## OFDM Communications with Cooperative Relays

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## 1. Introduction

## 1.1 Cooperative relay communications

Signal fading due to multi-path propagation is one of the major impairments to meet the demands of next generation wireless networks for high data rate services. To mitigate the fading effects, time, frequency, and spatial diversity techniques or their hybrid can be used. Among different types of diversity techniques, spatial diversity is of special interest as is does not incur system losses in terms of delay and bandwidth efficiency.

Recently, cooperative diversity in wireless network has received great interest and is regarded as a promising technique to mitigate multi-path fading, which results in a fluctuation in the amplitude of the received signal. The cooperative communications is a new communication paradigm which generates independent paths between the user and the base station by introducing a relay channel. The relay channel can be thought of as an auxiliary channel to the direct channel between the source and destination. The basic idea behind cooperation is that several users in a network pool their resources in order to form a virtual antenna array which creates spatial diversity (Laneman et al., 2004; Sendonaris et al., Part I, 2003). Since the relay node is usually several wavelengths distant from the source, the relay channel is guaranteed to fade independently from the direct channel, which introduces a full-rank Multiple-input-multiple-output (MIMO) channel between the source and the destination. This cooperative spatial diversity leads to an increased exponential decay rate in the error probability with increasing signal-to-noise ratio (SNR) (Liu et al., 2009).

Before discussing cooperative OFDM, let us first review some fundamental knowledge of OFDM and MIMO, which is associated with the cooperative OFDM study in this chapter.

#### 1.2 Physical layer of cooperative wireless networks (OFDM & MIMO) 1.2.1 OFDM basics

In the modern wireless communication, OFDM technology has been widely used due to its spectral efficiency and inherent flexibility in allocating power and bit rate over distinct subcarriers which are orthogonal to each other. Different from a serial transmission, OFDM Source: Communications and Networking, Book edited by: Jun Peng.

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is a multi-carrier block transmission, where, as the name suggests, information-bearing symbols are processed in blocks at both the transmitter and the receiver.



Fig. 1. Discrete-time block equivalent models of CP-OFDM, top: transmitter & channel, bottom: receiver.

A number of benefits the OFDM brings to cooperative relay systems originate from the basic features that OFDM possesses. To appreciate those, we first outline Cyclic Prefix (CP)-OFDM's operation using the discrete-time baseband equivalent block model of a singletransceiver system depicted in Fig.1, where  $X_i$  is the so-called frequency signal at the *i*-th time symbol duration in one OFDM frame, then it will be transferred as  $\tilde{\mathbf{x}}_i$  in the time domain by the *M*-point inverse fast Fourier transform (IFFT) matrix  $\mathbf{F}_{M}^{-1} = \mathbf{F}_{M}^{H}$  with (m, k)-th entry  $\exp(j2\pi mk/M)/\sqrt{M}$ , i.e.,  $\tilde{\mathbf{x}}_i = \mathbf{F}_M^H \mathbf{X}_i$ ,  $\mathbf{F}_M$  is the *M*-point fast Fourier transform (FFT) matrix where  $(\cdot)^{H}$  denotes conjugate transposition,  $(\cdot)^{\dagger}$  denotes matrix pseudoinverse, and  $(\cdot)^{-1}$  denotes matrix inversion and *m*, *k* denote the index in frequency and time domain, respectively. Applying the triangle inequality to the M-point IFFT definition shows that the entries of  $\mathbf{F}_{M}^{H}\mathbf{X}_{i}$  have magnitudes that can exceed those of  $\mathbf{X}_{i}$  by a factor as high as M. In other words, IFFT processing can increase the peak to average power ratio (PAPR) by a factor as high as the number of subcarriers (which in certain applications can exceed 1000). Then a CP of length *D* is inserted between each  $\tilde{\mathbf{x}}_i$  to form the redundant OFDM symbols  $ilde{\mathbf{x}}_{_{cp,i}}$  , which are sequentially transmitted through the channel. The total number of the time domain signals in each OFDM symbol is, thus, C = M + D. If we define  $\mathbf{F}_{cr} := [\mathbf{F}_D, \mathbf{F}_M]^H$  as the  $C \times M$  expanded IFFT matrix, where  $\mathbf{F}_D$  is the last D columns of  $\mathbf{F}_{M_t}$  that way, the redundant OFDM symbol to be transmitted can also be expressed as  $\tilde{\mathbf{x}}_{cp,i} = \mathbf{F}_{cp} \mathbf{X}_i$ . With  $(\cdot)^T$  denotes transposition, and assuming no channel state information (CSI) to be available at the transmitter, then the received symbol  $\tilde{\mathbf{y}}_{cp,i}$  at the *i*-th time symbol duration can be written as:

$$\tilde{\mathbf{y}}_{cp,i} = \mathbf{H}\mathbf{F}_{cp}\mathbf{X}_{i} + \mathbf{H}_{ISI}\mathbf{F}_{cp}\mathbf{X}_{i-1} + \tilde{\mathbf{n}}_{C,i}$$
(1)

where **H** is the  $C \times C$  lower triangular Toeplitz filtering matrix with first column  $[h_1 \cdots h_L \quad 0 \cdots 0]^T$ , where *L* is the channel order (i.e.,  $h_i = 0, \forall i > L$ ), **H**<sub>*ISI*</sub> is the  $C \times C$  upper triangular Toeplitz filtering matrix with first row  $[0 \cdots 0 \quad h_1 \cdots h_2]$ , which captures inter-

