Satellite Motion

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1. Introduction

What is satellite?
The word satellite is coming from the Latin language (Latin *satelles* – escort, companion). Satellites are objects that rotate around the planets under the influence of the gravitational force. For example, the Moon is natural satellite of the Earth.
What is the artificial Earth satellite?
The artificial Earth satellites are artificial objects which are launched into orbit around the Earth by a rocket vehicle. This kind of satellites was named, the Human-Made Earth Satellites.
How do rockets function?
Aeroplanes work on the principle of buoyancy difference on their wings. This is the reason why aeroplanes can fly only in the air but not in the vacuum. Thus, an airplane cannot be used for launching satellites in their orbit around the Earth (Fig. 1).

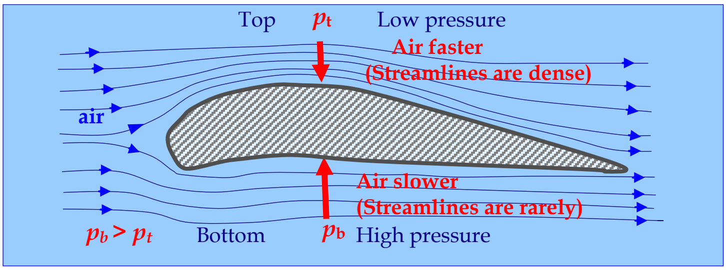


Fig. 1. When the aeroplanes have velocity in the air on the bottom of his wings they have higher pressure then on the top of wings. This difference of pressure is giving the force of buoyancy and the aeroplanes can fly.

It is also not possible to launch a human-made satellite into the orbit around the Earth with a cannon or a gun because a cannon-ball has the velocity of about 0.5 km/s. It means that this velocity of cannon-ball is about 15 times smaller than the first cosmic velocity (7.9 km/s). So for it has been possible to launch a satellite into an orbit around the Earth where there is vacuum is possible only with using rockets.
The word *rocket* comes from the Italian *Rocchetta* (i.e. *little fuse*), a name of a small firecracker. It is commonly accepted that the first recorded use of a rocket in battle was by the Chinese

in 1232 against the Mongol hordes at Kai Feng Fu. The Mongols were the first to have applied rocket technology in Europe as they conquered some parts of China and of Russia, Eastern and Central Europe.

Konstantin Tsiolkovsky (1857-1935) (Fig. 2) from the Imperial Russia and after from the Soviet Union published the first serious scientific work on space travels titled ''*The exploration of Cosmic Space by Means of Reaction Devices*'' in 1903. He is considered by many to be the *father of theoretical astronautics.* He also advocated the use of liquid hydrogen and oxygen for propellant, calculating their maximum exhaust velocity. His work inspired further research, experimentation and the formation of Society for Studies of Interplanetary Travel in 1924. Also in 1924, Tsiolkovsky wrote about multi-stage rockets, in ''*Cosmic Rocket Trains*''.

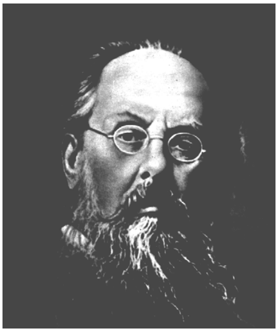


Fig. 2. Konstantin Tsiolkovsky


Fig. 3. Robert Goddard


Fig. 4. Sergei Korolev


Fig. 5. Wernher von Braun

In the USA, Robert Goddard (Fig. 3) began a serious analysis of rockets in 1912. It can thus be concluded that conventional solid-fuel rockets needed to be improved in three ways. One of them is that rockets could be arranged in stages. He also independently developed the mathematics of rocket flight. For his ideas, careful research, and great vision, Goddard was called the *father of modern astronautics.*

After the World War II in the USA Wernher von Braun (Fig. 5) and Sergei Korolev (Fig. 4) in the Soviet Union were the leaders in the advancing rockets technology.

The operational principle a rocket can be explained by means of a balloon (Fig. 6).

In a balloon the pressure of gas is practically equal on all sides (Fig. 6 a). When this balloon has an aperture then a particle of gas on this aperture will under pressure of gas be thrown out with velocity v_g (Fig. 6 b). The pressure in the balloon in opposite direction of the aperture will produce the pressure on the balloon and will give it the velocity v_B. The pressures in the other directions will be mutually cancelled in opposite direction. The rocket operates on this principle.

A rocket travelling in vacuum is accelerated by the high-velocity expulsion of a small part of its mass (gas). The Fig. 7. represents a rocket with the situations before and after the explosion. This is closed material system and for this system linear momentum needs to be conserved. So we can say that the momentum in the beginning position ($M{\cdot}\mathbf{v}$) is equal to momentum of this system after the explosion when a particle with the mass (dm) is be thrown out with velocity ($\mathbf{u}_{rel}$) in the opposite direction of this rocket velocity.

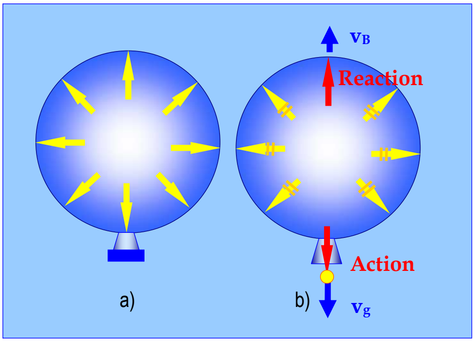


Fig. 6. A balloon closed under gas pressure and after opening an aperture.

