), where the new variables G_i are related to matrices C_i instead of the matrix X_i.

Corollary 2.1. *If there exist symmetric matrices X_i, matrices G_i, V_i, Y_i and Z_i solutions of the following LMIs:*

$$\begin{bmatrix} G_i + G_i^T - X_i & \Psi_{is}^T \\ * & X_j \end{bmatrix} \quad > \quad 0, \tag{47}$$

$$\begin{bmatrix} 1, & Z_{il} \\ * & G_i + G_i^T - X_i \end{bmatrix} \quad > \quad 0, \tag{48}$$

$$V_iC_i \quad = \quad C_iG_i \tag{49}$$

$$\forall(i,j) \in \mathcal{I}^2, \ \forall s \in [1,\eta], \ \forall l \in [1,m]$$

where $\Psi_{is} = (A_iG_i + B_iD_{is}Y_iC_i + B_iD_{is}^-Z_i)$, then the closed-loop saturated switching system (3) with

$$K_i = Y_iV_i^{-1}, H_i = Z_iG_i^{-1}, P_i = X_i^{-1} \tag{50}$$

is asymptotically stable $\forall x_0 \in \Omega$ and all switching sequences $\alpha(k)$.

Proof: The proof uses the equivalent LMI (17) and is similar to that of Theorem 2.6. However, the inclusion condition $\varepsilon(P_i, 1) \subset \mathcal{L}(H_i) \quad \forall i \in \mathcal{I}$ holds if $1 - H_{il}X_iH_{il}^T > 0, \forall l \in [1, m]$, which is equivalent to,

$1 - (H_iG_i)_l(G_i^TX_i^{-1}G_i)^{-1}(H_iG_i)_l^T > 0.$ That is, by virtue of (50) $1 - (Z_{il})(G_i^TX_i^{-1}G_i)^{-1}(Z_{il})^T > 0.$ Since, $(G_i - X_i)^TX_i^{-1}(G_i - X_i) \geq 0$, then $G_iX_i^{-1}G_i^T \geq G_i + G_i^T - X_i$. It follows that, $1 - Z_{il}(G_i + G_i^T - X_i)Z_{il}^T > 0$ sufficient to have $\varepsilon(P_i, 1) \subset \mathcal{L}(H_i)$. By Schur complement, the LMI (48) is obtained. □

To achieve a domain of attraction as large as possible, we can solve similar optimization problem as for state feedback control.

Another way to deal with the problem by relaxing the more constraining relations (45) of Theorem 2.6, especially in MIMO case, is presented by the following result which uses the same idea as in (El Ghaoui *et al.*, 1997) and (Chadli *et al.*, 2002).

Theorem 2.7. *If there exist symmetric matrices X_i and S_i, matrices Y_i and Z_i solutions of the following LMIs:*

$$
\begin{bmatrix} X_i & (A_i + B_iD_{is}Y_iC_i + B_iD_{is}^-Z_i)^T \\ * & S_j \end{bmatrix} > 0, \tag{51}
$$

$$
\begin{bmatrix} 1 & Z_{il} \\ * & X_i \end{bmatrix} > 0, \tag{52}
$$

$$
\begin{bmatrix} X_i, & \mathbb{I} \\ * & S_i \end{bmatrix} \geq 0 \tag{53}
$$

$$
\forall(i,j) \in \mathcal{I}^2, \ \forall s \in [1, \eta], \ \forall l \in [1, m]
$$

such that $Trace(X_iS_i) = n$, then the closed-loop saturated switching system (3) with

$$
K_i = Y_i, H_i = Z_i, P_i = X_i \tag{54}
$$

is asymptotically stable $\forall x_0 \in \Omega$ and all switching sequences $\alpha(k)$.

Proof: The inequality (28) can be equivalently transformed via the Schur Complement to the following:

$$
\left[A_i + B_i(D_{is}K_iC_i + D_{is}^-H_i)\right]^T P_j \left[A_i + B_i(D_{is}K_iC_i + D_{is}^-H_i)\right] - P_i < 0 \tag{55}
$$

The use of the Schur complement a second time leads to

$$
\begin{bmatrix} P_i & (A_i + B_iD_{is}K_iC_i + B_iD_{is}^-H_i)^T \\ * & P_j^{-1} \end{bmatrix} > 0
$$

$$
\tag{56}
$$

By letting $X_i = P_i$, $S_j = P_j^{-1}$, $Y_i = K_i$ and $Z_i = H_i$, the LMI (51) follows together with relations (54). The inclusion condition (29) can also be transformed to the equivalent LMI (52) (Boyd *et al.*, 1994): $H_{il}P_i^{-1}H_{il}^T \leq 1$ can be rewritten by Schur complement, while using $Z_i = H_i$ as (52). Note that the LMI (51) ensures that the obtained solutions X_i and S_i are positive definite. Finally, the LMI (53) is equivalent to $S_iX_i \geq \mathbb{I}$. □