symbol interference (ISI),  $\tilde{\mathbf{n}}_{C,i}$  denotes the additive white Gaussian noise (AWGN) vector with variance  $N_0$  and Length *C*. After removing the CP at the receiver, ISI is also discarded, and (1) can be rewritten as:

$$\tilde{\mathbf{y}}_{i} = \mathbf{C}_{M}(\mathbf{h})\mathbf{F}_{M}^{H}\mathbf{X}_{i} + \tilde{\mathbf{n}}_{M,i}$$
(2)

where  $\mathbf{C}_M$  (**h**) is  $M \times M$  circulant matrix with first row  $[h_1 \quad 0 \cdots 0 \quad h_L \cdots h_2]$ , and  $\tilde{\mathbf{n}}_{M,i}$  is a vector formed by the last M elements of  $\tilde{\mathbf{n}}_{C,i}$ .

The procedure of adding and removing CP forces the linear convolution with the channel impulse response to resemble a circular convolution. Equalization of CP-OFDM transmissions ties to the well known property that a circular convolution in the time domain, is equivalent to a multiplication operation in the frequency domain. Hence, the circulant matrix can be diagonalized by post- (pre-) multiplication by (I)FFT matrices, and only a single-tap frequency domain equalizer is sufficient to resolve the multipath effect on the transmitted signal. After demodulation with the FFT matrix, the received signal is given by:

$$\mathbf{Y}_{i} = \mathbf{F}_{M} \mathbf{C}_{M} (\mathbf{h}) \mathbf{F}_{M}^{H} \mathbf{X}_{i} + \mathbf{F}_{M} \tilde{\mathbf{n}}_{M,i}$$
$$= \operatorname{diag} (H_{1} \cdots H_{M}) \mathbf{X}_{i} + \mathbf{F}_{M} \tilde{\mathbf{n}}_{M,i}$$
$$= \mathbf{D}_{M} (\mathbf{H}_{M}) \mathbf{X}_{i} + \mathbf{n}_{M,i}$$
(3)

where  $\mathbf{H}_{M} = \begin{bmatrix} H_{1} \cdots H_{M} \end{bmatrix}^{T} = \sqrt{M} \mathbf{F}_{M} \mathbf{h}$ , with

$$H_{k} \equiv H(2\pi k / M) := \sum_{l=1}^{L} h_{l} e^{-j2\pi k l / M}$$
(4)

denoting the channel's transfer function on the *k*-th subcarrier,  $\mathbf{D}_M$  ( $\mathbf{H}_M$ ) stands for the  $M \times M$  diagonal matrix with  $\mathbf{H}_M$  on its diagonal,  $\mathbf{n}_{M,i} := \mathbf{F}_M \tilde{\mathbf{n}}_{M,i}$ .

Equations (3) and (4) show that an OFDM system which relies on *M* subcarriers to transmit the symbols of each block  $X_i$ , converts an FIR frequency-selective channel to an equivalent set of *M* flat fading subchannels. This is intuitively reasonable since each narrowband subcarrier that is used to convey each information-bearing symbol per OFDM block "sees" a narrow portion of the broadband frequency-selective channel which can be considered frequency flat. This scalar model enables simple equalization of the FIR channel (by dividing (3) with the corresponding scalar subchannel  $H_M$ ) as well as low-complexity decoding across subchannels using (Muquet et al., 2009; Wang & Giannakis, 2000). Transmission of symbols over subcarriers also allows for a flexible allocation of the available bandwidth to multiple users operating with possibly different rate requirements imposed by multimedia applications, which may include communication of data, audio, or video. When CSI is available at the transmitter side, power and bits can be adaptively loaded per OFDM subcarrier, depending on the strength of the intended subchannel. Because of orthogonality of ODFM subcarriers, OFDM system exhibits robustness to the narrow band interference.

The price paid for OFDM's attractive features in equalization, decoding, and possibly adaptive power and bandwidth allocation is its sensitivity to subcarrier drifts and the high PAPR that IFFT processing introduces to the entries of each block transmitted. Subcarrier drifts come either from the carrier-frequency and phase offsets between transmit-receive oscillators or from mobility-induced Doppler effects, with the latter causing a spectrum of frequency drifts. Subcarrier drifts cause inter-carrier interference (ICI), which renders (3) invalid. On the other hand, high PAPR necessitates backing-off transmit-power amplifiers to avoid nonlinear distortion effects (Batra et al., 2004).

However, the same multipath robustness can be obtained by adopting ZP instead of CP (Lu et al., 2009). If the length of the zero-padding equals the length of CP, then the ZP-OFDM will achieve the same spectrum efficiency as CP-OFDM.