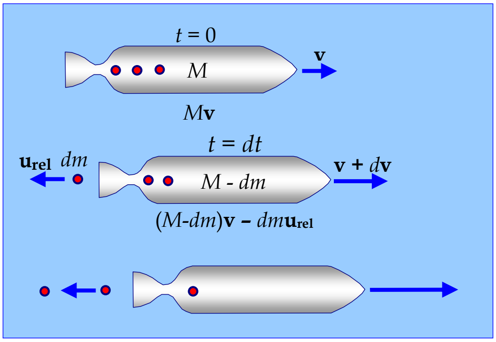
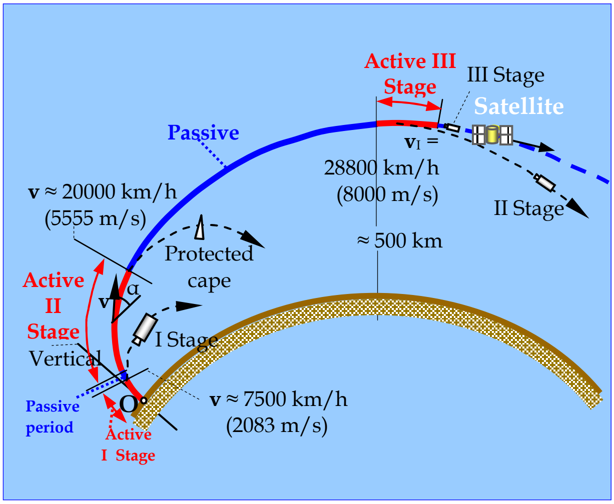


Fig. 7. The principle of a rocket operation.

It means that the momentum for a particle is negative ($-dm \cdot \mathbf{u}_{rel}$) and we can write the equation (Carton, 1965) and (Danby, 1989):

$$M \cdot \mathbf{v} = (M - dm) \cdot (\mathbf{v} + d\mathbf{v}) - dm \cdot \mathbf{u}_{rel}\,. \tag{1}$$



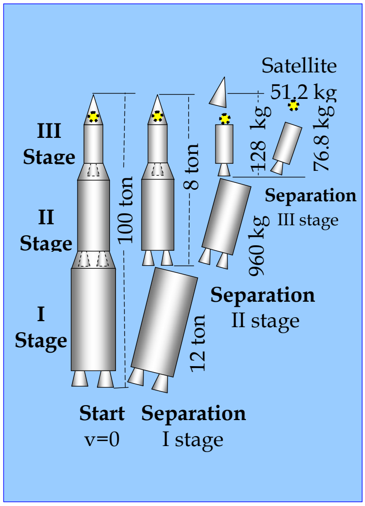


Fig. 8. Imagery of launching a satellite by rocket with the three stages.

Fig. 9. For example during the start a three stages rocket may have the mass 100 t but mass of a space vehicle will be only 51.2 kg.

It follows from this equation that the velocity of rocket increased for the elementary magnitude

$$d\mathbf{v} = \frac{dm(\mathbf{v} + \mathbf{u}_{\text{rel}})}{M - dm}. \tag{2}$$

From this equation it is possible to see that the increase of the rocket velocity is larger if the velocity $\mathbf{u}_{rel}$ of the particles of gas is maximally the greater when the mass of particle *dm* has some magnitude. This is the reason why the constructors of rockets like to make rockets with very high (maximal) velocity of the particles (gas) of the rocket.
Usual rockets have vertical start. Longer delaying of rockets in the Earth gravitation field causes the loss of velocity but also to big thrust during the start is not suitable. So rockets are usually made in same stages (Fig. 8 and 9). During the starts of rockets the consummation of fuel is very large so that a satellite or a space vehicle enters into the orbit with a mass practically next to nothing (see for example Table 1).

Table 1. Data on initial mass of a rocket in a start, spend fuel, thrown parts for a launching space vehicle of the mass 51.2 kg

Stage	1	2	3
Initial mass	100 t	8 t	640 kg
Spend fuel	80 t	6.4 t	512 kg
Thrown part	-	12 t	960 kg
Definitely mass	20 t	1.6 t	128 kg
Velocity	3860 (m/s)	7720 (m/s)	11580 (m/s)

2. Planet Motion

Until the 17th century people were thinking that the Sun and planets are rotated around the Earth by circles. Such opinions were practically usual until Johann Kepler.

2.1 Kepler's Laws of Planetary Motion

Johann Kepler (1546 -1601) discovered the laws of planetary motion empirically from Tycho Brahe's (1546 -1601) astronomical observations of the planet Mars. The first and the second laws he published in *Astronomia Nova (New Astronomy*) in 1609, and the third law in *Harmonices mundi libri V (Harmony of the World*) in 1619.

a) Kepler's First Law of Planetary Motion

This law can be expressed as follows:

The path of each planet describes an ellipse with the Sun located at one of its foci.
(The Law of Ellipse)

This first Kepler's Law (Fig. 10 and 11) is sometimes referred to as *the law of ellipse* because planets are orbiting around the Sun in a path described as an ellipse. An ellipse is a special curve in which the sum of the distances from every point on the curve to two other points (foci F_1 and F_2) is a constant.