This result presents the advantage of computing directly the matrices K_i and H_i. Neverthe-less, an optimization problem must be solved to achieve $X_i S_i \simeq \mathbb{I}$ by minimizing iteratively the trace of matrix $X_i S_i$ using an algorithm presented in (El Ghaoui *et al.*, 1997). This heuristic is based on a linear approximation of $Tr(X_i S_i)$ by $Tr(X_0 S_i + S_0 X_i)$ where X_0 and S_0 are particular solutions of the LMI constraints (51), (52) and (53).

$$(Pb.3) : \left\{ \begin{array}{c} \min_{(S_j, X_i, Y_i, Z_i)} Trace(X_i S_i) \\ s.t. \ (51), (52), (53) \\ i, j = 1, \ldots, N \end{array} \right.$$

An output feedback non saturating controller can be obtained, as noted in Comment 2.1, as a particular case of the saturating controller results by letting $D_{is} = \mathbb{I}$ and $D_{is}^- = 0$. It is also obvious that the inclusion condition becomes $\varepsilon(P_i, 1) \subset \mathcal{L}(K_i C_i)$. The following results are then directly obtained.

Corollary 2.2. *If there exist symmetric matrices X_i, matrices V_i and Y_i solutions of the following LMIs:*

$$\left[\begin{array}{cc} X_i & (A_i X_i + B_i Y_i C_i)^T \\ * & X_j \end{array} \right] > 0, \tag{57}$$

$$\left[\begin{array}{cc} 1 & (Y_i C_i)_l \\ * & X_i \end{array} \right] > 0, \tag{58}$$

$$V_i C_i = C_i X_i \tag{59}$$

$$\forall (i, j) \in \mathcal{I}^2, \forall l \in [1, m]$$

then the closed-loop saturated switching system (3) with

$$K_i = Y_i V_i^{-1}, P_i = X_i^{-1} \tag{60}$$

is asymptotically stable $\forall x_0 \in \Omega$ and for all switching sequences $\alpha(k)$.

Proof: This proof can be obtained easily by letting $D_{is} = \mathbb{I}$ and $D_{is}^- = 0$ in Theorem 2.6. However, the inclusion condition $\varepsilon(P_i, 1) \subset \mathcal{L}(K_i C_i)$ leads to $(K_i C_i)_l P_i^{-1} (K_i C_i)_l^T \leq 1$ which is equivalent to,

$$(K_i C_i P_i^{-1})_l P_i (K_i C_i P_i^{-1})_l^T \leq 1,$$

by using (59), $(K_i V_i C_i)_l X_i^{-1} (K_i V_i C_i)_l^T \leq 1$, can be rewritten by Schur complement, while using $Y_i = K_i V_i$ as (58). $\square$

In a similar way, the result of Theorem 2.7 can be applied for designing non saturating controls by letting $D_{is} = \mathbb{I}$ and $D_{is}^- = 0$.

Example 2.2. *In order to illustrate these results, consider a SISO saturated switching discrete-time system with two modes given by the following matrices:*

$$A_1 = \left[\begin{array}{cc} 1 & 1 \\ 0 & 1 \end{array} \right]; B_1 = \left[\begin{array}{c} 10 \\ 5 \end{array} \right]; C_1 = \left[\begin{array}{cc} 1 & 1 \end{array} \right]$$

$$A_2 = \left[\begin{array}{cc} 0 & -1 \\ 0.0001 & 1 \end{array} \right]; B_2 = \left[\begin{array}{c} 0.5 \\ -2 \end{array} \right]; C_2 = \left[\begin{array}{cc} 2 & 3 \end{array} \right]$$