The only difference between the transmission part of the ZP-OFDM and CP-OFDM, as shown in Fig. 2, is the CP replaced by *D* appending zeros at the end of the symbol. If we define  $\mathbf{F}_{zp} := [\mathbf{F}_M, \mathbf{0}]^H$ , and Z = C = M + D, the transmitted OFDM symbol can be denoted as  $\tilde{\mathbf{x}}_{zp,i} = \mathbf{F}_{zp} \mathbf{X}_i$ . The received symbol is now expressed as:

$$\tilde{\mathbf{y}}_{zp,i} = \mathbf{H}\mathbf{F}_{zp}\mathbf{X}_i + \mathbf{H}_{ISI}\mathbf{F}_{zp}\mathbf{X}_{i-1} + \tilde{\mathbf{n}}_{Z,i}.$$
(5)

The key advantage of ZP-OFDM relies on two aspects: first, the all-zero  $D \times M$  matrix **0** is able to take good care of the ISI, when the length of the padded zeros is not less than the maximum channel delay. Second, according to the Eq. (4), multipath channel will introduce 3 impact factors,  $h_l$ , k and l to the received signal, which stand for the amplitude, subcarriers (in frequency domain) and delay (in time domain), respectively. Therefore, different CP copies from multipath certainly pose stronger interference than ZP copies. Thus, without equalization or some pre-modulation schemes, like Differential-PSK, the ZP-OFDM has a natural better bit error rate (BER) performance than the CP-OFDM. Furthermore, the linear structure of the channel matrix in ZP-OFDM ensures the symbol recovery regardless of the channel zeros locations.



Fig. 2. Discrete-time block equivalent models of ZP-OFDM, top: transmitter & channel, bottom: receiver.

Nevertheless, because of the zero-padding and linear structure of ZP-OFDM, it outperforms CP-OFDM in terms of the lower PAPR (Batra et al., 2004; Lu et al., 2009). Similar to silent periods in TDMA, trailing zeros will not pose problems to high-power amplifiers (HPA). By adopting the proper filter, they will not give rise to out-of-band spectral leakage, either. The

circulant channel convolution matrix  $C_M$  (**h**) in the CP-OFDM is invertible if and only if the channel transfer function has no zeros on the FFT grid, i.e.,  $H_k \neq 0$ ,  $\forall k \in [1, M]$ , therefore, when channel nulls hit the transmitted symbols, the signal recovery becomes impossible. However, in the ZP-OFDM, the tall Toeplitz structure of equivalent channel matrix always guarantees its full rank (it only becomes rank deficient when the channel impulse response is identically zero, which is impossible in practice) (Muquet et al., 2009). In other words, the full rank property guarantees the detection of transmitted symbols.

In the blind channel estimation and blind symbol synchronization, ZP-OFDM also has its advantage in reducing the system complexity. Therefore, for more efficient utilization of the spectrum and low power transmission, a fast-equalized ZP-OFDM seems more promising than the CP-OFDM.

The above reviewed advantages and limitations of single-transceiver CP-OFDM and ZP-OFDM systems are basically present in the cooperative scenario which we present later under the name of cooperative OFDM.

#### 1.2.2 From MIMO to cooperative communications

MIMO systems have been constructed comprising multiple antennas at both the transmitter and receiver to offer significant increases in data throughput and link range without additional expenditure in frequency and time domain. The spatial diversity has been studied intensively in the context of MIMO systems (Barbarossa, 2005). It has been shown that utilizing MIMO systems can significantly improve the system throughput and reliability (Foschini & Gans, 1998).

In the fourth generation wireless networks to be deployed in the next couple of years, namely, mobile broadband wireless access (MBWA) or IEEE 802.20, peak date rates of 260 Mbps can be achieved on the downlink, and 60 Mbps on the uplink (Hwang et al., 2007). These data rates can, however, only be achieved for full-rank MIMO users. More specifically, full-rank MIMO users must have multiple antennas at the mobile terminal, and these antennas must see independent channel fades to the multiple antennas located at the base station. In practice, not all users can guarantee such high rates because they either do not have multiple antennas installed on their small-size devices, or the propagation environment cannot support MIMO because, for example, there is not enough scattering. In the latter case, even if the user has multiple antennas installed full-rank MIMO is not achieved because the paths between several antenna elements are highly correlated.

To overcome the above limitations of achieving MIMO gains in future wireless networks, we must think of new techniques beyond traditional point-to-point communications. The traditional view of a wireless system is that it is a set of nodes trying to communicate with each other. From another point of view, however, because of the broadcast nature of the wireless channel, we can think of those nodes as a set of antennas distributed in the wireless system. Adopting this point of view, nodes in the network can cooperate together for a distributed transmission and processing of information. The cooperating node acts as a relay node for the source node. Since the relay node is usually several wavelengths distant from the source, the relay channels are guaranteed to fade independently from the direct channels, as well as each other which introduces a full-rank MIMO channel between the source and the destination. In the cooperative communications setup, there is a-priori few constraints to different nodes receiving useful energy that has been emitted by another transmitting node. The new paradigm in user cooperation is that, by implementing the appropriate signal

processing algorithms at the nodes, multiple terminals can process the transmissions overheard from other nodes and be made to collaborate by relaying information for each other. The relayed information is subsequently combined at a destination node so as to create spatial diversity. This creates a network that can be regarded as a system implementing a distributed multiple antenna where collaborating nodes create diverse signal paths for each other (Liu et al., 2009). Therefore, we study the cooperative relay communication system, and consequently, a cooperative ZP-OFDM to achieve the full diversity is investigated.

The rest of the chapter is organized as follows. In Section II, we first provide and discuss the basic models of AF, DF and their hybrid scheme. The performance analysis of the hybrid DF-AF is presented in Section III. The cooperative ZP-OFDM scheme, which will be very promising for the future cooperative Ultra Wide Band (UWB) system, is addressed in Section IV, the space time frequency coding (STFC) scheme for the full diversity cooperation is proposed as well. The conclusions of the chapter appear in Section VI.