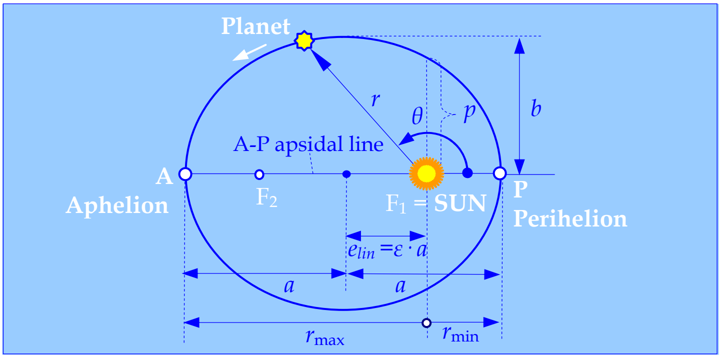


Fig. 10. The Orbit of a planet is an ellipse and its elements.

Before Kepler the Greek astronomer Ptolemy and many others after him were thinking that the Sun and planets travel in circles around the Earth. The ellipse can be mathematically expressed in the polar coordinate system by this equation:

$$r = \frac{p}{1 + \varepsilon \cos\theta}, \tag{3}$$

where (r, θ) are heliocentric polar coordinates for the orbit of planet (r – the distance between the Sun and a planet, θ – angle from the perihelion to the planet as seen from the Sun, respectively known as *the true anomaly*), p is the semi-latus rectum, and ε is the numerical eccentricity.

At $\theta = 0°$ the minimum distance is equal to

$$r_{\min} = \frac{p}{1+\varepsilon}. \tag{4}$$

At $\theta = 90°$ the distance is equal p.

At $\theta = 180°$ the maximum distance is

$$r_{\max} = \frac{p}{1-\varepsilon}. \tag{5}$$

The semi-major axis is the arithmetic mean between $r_{\min}$ and $r_{\max}$:

$$a = \frac{r_{\max} + r_{\min}}{2} = \frac{p}{1-\varepsilon^2}. \tag{6}$$

The semi-minor axis is the geometric mean between $r_{\min}$ and $r_{\max}$:

$$b = \sqrt{r_{\min} \cdot r_{\max}} = \frac{p}{\sqrt{1-\varepsilon^2}} = a\sqrt{1-\varepsilon^2}\,. \tag{7}$$

The semi-latus rectum p is equal to

$$p = \frac{b^2}{a}\,. \tag{8}$$

The area A of an ellipse is

$$A = \pi ab\,. \tag{9}$$

In the special case when $\varepsilon = 0$ then an ellipse turns into a circle where $r = p = r_{\min} = r_{\max} = a = b$ and $A = \pi r^2$.

Using ellipse-related equations Kepler's procedure for calculating heliocentric polar coordinates r, θ, for planetary position as a function of the time t from Perihelion, and the orbital period P, follows four steps:

1. Compute the *mean anomaly* M_a from the equation $M_a = \frac{2\pi t}{P}$. (10)
2. Compute the *eccentric anomaly* E by numerically solving Kepler's equation:

$$M_a = E - \varepsilon \sin E\,. \tag{11}$$

3. Compute the *true anomaly* θ by the equation $\tan\frac{\theta}{2} = \sqrt{\frac{1+\varepsilon}{1-\varepsilon}}\tan\frac{E}{2}$. (12)
4. Compute the *heliocentric distance* r from the equation $r = \frac{p}{1+\varepsilon\cos\theta}$. (13)

For the circle $\varepsilon = 0$ we have simple dependence $\theta = E = M_a$.

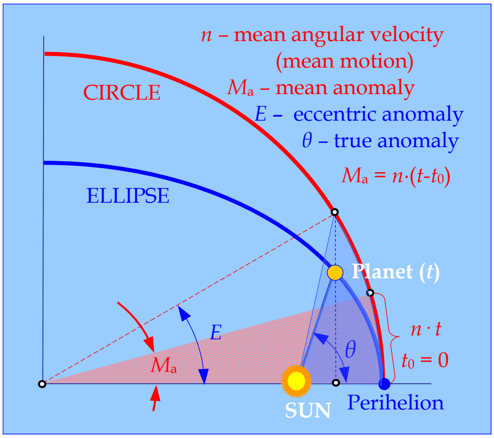


Fig. 11. Elements of parameters of a satellite orbit.

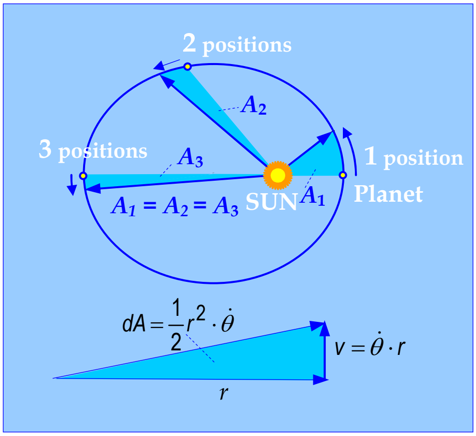


Fig. 12. The radius vector drawn from the Sun to a planet covers equal areas in equal times.

b) The Second Kepler's Law of Planetary Motion
This law can be expressed as follows:

The radius vector drawn from the Sun to a planet covers equal areas in equal times. (The Law of equal areas)

Mathematically this law can be expressed with the equation:

$$\frac{d}{dt}\left(\frac{1}{2}r^2\dot{\theta}\right)=0\,, \tag{14}$$

where $\dot{\theta}$ is angular velocity of true anomaly, $\frac{1}{2}r^2\dot{\theta}$ is the ''*areal velocity*'' that the radius vector r drawn from the Sun to the planet sweeps in one second (Fig. 12).