For this example with $n = 2$, $m = p = 1$ and $N = 2$, we have to solve 13 LMIs with 9 variables by using the LMIs of Corollary 2.1. Let the scalar $\rho = 1$. The use of the Matlab LMI Toolbox yields the following results:

$$P_1 = \begin{bmatrix} 0.0016 & 0.0006 \\ 0.0006 & 0.0025 \end{bmatrix}; H_1 = \begin{bmatrix} -0.0196 & -0.0479 \end{bmatrix};$$

$$K_1 = -0.0743; Ac_1 = \begin{bmatrix} 0.2574 & 0.2574 \\ -0.3713 & 0.6287 \end{bmatrix};$$

$$P_2 = \begin{bmatrix} 0.0010 & 0.0004 \\ 0.0004 & 0.0027 \end{bmatrix}; H_2 = \begin{bmatrix} 0.000007 & 0.0483 \end{bmatrix};$$

$$K_2 = 0.0347; Ac_2 = \begin{bmatrix} 0.0615 & -0.9077 \\ -0.2460 & 0.6309 \end{bmatrix}$$

The main conditions (12) are also satisfied:

$$Ac_1^T P_2 Ac_1 - P_1 = 10^{-3} \begin{bmatrix} -0.0012 & -0.0011 \\ -0.0011 & -0.0013 \end{bmatrix};$$

$$Ac_2^T P_1 Ac_2 - P_2 = 10^{-3} \begin{bmatrix} -0.0008 & -0.0007 \\ -0.0007 & -0.0011 \end{bmatrix}$$

Figure 3 shows the level set as the union of two ellipsoid sets of invariance and contractivity for
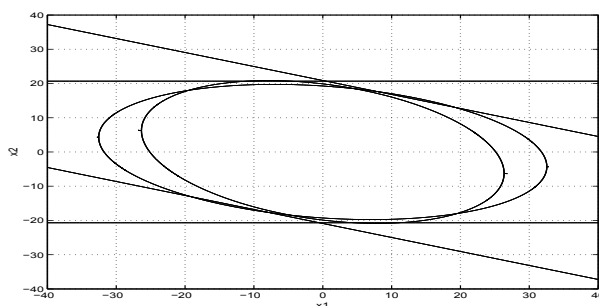

Fig. 3. The level set as the union of two ellipsoid sets of invariance and contractivity for the switching discrete-time linear system obtained by Corollary 2.1.

the switching discrete-time linear system obtained by Corollary 2.1. Each ellipsoid set is contained inside the set of admissible saturations $\mathcal{L}(H_i)$. Inside this set, the asymptotic stability of the saturated switching system is guaranteed for any arbitrary switch from any linear subsystem to another. This SISO example is studied only in order to illustrate the results of this work by plotting the level set in the plane. The use of Theorem 2.7 leads to the following results for $\rho = 1$:

$$P_1 = \begin{bmatrix} 1.0961 & 0.0848 \\ 0.0848 & 1.0290 \end{bmatrix}; P_2 = \begin{bmatrix} 1.5366 & 0.1255 \\ 0.1255 & 1.4821 \end{bmatrix};$$

$$S_1 = \begin{bmatrix} 0.6553 & -0.0555 \\ -0.0555 & 0.6794 \end{bmatrix}; S_2 = \begin{bmatrix} 0.9182 & -0.0756 \\ -0.0756 & 0.9781 \end{bmatrix};$$

$H_1 = \begin{bmatrix} -0.0786 & -0.1208 \end{bmatrix}; K_1 == -0.1007; H_2 = \begin{bmatrix} 0.00004 & 0.5473 \end{bmatrix}; K2 = 0.1812$

The level set obtained with this approach is plotted in Figure 4 together with few trajectories.
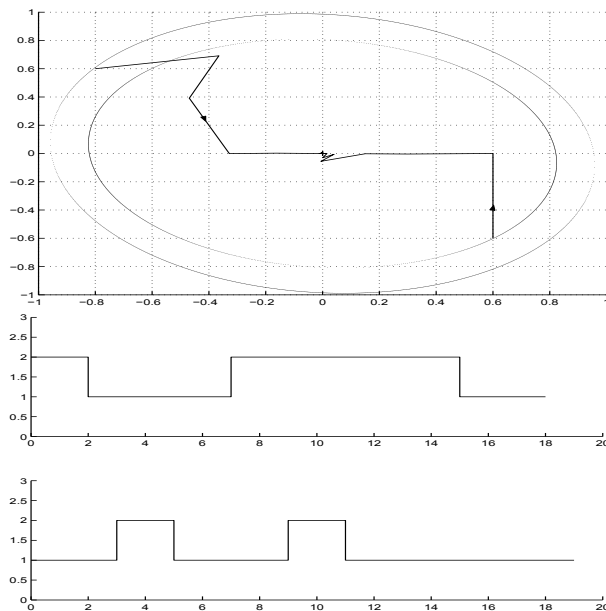

Fig. 4. The level set as the union of two ellipsoid sets of invariance and contractivity for the switching discrete-time linear system obtained by Theorem 2.7 and the corresponding switching sequences.