#### 2. System model

Cooperative communications is a new paradigm shift for the fourth generation wireless system that will guarantee high data rates to all users in the network, and we anticipate that it will be the key technology aspect in the fifth generation wireless networks (Liu et al., 2009).

In terms of research ascendance, cooperative communications can be seen as related to research on relay channel and MIMO systems. The concept of user cooperation itself was introduced in two-part series of papers (Sendonaris et al., Part I, 2003; Sendonaris et al., Part I, 2003). In these works, Sendonaris *et al.* proposed a two-user cooperation system, in which pairs of terminals in the wireless network are coupled to help each other forming a distributed two-antenna system. Cooperative communications allows different users or nodes in a wireless network to share resources and to create collaboration through distributed transmission/processing, in which each user's information is sent out not only by the user but also by the collaborating users (Nosratinia et al., 2004). Cooperative communications promises significant capacity and multiplexing gain increase in the wireless system (Kramer et al., 2005). It also realizes a new form of space diversity to combat the detrimental effects of severe fading. There are mainly two relaying protocols: AF and DF.

#### 2.1 Amplify and forward protocol

In AF, the received signal is amplified and retransmitted to the destination. The advantage of this protocol is its simplicity and low cost implementation. But the noise is also amplified at the relay. The AF relay channel can be modeled as follows. The signal transmitted from the source *x* is received at both the relay and destination as

$$y_{s,r} = \sqrt{E_s} h_{s,r} x + n_{s,r}$$
, and  $y_{s,D} = \sqrt{E_s} h_{s,D} x + n_{s,D}$  (6)

where  $h_{S,r}$  and  $h_{S,D}$  are the channel gains between the source and the relay and destination, respectively, and are modeled as Rayleigh flat fading channels. The terms  $n_{S,r}$  and  $n_{S,D}$  denote the additive white Gaussian noise with zero-mean and variance  $N_0$ ,  $E_S$  is the average transmission energy at the source node. In this protocol, the relay amplifies the signal from the source and forwards it to the destination ideally to equalize the effect of the channel

fading between the source and the relay. The relay does that by simply scaling the received signal by a factor  $A_r$  that is inversely proportional to the received power, which is denoted by

$$A_r = \sqrt{\frac{E_s}{E_s h_{s,r} + N_0}} \tag{7}$$

The destination receives two copies from the signal *x* through the source link and relay link. There are different techniques to combine the two signals at the destination. The optimal technique that maximizes the overall SNR is the maximal ratio combiner (MRC). Note that the MRC combining requires a coherent detector that has knowledge of all channel coefficients, and the SNR at the output of the MRC is equal to the sum of the received signal-to-noise ratios from all branches.

## 2.2 Decode and forward protocol

Another protocol is termed as a decode-and-forward scheme, which is often simply called a DF protocol. In the DF, the relay attempts to decode the received signals. If successful, it reencodes the information and retransmits it. Although DF protocol has the advantage over AF protocol in reducing the effects of channel interferences and additive noise at the relay, the system complexity will be increased to guarantee the correct signal detection.

Note that the decoded signal at the relay may be incorrect. If an incorrect signal is forwarded to the destination, the decoding at the destination is meaningless. It is clear that for such a scheme the diversity achieved is only one, because the performance of the system is limited by the worst link from the source-relay and source-destination (Laneman et al., 2004).

Although DF relaying has the advantage over AF relaying in reducing the effects of noise and interference at the relay, it entails the possibility of forwarding erroneously detected signals to the destination, causing error propagation that can diminish the performance of the system. The mutual information between the source and the destination is limited by the mutual information of the weakest link between the source–relay and the combined channel from the source-destination and relay-destination.

Since the reliable decoding is not always available, which also means DF protocol is not always suitable for all relaying situations. The tradeoff between the time-consuming decoding, and a better cooperative transmission, finding the optimum hybrid cooperative schemes, that include both DF and AF for different situations, is an important issue for the cooperative wireless networks design.

#### 2.3 Hybrid DF-AF protocol

In this chapter, we consider a hybrid cooperative OFDM strategy as shown in Fig. 3, where we transmit data from source node S to destination node D through R relays, without the direct link between S and D. This relay structure is called 2-hop relay system, i.e., first hop from source node to relay, and second hop from relay to destination. The channel fading for different links are assumed to be identical and statistically independent, quasi-statistic, i.e., channels are constant within several OFDM symbol durations. This is a reasonable assumption as the relays are usually spatially well separated and in a slow changing environment. We assume that the channels are well known at the corresponding receiver

sides, and a one bit feedback channel from destination to relay is used for removing the unsuitable AF relays. All the AWGN terms have equal variance  $N_0$ . Relays are re-ordered according to the descending order of the SNR between *S* and *Q*, i.e.,  $SNR_{SQ_1} > \cdots > SNR_{SQ_R}$ , where  $SNR_{SQ_1}$  denotes the *r*-th largest SNR between *S* and *Q*.



Fig. 3. Hybrid relay cooperation with dynamic optimal combination of DF-AF relays (S: Source, D: Destination,  $Q_r$ : *r*-th Relay)

In this model, relays can determine whether the received signals are decoded correctly or not, just simply by comparing the SNR to the threshold, which will be elaborated in Section 3.1. Therefore, the relays with SNR above the threshold will be chosen to decode and forward the data to the destination, as shown with the white hexagons in Fig.3. The white circle is the removed AF relay according to the dynamic optimal combination strategy which will be proposed in Section 3.2. The rest of the relays follow the AF protocol, as shown with the white hexagons in Fig. 3 (Lu & Nikookar, 2009; Lu et al., 2010).