From this law it follows that the speed at which any planet moves through space is continuously by changing. A planet moves most quickly when it's closer to the Sun and more slowly when it is further from the Sun.

c) The Third Kepler's Law of Planetary Motion
This law can be expressed as follows:

The squares of the periodic times of the planets are proportional to the cubes of the semi-major axes of their orbits. (The harmonic law)

This law is giving the relationship between the distance of planets from the Sun and their orbital periods. Mathematically and symbolically it's possible to express as follows:

$$P^2 \propto a^3\,,$$

where P is the orbital period of circulate planet around the Sun and a is the semi-major axis of this orbit. Because this proportionality is the same for any planet which rotates around the Sun it's possible to write the next equation:

$$\frac{P_{planet1}^2}{a_{planet1}^3}=\frac{P_{planet2}^2}{a_{planet2}^3}\,,\quad \text{namely} \quad \frac{P_{planet1}^2}{P_{planet2}^2}=\frac{a_{planet1}^3}{a_{planet2}^3}\,. \tag{15}$$

3. The Physical Laws of Motions

Sir Isaac Newton's formulated three fundamental laws of the classical mechanics and the law of gravitation in his great work *Philosophieæ Naturalis (Principia Mathematica)* published on July 5, 1687. Before Isaac Newton the great contribution to the advance of mechanic was given by Galileo, Kepler and Huygens.

3.1 The First Law of Motion – Law of Inertia

This law can be expressed as follows:

Everybody persists in its state of being at rest or moving uniformly straight forward, except insofar as it is compelled to change its state by force impressed.

Newton's first law of motion is also called *the law of inertia*. It states that if the vector sum of all forces acting on an object is zero, then the acceleration of the object is zero and its velocity is constant. Consequently:

- An object that is at rest will stay at rest until a balanced force acts upon it.
- An object that is in motion will not change its velocity until a balanced force acts upon it.

If the resultant force acting on a particle is zero, the particle will remain at rest (if originally at rest) or will move with constant speed in a straight line (if originally in motion).

3.2 The Second Law of Motion – Law of Force

This law can be expressed as follows:

Force equals mass times acceleration.
If the resultant force acting on a particle is not zero, the particle will have acceleration proportional to the magnitude of the resultant and in the direction of this resultant force.

This law may be expressed by the equation:

$$\mathbf{F} = m\mathbf{a}\,, \tag{16}$$

where $\mathbf{F}$ is the vector of force, m the mass of particle and $\mathbf{a}$ is the vector of acceleration (Fig. 13. a).

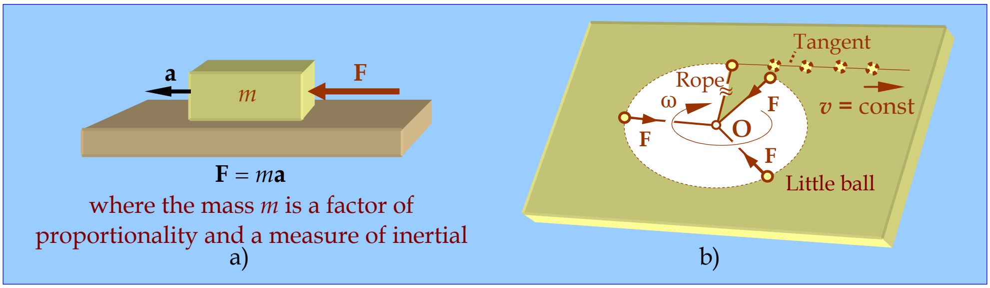


Fig. 13. a) An acceleration **a** of a free body on a horizontal plane under influence of a force **F**, b) In the rotation when the rope is broken, a little ball shall start moving with constant velocity along the line of tangent in the horizontal plane.

Really, this is differential equation which represents a basic equation of motion or basic equation of dynamic.
Alternatively this law can be expressed by the equation:

$$\mathbf{F} = \frac{d}{dt}(m\mathbf{v})\,, \tag{17}$$

where the product $m\mathbf{v}$ is the momentum: m - the mass of particle and $\mathbf{v}$ - the velocity. So, we can say:

The force is equal to the time derivative of the body's momentum.

3.3 The Third Law – Law of action and reaction

This law can be expressed in the following way:

To every action there is an equal by magnitude and opposite reaction (Fig. 14. a).

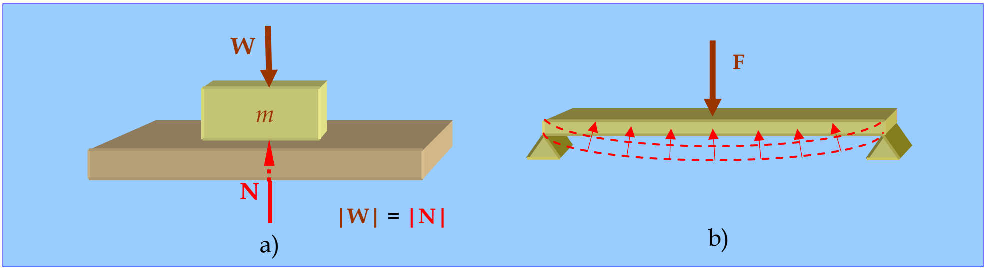


Fig. 14. a) Under the influence of the weight **W** of a body, a normal reaction of its support occurs, b) A beam under loading by a force **F** will be deformed as reaction to an active force **F**.