One can notice that the obtained level sets with Corollary2.1 are the most larger. One can conclude, via this example, that the result of Corollary 2.1 is the less conservative among all the other results due to the introduced slack variables G_i. Combining the comparisons made for state feedback control and output feedback control, one can expect that the introduction of slack variables in the Corollary 2.2 can lead to the more less conservative results.

In this section, two main different sufficient conditions of asymptotic stability are obtained for switching discrete-time linear systems subject to actuator saturations for each case: state feedback and output feedback control. The first allows the synthesis of stabilizing controllers inside a large region of linear behavior while the second applies the idea of Lemma 2.1 which rewrites the saturation function under a combination of 2^m elements to obtain stabilizing controllers tolerating saturations to take effect.

3. Stabilization of saturated switching systems with polytopic uncertainties

The objective of this section is to extend the results of (Benzaouia *et al.,* 2006) to uncertain switching system subject to actuator saturations. The uncertainty type considered in this work is the polytopic one. This type of uncertainty was also studied, without saturation, in (Hetel *et al.,* 2006). Thus, in this work, two directions are explored: the first concerns the synthesis of non saturating controllers, while the second direction deals with controllers

tolerating saturations to take effect under polytopic uncertainties. The main results of this section are published in (Benzaouia *et al.,* 2009b).

3.1 Problem presentation

Let us consider the uncertain saturated switching discrete-time linear system described by:

$$\begin{cases} x_{t+1} = A_\alpha(q_\alpha(t))x_t + B_\alpha(q_\alpha(t))sat(u_t) \\ y_t = C_\alpha(q_\alpha(t))x_t \end{cases} \tag{61}$$

where $x_t \in \mathbb{R}^n$, $u_t \in \mathbb{R}^m$, $y_t \in \mathbb{R}^p$ are the state, the input and the output respectively, $sat(.)$ is the standard saturation (assumed here to be normalized, *i.e.,* $|sat(u_t)| = min(1, |u_t|)$), function $\alpha(t) : \mathbb{N} \longmapsto \mathcal{I}$ is a switching rule taking its values $\alpha(t) = i$ in the finite set $\mathcal{I} = \{1, ..., N\}$ and $q_\alpha(t) \in \Gamma_\alpha \subset \mathbb{R}^{d_\alpha}$ are the bounded uncertainties that affect the system parameters in such a way that

$$A_\alpha(q_\alpha(t)) = A_\alpha + \sum_{h=1}^{d_\alpha} A_{\alpha h}q_{\alpha h}(t) \tag{62}$$

$$B_\alpha(q_\alpha(t)) = B_\alpha + \sum_{h=1}^{d_\alpha} B_{\alpha h}q_{\alpha h}(t) \tag{63}$$

$$C_\alpha(q_\alpha(t)) = C_\alpha + \sum_{h=1}^{d_\alpha} C_{\alpha h}q_{\alpha h}(t) \tag{64}$$

where matrices A_α, B_α, C_α represent the nominal matrices and $q_{\alpha h}(t)$ the hth component of vector $q_\alpha(t)$:

$$q_\alpha(t) = [q_{\alpha 1}(t) \quad q_{\alpha 2}(t) \quad ... \quad q_{\alpha h}(t) \quad ... \quad q_{\alpha d_\alpha}(t)]^T.$$

The following additional assumption is required:

- Γ_α are compact convex sets.

Let the control be obtained by an output feedback control law:

$$\begin{aligned} u_t &= K_\alpha y_t = F_\alpha x_t, \\ F_\alpha &= K_\alpha C_\alpha. \end{aligned} \tag{65}$$

Thus, the closed-loop system is given by:

$$x_{t+1} = A_\alpha(q_\alpha(t))x_t + B_\alpha(q_\alpha(t))sat(K_\alpha C_\alpha(q_\alpha(t))x_t). \tag{66}$$

Defining the indicator function:

$$\xi(t) := [\xi_1(t), ..., \xi_N(t)]^T, \tag{67}$$

where $\xi_i(t) = 1$ if the switching system is in mode i and 0 otherwise, yields the following representation for the closed-loop system:

$$x_{t+1} = \sum_{i=1}^{N} \xi_i(t)[A_i(q_i(t))x_t + B_i(q_i(t))sat(K_i C_i(q_i(t))x_t]. \tag{68}$$

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