The received SNR at the destination in the hybrid cooperative network can be denoted as

$$\gamma_{h} = \sum_{Q_{i} \in DF} \frac{E_{Q} h_{Q_{i},D}}{N_{0}} + \sum_{Q_{j} \in AF} \frac{\frac{E_{S} h_{S,Q_{j}}}{N_{0}} \frac{E_{Q} h_{Q_{j},D}}{N_{0}}}{\frac{E_{S} h_{S,Q_{j}}}{N_{0}} + \frac{E_{Q} h_{Q_{j},D}}{N_{0}} + 1}$$
(8)

where  $h_{Q_i,D}$ ,  $h_{S,Q_j}$  and  $h_{Q_j,D}$  denote the power gains of the channel from the *i*-th relay to the destination in DF protocol, source node to the *j*-th relay in AF protocol and *j*-th relay to the destination in AF protocol, respectively.  $E_S$  and  $E_Q$  in (8) are the average transmission energy at the source node and at the relays, respectively. By choosing the amplification factor  $A_{Q_i}$  in the AF protocol as:

$$A_{Q_{j}}^{2} = \frac{E_{S}}{E_{S}h_{S,Q_{j}} + N_{0}}$$
(9)

and forcing the  $E_Q$  in DF equal to  $E_s$ , it will be convenient to maintain constant average transmission energy at relays, equal to the original transmitted energy at the source node.

In this chapter, OFDM is used as a modulation technique in the cooperative system to gain from its inherent advantages and combat frequency selective fading of each cooperative link, with  $W_{r}$ , r = 1, 2, ..., R independent paths. Later, we also show that, by utilizing the space-frequency coding, hybrid DF-AF cooperative OFDM can also gain from the frequency selective fading and achieve the multi-path diversity with a diversity gain of  $W_{min}$  = min ( $W_r$ ). As shown in the Fig.4, the *r*-th relay first decides to adopt DF or AF protocol according to the SNR threshold. For the DF-protocol, the symbols are decoded at the relays, and then an IFFT operation is applied on these blocks to produce the OFDM symbol. Before transmission, a prefix (CP or ZP) is added to each OFDM symbol. For the AF-protocol, relays which undergo the deep fading will be removed by using the dynamic optimal combination strategy discussed later in this section. Other AF relays are proper relays, amplify and forward the data to the destination. At the destination node, after the prefix removal, the received OFDM symbols are fast-Fourier-transformed, and the resulting symbols at the destination are used for the combination and detection.



Fig. 4. Relay selection in the hybrid DF-AF cooperative OFDM wireless transmission strategy (top: source, middle: relay, bottom: destination)

The receiver at the destination collects the data from DF and AF relays with a MRC. Because of the amplification in the intermediate stage in the AF protocol, the overall channel gain of the AF protocol should include the source to relay, relay to destination channels gains and amplification factor. The decision variable *u* at the MRC output is given by

$$u = \sum_{Q_i \in DF} \frac{\left(H_{Q_i,D}\right)^* Y_{Q_i}}{\left(H_{Q_i,D}\right)^* H_{Q_i,D}} + \sum_{Q_j \in AF} \frac{\left(H_{S,Q_j} A_{Q_j} H_{Q_j,D}\right)^* Y_{Q_j}}{\left(H_{S,Q_j} A_{Q_j} H_{Q_j,D}\right)^* \left(H_{S,Q_j} A_{Q_j} H_{Q_j,D}\right)}$$
(10)

where  $Y_{Q_i}$  and  $Y_{Q_j}$  are the received signal from DF *i*-th relay and AF *j*-th relays, respectively, and  $(\cdot)^*$  denotes the conjugate operation.  $H_{Q_i,D}$ ,  $H_{s,Q_j}$  and  $H_{Q_j,D}$  are frequency response of the channel power gains, respectively.

In the proposed hybrid DF-AF cooperative network, DF plays a dominant role in the whole system. However, switching to AF scheme for the relay nodes with SNR below the threshold often improves the total transmission performance, and accordingly AF plays a positive compensating role.

## 3. Performance analysis of Hybrid DF-AF protocol

## 3.1 Threshold for DF and AF relays

In general, mutual information *I* is the upper bound of the target rate *B* bit/s/Hz, i.e., the spectral efficiency attempted by the transmitting terminal. Normally,  $B \le I$ , and the case B > I is known as the outage event. Meanwhile, channel capacity, *C*, is also regarded as the maximum achievable spectral efficiency, i.e.,  $B \le C$ .