This law can be also expressed:

The forces of action and reaction between bodies in contact have the same magnitude, same line of action and opposite sense.

3.4 The Law of Gravitation – The Law of Universal Gravitation

Isaac Newton stated that two particles at the distance r from each other and, respectively, of mass M and m, attract each other with equal and opposite forces $\mathbf{F_1}$ and $\mathbf{F_2}$ directed along the line joining the particles (Fig. 15). The common magnitude F of these two forces is:

$$F = G\frac{Mm}{r^2}, \tag{18}$$

where G is the *universal constant of gravitation* G ≈ 6.67428 × 10^{-11} $m^3kg^{-1}s^{-2}$),or approximately $3.44\times10^{-8}ft^4/lb\text{-}sec^4$ in British gravitational system of units (Beer & Johnston 1962).

The force of attraction exerted by the Earth on body of the mass m located on or near its surface is defined as the weight (W) of the body (Fig. 16)

$$W = mg, \tag{19}$$

where g is the acceleration of gravity, being also the acceleration of force of weight.

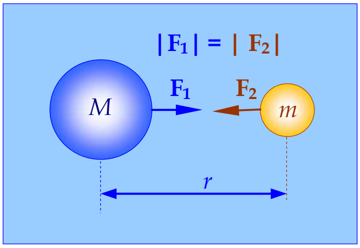


Fig. 15. Newton's Law of the universal gravitation.

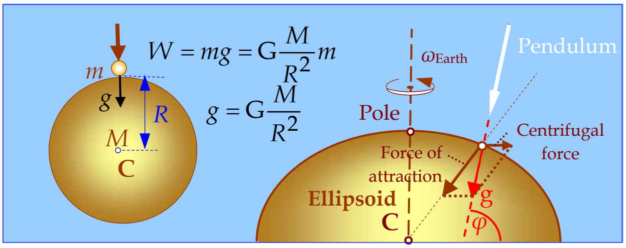


Fig. 16. The weight of a body on the surface of the Earth and the influence of centrifugal forces.

Because this force is really the force of universal gravitation it's possible to say

$$\mathrm{W} = mg = \mathrm{G}\frac{M}{R^2}m\,. \tag{20}$$

From this equation it follows that the acceleration of gravity is

$$g = \frac{\mathrm{G}M}{R^2}\,. \tag{21}$$

The Earth is not truly spherical so the distance R from the centre of the Earth depends on the point selected on its surface. This will be the reason why the weight of the same body will also not be the same weight on different geographical latitude and altitude of the considered point. For more accurate definition of the weight of a body it's necessary to include a component representing the centrifugal force due to the rotation of the Earth. So, the values of g for a body in rest at the sea level vary from 9.780 m/s² (32.09 ft/s²) at the equator to 9.832 m/s² (32.26 ft/s²) at the poles.

3.5 D'Alembert's Principle

Jean le Rond d' Alembert (1717-1783) postulated the principle called by his name from the basic equation of dynamic:

$$\mathbf{F} = m \cdot \mathbf{a}\,. \tag{22}$$

This equation can be written in this form:

$$\mathbf{F} + (-m \cdot \mathbf{a}) = 0\,. \tag{23}$$

From this equation the magnitude $(-m \cdot \mathbf{a})$ is called *inertial force*. So, this equation represents an equation of fictive equilibrium where **F** represents resultant of all active and reactive forces, and inertial force which has the magnitude $m{\cdot}a$, but in opposite direction of the acceleration **a**. This equation is named *equation of dynamic equilibrium*.

For example, when a body is rotating in a circle with constant velocity v restrained by a rope length R then centrifugal force appears (Fig. 17).

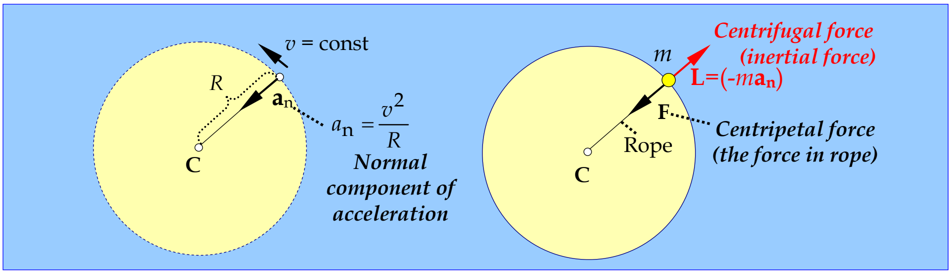


Fig. 17. Centrifugal force at rotation with constant magnitude of velocity v along a circle, and then centrifugal force **L** appears.

3.6 Potential due to a Spherical Shell

A basic result proved by Isaac Newton is that spherical shell which is homogeneous (with constant density) attracts an exterior point with mass m=1 as if all of the mass M of the spherical shell concentrated at its centre C (Fig. 18). This is the same, as if we have homogenous concentric layers but with different densities and whole masses M then an exterior mass point m attracts as if all of the mass M of the spheres was concentrated at its centre (Fig. 19).