Conventionally, the maximum average mutual information of the direct transmission between source and destination, i.e.,  $I_D$ , achieved by independent and identically distributed (i.i.d) zero-mean, circularly symmetric complex Gaussian inputs, is given by

$$I_D = \log_2(1 + \text{SNR}\,h_{S,D}) \tag{11}$$

as a function of the power gain over source and destination,  $h_{S,D}$ . According to the inequality  $B \le I$ , we can derive the SNR threshold for the full decoding as

$$SNR \ge \frac{2^B - 1}{h_{S,D}}$$
(12)

Then, we suppose all of the X relays adopt the DF cooperative transmission without direct transmission. The maximum average mutual information for DF cooperation  $I_{DF_{-co}}$  is shown (Laneman et al., 2004) to be

$$I_{DF_{co}} = \frac{1}{X} \min\left\{ \log_2\left(1 + \sum_{r=1}^R \text{SNR} h_{S,Q_r}\right), \log_2\left(1 + \sum_{r=1}^R \text{SNR} h_{Q_r,D}\right)\right\}$$
(13)

which is a function of the channel power gains. Here, *R* denotes the number of the relays. For the *r*-th DF link, requiring both the relay and destination to decode perfectly, the maximum average mutual information  $I_{DF \ li}$  can be shown as

$$I_{DF_{li}} = \min\left\{\log_2\left(1 + \text{SNR } h_{S,Q_r}\right), \log_2\left(1 + \text{SNR } h_{Q_r,D}\right)\right\}$$
(14)

The first term in (14) represents the maximum rate at which the relay can reliably decode the source message, while the second term in (14) represents the maximum rate at which the destination can reliably decode the message forwarded from relay. We note that such mutual information forms are typical of relay channel with full decoding at the relay (Cover & El Gamal, 1979). The SNR threshold of this DF link for target rate *B* is given by  $I_{DF_{li}} \ge B$  which is derived as

$$\operatorname{SNR} \ge \frac{2^{B} - 1}{\min(h_{S,O_{*}}, h_{O_{*},D})}$$
(15)

In the proposed hybrid DF-AF cooperative transmission, we only consider that a relay can fully decode the signal transmitted over the source-relay link, but not the whole DF link. Thus, the SNR threshold for the full decoding at the *r*-th relay reaches its lower bound as

$$\gamma_{th} \ge \frac{2^B - 1}{h_{S,O_t}} \tag{16}$$

For the DF protocol, let *R* denote the number of the total relays, *M* denote the set of participating relays, whose  $SNR_S$  are above the SNR threshold, and the reliable decoding is available. The achievable channel capacity,  $C_{DF}$ , with SNR threshold is calculated as

$$C_{DF} = \sum_{M} \frac{1}{R} \mathbf{E} \left( \log_2 \left( 1 + y | M \right) \right) \Pr(M)$$
(17)

where  $E(\cdot)$  denotes the expectation operator,  $y|M = (R - K)\gamma_{S,D} + \sum_{Q \in M} \gamma_{Q,D}$  denotes the instantaneous received SNR at the destination given set M with K participating relays, where  $\gamma_{n,m}$  denotes the instantaneous received SNR at node m, which is directly transmitted from n to m. Since y|M is the weighted sum of independent exponential random variables (Farhadi & Beaulieu, 2008), the probability density function (PDF) of y|M can be obtained using its moment generating function (MGF) and partial fraction technique for evaluation of the inverse Laplace transform, see Eq. (8d) and Eq. (8e) in (Farhadi & Beaulieu, 2008). Pr(M) in (17) is the probability of a particular set of participating relays which are obtained as

$$\Pr(M) = \prod_{Q \in M} \exp\left(-\frac{R\gamma_{th}}{\Gamma_{S,Q \in M}}\right) \prod_{Q \notin M} \left(1 - \exp\left(-\frac{R\gamma_{th}}{\Gamma_{S,Q \notin M}}\right)\right)$$
(18)

where  $\Gamma_{u,v}$  denotes the average SNR over the link between nodes *u* and *v*.

Combining (13), (17) and (18) with the inequality  $I_{DF\_co} \leq C_{DF}$ , since the maximum average mutual information, *I*, is upper bound by the achievable channel capacity, *C*, we can calculate the upper bound of SNR threshold  $\gamma_{th}$  for fully decoding in the DF protocol.

Now, we can obtain the upper bound and the lower bound of the SNR threshold  $\gamma_{th}$  for the hybrid DF-AF cooperation. However, compared to the upper bound, the lower bound as shown in the (16) is more crucial for improving the transmission performance. This is because the DF protocol plays a dominant role in the hybrid cooperation strategy, and accordingly we want to find the lower bound which provides as much as possible DF relays. We will elaborate this issue later. Fully decoding check can also be guaranteed by employing the error detection code, such as cyclic redundancy check. However, it will increase the system complexity (Lin & Constello, 1983).

#### 3.2 Dynamic optimal combination scheme

In the maximum ratio combining the transmitted signal from *R* cooperative relays nodes, which underwent independent identically distributed Rayleigh fading, and forwarded to

the destination node are combined. In this case the SNR per bit per relay link  $\gamma_r$  has an exponential probability density function (PDF) with average SNR per bit  $\overline{\gamma}$ :

$$p_{\gamma_r}(\gamma_r) = \frac{1}{\overline{\gamma}} e^{-\gamma_r/\overline{\gamma}}$$
(19)