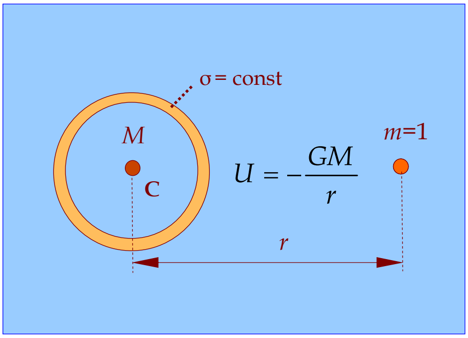


Fig. 18. The potential due to the solid spherical shell.

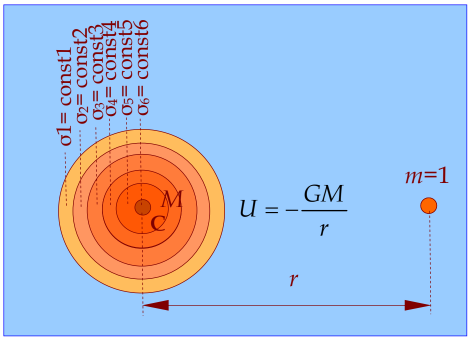


Fig. 19. The potential due to the concentric solid homogeneous spherical shells.

This fundamental result allows us to consider that the attraction between the Earth and the Sun, for example, to be equivalent to that between two mass points.
So we can say:

The solid sphere of constant density attracts an exterior unit mass though all of its masses were concentrated at the centre.

The potential, therefore, due to a spherical body homogeneous in concentric layers, for a point outside the sphere is

$$U = -\frac{GM}{r}, \tag{24}$$

where r is the distance from the point with mass m to the centre C of the mass of the homogeneous sphere or to the centre C of the concentric homogeneous spheres.

4. Determination of Orbits

Jacques Philippe Marie Binet (1786-1856) derived the differential equation in the polar coordinate system of the motion free material particle under action of the central force when areal velocity by the second Kepler's law is constant. This Binet's differential equation can be put down in writing

$$-mC^2u^2\left[\frac{d^2u}{d\theta^2}+u\right] = F_{\mathrm{rad}}, \tag{25}$$

where is $u = \frac{1}{r}$, C - double areal velocity ($C = \dot{\theta}r^2$), F_{rad} gravitation force of the central body with mass M on the free particle with mass m ($F_{\mathrm{rad}} = -GMmu^2$), where minus sign indicates that this is an attracting force, and plus sign stands for the repulsive force.
This differential equation is equation of free particle motion in a plane displayed in the polar coordinate system. Thus, inhomogeneous differential equation is obtained

$$\underbrace{\left[\frac{d^2u}{d\theta^2}+u\right]}_{\text{Homogeneous Part}} = \underbrace{\frac{GM}{C^2}}_{\text{Inhomogeneous Part}}. \tag{26}$$

The solution for the homogeneous part of this equation is

$$u_1 = B\cos(\theta-\theta_0), \tag{27}$$

where B and θ_0 are the constants of integration. Choosing the polar axis so that $\theta_0 = 0$ we can write

$$u_1 = B\cos(\theta) \tag{28}$$

and for the inhomogeneous part of the equation

$$u_2 = \frac{GM}{C^2} = \frac{1}{p}. \tag{29}$$

The solution of this inhomogeneous differential equation (26) is

$$u = u_2 + u_1 = \frac{1}{p}\left[1 + \frac{C^2}{GM} B\cos(\theta)\right]. \tag{30}$$

The equation for the ellipse and for the other conic section in the polar coordinate system can be written in the form

$$\frac{1}{r} = \frac{1}{p}\left[1 + \varepsilon\cos\theta\right]. \tag{31}$$

After comparing the equations (30) and (31) it is possible to see that the equation (30) truly represents the equation of a conic section. The product of the constants B and $C^2/(GM)$ defines the *eccentricity* ε of the conic section. So it can be expressed by the equation (Beer & Johnston, 1962)

$$\varepsilon = \frac{B}{\frac{GM}{C^2}} = \frac{BC^2}{GM}. \tag{32}$$

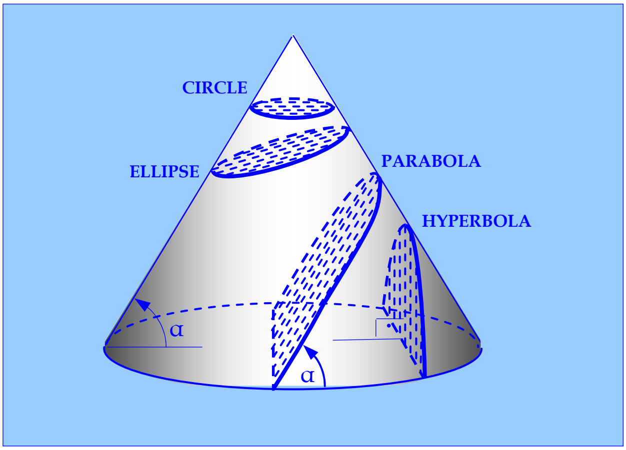


Fig. 20. The conic sections: circle, ellipse, parabola and hyperbola.

Four cases may be distinguished for different eccentricities (Fig. 20).

1) The conic section is a *circle* when is $\varepsilon = 0$.
2) The conic section is an *ellipse* when $0 < \varepsilon < 1$.
3) The conic section is a *parabola* when $\varepsilon = 1$.
4) The conic section is a *hyperbola* when $\varepsilon > 1$.

Of cause for the planets and for the satellites orbits can be only circulars or ellipses.

5. The Two-Body Problem

It is possible to investigate the motion of two bodies that are only under their mutual attraction. It can also be assumed that the bodies are symmetrical and homogeneous and that they can be considered to be point masses. So we can do analysis of the motion of planets and the Sun.

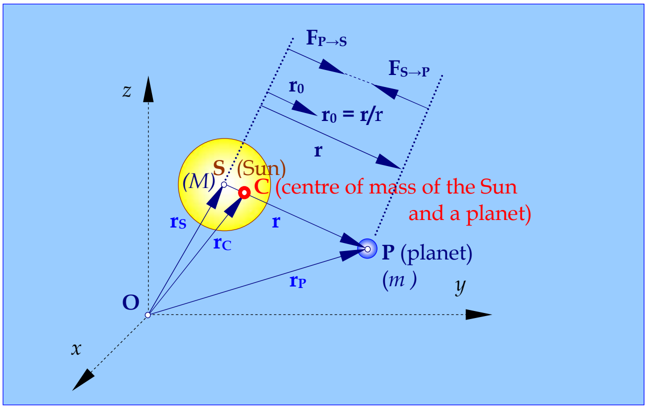


Fig. 21. Motion of the Sun and a planet in two-body problem.

The differential equation of the Sun motion (Fig. 21) is

$$M\frac{d^2\mathbf{r}_S}{dt^2} = +G\frac{Mm}{r^2}\frac{\mathbf{r}}{r}. \tag{33}$$

The sign + is because the force $\mathbf{F}_{P\to S}$ has the same orientation as the vector $\mathbf{r}_0$.
The differential equation of the planet motion is

$$m\frac{d^2\mathbf{r}_P}{dt^2} = -G\frac{Mm}{r^2}\frac{\mathbf{r}}{r}. \tag{34}$$

The sign – is because the force $\mathbf{F}_{S\to P}$ has the same orientation as the vector $\mathbf{r}_0$.
After summing up the equations (33) and (34) we can write

$$M\frac{d^2\mathbf{r}_S}{dt^2} + m\frac{d^2\mathbf{r}_P}{dt^2} = 0 \quad \text{or} \quad \frac{d^2}{dt^2}\left(M\mathbf{r}_S + m\mathbf{r}_P\right) = 0. \tag{35}$$

From the static it is known that the sum of the moment forces is equal to the moment of resultant. So we can say that the sum of the moment masses is equal to the moment of resultant mass. Now it is possible to write

$$M\mathbf{r}_S + m\mathbf{r}_P = (M+m)\mathbf{r}_C. \tag{36}$$

After the first and the second derivation we have

$$\frac{d}{dt}(M\mathbf{r}_S + m\mathbf{r}_P) = \frac{d}{dt}[(M+m)\mathbf{r}_C] \qquad \text{and}$$

$$\frac{d^2}{dt^2}(M\mathbf{r}_S + m\mathbf{r}_P) = \frac{d^2}{dt^2}[(M+m)\mathbf{r}_C] = \frac{d^2\mathbf{r}_C}{dt^2}(M+m). \tag{37}$$

From the equations (35) and (37) it follows

$$\frac{d^2\mathbf{r}_C}{dt^2}(M+m) = 0 \quad \text{or} \quad \frac{d^2\mathbf{r}_C}{dt^2} = 0\,. \tag{38}$$

Therefore the centre of this material system has no acceleration, namely this material system is in the inertial motion with the possibility to move with constant velocity $\mathbf{v}_C$, or remain at rest.
The equation (33) may be multiplied by m, and equation (34) by M and after subtracting the equation (33) from (34) we can write

$$Mm\left[\frac{d^2\mathbf{r}_P}{dt^2} - \frac{d^2\mathbf{r}_S}{dt^2}\right] = -G\frac{mM}{r^2}\frac{\mathbf{r}}{r}(M+m), \tag{39}$$

$$Mm\frac{d^2}{dt^2}[\mathbf{r}_P - \mathbf{r}_S] = -G\frac{mM}{r^2}\frac{\mathbf{r}}{r}(M+m) \tag{40}$$

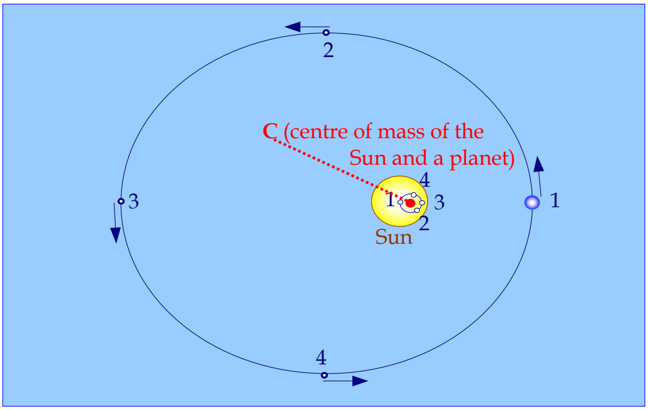


Fig. 22. The Sun is moving also around the centre C of mass of the Sun and a planet by a small ellipse and a planet is moving about the same centre C by the bigger ellipse, not around the geometrical centre of Sun.