Since the fading on the *R* paths is identical and mutually statistically independent, the SNR per bit of the combined SNR  $\gamma_c$  will have a Chi-square distribution with 2*R* degrees of freedom. The PDF  $p_{\gamma_c}(\gamma_c)$  is

$$p_{\gamma_c}(\gamma_c) = \frac{1}{(R-1)! \overline{\gamma_c}^R} \gamma_c^{R-1} e^{-\gamma_c/\overline{\gamma_c}}$$
(20)

where  $\overline{\gamma}_c$  is the average SNR per channel, then by integrating the conditional error probability over  $\overline{\gamma}_c$ , the average probability of error  $P_e$  can be obtained as

$$P_e = \int_0^\infty \mathbb{Q}\left(\sqrt{2g\gamma_c}\right) p_{\gamma_c}(\gamma_c) d\gamma_c \tag{21}$$

where g = 1 for coherent BPSK,  $g = \frac{1}{2}$  for coherent orthogonal BFSK, g = 0.715 for coherent BFSK with minimum correlation, and  $\mathbb{Q}(\cdot)$  is the Gaussian *Q*-function, i.e.,  $\mathbb{Q}(x) = \frac{1}{\sqrt{2\pi}} \int_x^{\infty} \exp(-t^2/2) dt$ . For the BPSK case, the average probability of error can be found in the closed form by successive integration by parts (Proakis, 2001), i.e.,

$$P_{e} = \left(\frac{1-\mu}{2}\right)^{R} \sum_{k=0}^{R-1} \binom{R-1+k}{k} \left(\frac{1+\mu}{2}\right)^{R}$$
(22)

where

$$\mu = \sqrt{\frac{\overline{\gamma_c}}{1 + \overline{\gamma_c}}} \tag{23}$$

In the hybrid DF-AF cooperative network with two hops in each AF relay, the average SNR per channel  $\overline{\gamma}_c$  can be derived as

$$\overline{\gamma}_c = \frac{\gamma_h}{K + 2 \times J} \tag{24}$$

where *K* and *J* are the numbers of the DF relays and AF relays, respectively.  $\gamma_h$  can be obtained from (8). In the DF protocol, due to the reliable detection, we only need to consider the last hops, or the channels between the relay nodes and destination node.

As the average probability of error  $P_e$  is a precise indication for the transmission performance, we consequently propose a dynamic optimal combination strategy for the hybrid DF-AF cooperative transmission. In this algorithm the proper AF relays are selected to make  $P_e$  reach maximum.

First of all, like aforementioned procedure, relays are reordered according to the descending order of the SNR between source and relays, as shown in the Fig.3. According to the proposed SNR threshold, we pick up the DF relays having SNR greater than threshold. Then, we proceed with the AF relay selection scheme, where the inappropriate AF relays are removed. The whole dynamic optimal combination strategy for the hybrid DF-AF cooperation is shown in the flow chart of Fig. 5.



Fig. 5. Flow chart of the dynamic optimal combination strategy for the hybrid DF-AF cooperation

By exploiting the space-frequency coding proposed in (Li et al., 2009), we can further gain from the hybrid DF-AF cooperative OFDM in the frequency selective channel by coding across relays and OFDM tones, and obtain the multi-path diversity. According to the Eq. (14) in (Li et al., 2009), the multi-path diversity of the hybrid DF-AF cooperative OFDM can be shown by the upper bound of the error probability as:

$$P_{e} < G_{c}^{-RW_{\min}} \left( \frac{\log \gamma_{h}}{\gamma_{h}} \right)^{RW_{\min}}$$
$$\approx \left( G_{c} \gamma_{h} \right)^{-RW_{\min}} \text{ as } \gamma_{h} \to \infty$$
(25)

where  $G_c$  is a constant, which can be shown as Eq. (35) in (Li et al., 2009),  $\gamma_h$  is the average SNR at the destination in the hybrid cooperative network, and can be calculated by (8) in this chapter.

It can be seen from (25) that the achievable diversity gain is  $RW_{min}$ , i.e., the product of the cooperative (relay) diversity R and the multi-path diversity  $W_{min}$ . Here  $W_{min} = \min(W_r)$ , where  $W_r$ ,  $r = 1, 2, \dots, R$  is the number of independent paths in each relay-destination link.

#### 3.3 Simulation results

First, we simulated BPSK modulation, Rayleigh channel, flat fading, without OFDM, and supposed the SNR threshold for correct decoding is  $4E_b/N_0$ , then we assumed  $h_{Q_i,D} = h_{S,Q_j} = h_{Q_j,D} = 1$ , for all branches, to verify proposed analytical BER expression. The resulting average BERs were plotted against the transmit SNR defined as SNR =  $E_b/N_0$ . As shown in the Fig. 6, the theoretical curves of multi-DF cooperation derived from our analytical closed-form BER expression clearly agree with the Monte Carlo simulated curves, while the theoretical curves of 2-AF and 3-AF cooperation match the simulation result only at the low SNR region.

Fig. 7 shows the BER performance for hybrid DF-AF cooperation. For the DF-dominant hybrid cooperation, the theoretical curves exhibit a good match with the Monte Carlo simulation results curves. The slight gap between theoretical and simulation BER results for the hybrid case of 1-DF + multi-AF can be explained by the AF relay fading which was considered as a double Gaussian channel, a product of two complex Gaussian channel (Patel et al., 2006). Obviously, the distribution of combined SNR (i.e.,  $\gamma_c$ ) will no longer follow the chi-square distribution giving rise to this slight difference.



BER for BPSK modulation with DF or AF cooperation in Rayleigh channel

In this proposed hybrid cooperation protocol, DF is dominant. We show this characteristic of the hybrid DF-AF cooperation by the following theorem:

*Theorem 1:* For the *F*-hop relay link, and the full decoding in DF protocol, as long as the SNR of the last hop is larger than 1/F times of the arithmetic mean of the whole link SNR, DF always plays a more important role than AF in improving the BER performance.



Fig. 7. BER performance for hybrid DF+AF cooperation.

*Proof:* According to the (22) and (23), the average probability of error  $P_e$  is a decreasing function w.r.t. combined SNR,  $\gamma_c$ . The SNR of the *F*-hop AF relay link,  $\gamma_{AF}$ , is the 1/F times of the harmonic mean of  $\gamma_t$ ,  $\forall i \in [1, F]$ , i.e. (Hasna & Alouini, 2002),

$$\gamma_{\rm AF} = \frac{\gamma_1 \gamma_2, \dots, \gamma_L}{\sum_{i=1}^L \gamma_1 \gamma_2, \dots, \gamma_{i-1} \gamma_{i+1}, \dots, \gamma_L}$$
(26)

Using Pythagorean means theorem, the harmonic mean is always smaller than the arithmetic mean.  $\blacksquare$ 

For instance, in the high SNR region, the second term of (8) can be approximated as the  $\frac{1}{2}$  times the harmonic mean of the 2-hop SNR in AF relay link (i.e., 1 is negligible in the denominator). As in practice, it is very easy for the last hop relay to achieve a SNR larger than 1/L times of the arithmetic mean of the whole link SNR, we can only consider the last hop of the reliably decoded DF protocol. Therefore, under the condition of the correct decoding, DF can enhance the error probability performance better than AF in the cooperative relay network.

This DF dominant hybrid cooperative networks strategy can be verified by the above simulation results as well. Comparing 2-DF to 2-AF in Fig. 6, or 2-DF plus 1-AF to 1-DF plus 2-AF in Fig. 7, or other hybrid DF-AF protocols with the same *R*, we can see that the fully decoded DF protocols always show a better BER performance than AF protocols. Therefore, DF protocols with a reliable decoding play a more important role in hybrid cooperative networks than AF protocols. Meanwhile, we can see from the figure that, changing to the AF scheme for the relay nodes with SNR below the threshold also improves the BER performance, as well as the diversity gain of the whole network. In fact, this is a better way than just discarding these relay nodes.



Fig. 8. hybrid DF-AF cooperation and DF cooperation architectures with different average power gains. (a) hybrid DF-AF cooperation, (b) dual DF cooperation, (c) single DF cooperation. (*S*: Source, *D*: Destination, *h*: average power gain between two nodes).

Reference (Louie et al., 2009) proposes a closed-form BER expression for two-hop AF protocol, which includes Gauss' hypergeometric and Gamma functions. This closed-form BER expression needs more computational burden to derive the cooperative analytical expression. In (Sadek et al., 2007), the analytical expression for multi-node DF protocol is provided with a complicated form as well. Instead, the compact closed-form BER expression for hybrid DF-AF cooperation proposed in this chapter allows us to achieve insight into the results with relatively low computations. The simple expressions can also help understanding the factors affecting the system performance. It can also be used for designing different network functions such as power allocation, scheduling, routing, and node selection.

In order to study the effect of the channel gains between source, relay and destination, we compare the hybrid DF-AF with the dual DF as well as the single DF cooperation in Fig. 8. In this figure,  $h_1$ ,  $h_2$ ,  $h_3$  and  $h_4$  stand for the average power gain between corresponding two nodes. In this simulation, the SNR threshold for correct decoding is assumed to be  $4E_b/N_0$ , and we set the first hop average power gain in DF protocol, i.e.,  $h_1$  in Fig. 8 (a) and Fig. 8 (c), and  $h_1$ ,  $h_3$  in Fig. 8 (b) as 4, which means that the relay in DF protocol can fully decode the signal. The average power gains of the first hop in AF protocol, i.e.,  $h_3$  in Fig. 8 (a) increases

from 0.25 to 20. It can be seen from the Fig. 9 that the dual DF cooperation with reliable decoding outperforms the hybrid DF-AF cooperation, when corresponding average power gains are the same, i.e., diamond marked curve is better than square marked curve in Fig. 9. Meanwhile, the comparison of the curves shows that, the AF relay which undergoes the deep fading deteriorates the BER performance of hybrid DF-AF cooperation in the low SNR region. Thus, this AF relay should be removed according to the proposed dynamic optimal combination strategy to improve the transmission performance. Sum up the above discussion, due to power control, long transmission range, serious attenuation, etc., high SNR at relay and full decoding for DF protocol is not always available. In this case, relays can change to AF protocol with enough SNR to gain from the cooperative diversity.





Fig. 9. BER performance for hybrid DF-AF cooperation and DF cooperation with different path gains.

Finally, we illustrate the validity of the theoretical results for the OFDM cooperation via simulations. An OFDM system with 64-point FFT and a CP length of 16 samples, which accounts for 25% of the OFDM symbol was considered. In the simulation, a more practical scenario was considered with a 3-path Rayleigh fading between each source node and relay node or relay node and destination node, i.e.,  $W_r = 3$ . The 3-path delays were assumed at 0, 1, 2 samples, respectively. As illustrated in the Fig. 10, OFDM with CP can nicely cope with the multi-path, and the theoretical curves derived from (22) clearly agree with the Monte Carlo simulation curves. The simulation results indicate that under the condition of ISI resolved by OFDM and reliable decoding, the cooperative diversity gains from the increasing *R*, which is also shown by (8).

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