After dividing the equation (40) by M and taking from Fig. 21 that $\mathbf{r}=\mathbf{r}_P-\mathbf{r}_S$ we can write

$$m\frac{d^2\mathbf{r}}{dt^2} = -G\frac{m(M+m)}{r^2}\frac{\mathbf{r}}{r}\,. \tag{41}$$

This is differential equation (41) of the planet motion when taken into account and the planet acting on the Sun. It is easy to prove that this planet is really rotating around the centre of mass (C) of the Sun and the planet. Also the Sun's geometrical center is rotating by the small ellipse around the centre of mass (C) (Fig. 22).
Hence:

The planet is rotating around the centre of masses (C) of the Sun and the planet by the bigger ellipse. The geometrical centre of the Sun also rotates around the centre of masses (C) by a small ellipse.

6. Satellite Motion

The problem, of two bodies is solved exactly in the celestial mechanics, but only in the special case if both bodies are having small dimensions, i.e. if the Sun and a planet can be thought of as particles. In this special case the motions of particles around the body with finite dimensions is also included, if this body with finite dimensions has the central spherical field of forces. (For example, as a homogeneous ball (Fig. 18) or concentric solid homogeneous spherical shells with different densities (Fig. 19)). Just because our Earth is not a ball and with homogeneous masses some discrepancies appear at satellite motions around the Earth from the exact solutions of two bodies when we imagine whole mass of the Earth as concentrated in it the centre of mass. For the solution of the problem of the motion of bodies (two particles) exactly valuable are three Kepler's laws from which fallow that the satellite would be moving constantly in the same plane by the ellipse with constant areal velocity.

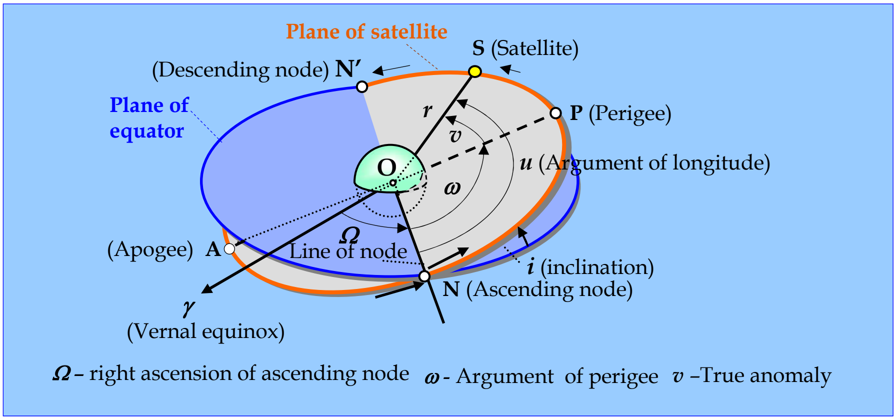


Fig. 23. Keplerian orbital parameters.

The positions of satellites are determined with six Keplerian orbital parameters: Ω, i, ω, a, e and v or t (Fig. 23):

- *The orientation of orbits* in space is determined by:

Ω - *the right ascension of ascending node* (the angle measured in the equator plane between the directions to the vernal equinox and ascending node N where the satellite crosses equatorial plane from the south to the north celestial sphere), i – *the inclination of orbit*, (the angle between the equatorial plane and orbital plane) and ω – *the argument of perigee* (the angle between the ascending node and the direction to perigee (as the nearest point of satellite)).

- *The dimensions of orbit* are determined by: a - *the semi-major axis* and ε – *the numerical eccentricity* of an ellipse.
- *The position of satellite on its orbits* is determined by: v - *the true anomaly* (as the angle between the directions to perigee and instantaneous position of satellite) or by t - the difference of time in instantaneous position and the time in perigee.

All Kepler's laws and Newton's laws for a planet motion are valued also for the Earth's satellites motion but at satellites there are some more perturbations.

6.1 Required Velocity for a Satellite

A body will be a satellite in a circular orbit around the Earth if it has velocity in the horizontal line so that centrifugal force is equal to centripetal force which is produced by the Earth's gravitation attraction (Fig. 24). So it can be expressed with the equation:

$$m\frac{v_I^2}{R+H_s} = G\frac{mM}{(R+H_s)^2}, \tag{42}$$

where: m - mass of a satellite, M – the Earth's mass, R - radius of the Earth, H_s - altitude of a satellite above the surface of the Earth, G constant of universal gravitation and v_I velocity of a body which will become the satellite.

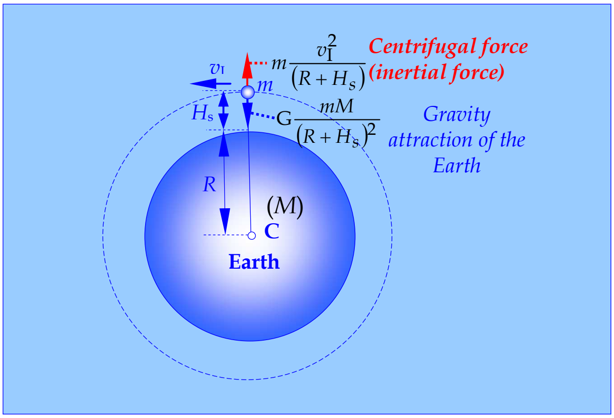


Fig. 24. On the satellite in orbit act the Earth's gravity attraction and the centrifugal force.

From this equation (42) next the equation follows

$$v_I = \sqrt{\frac{GM}{R+H_s}}. \tag{43}$$