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Floating Solar Chimney Technology

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1. Introduction

1.1 Floating Solar Chimney technology description

The purpose of this chapter is to present the Floating solar chimney (FSC) technology, look for the site www.floatingsolarchimney.gr, in order to explain its principles of operation and to point out its various significant benefits. This technology is the advisable one for candidacy for large scale solar electricity generation especially in desert or semi desert areas of our planet and a major technology for the global warming elimination.

The solar chimney power plants are usually referred to as solar updraft towers (http://en.wikipedia.org/wiki/Solar_updraft_tower) and the related solar chimneys are huge reinforced concrete structures. However due to the high construction cost of the concrete solar chimneys the solar up-draft tower technology is expensive demanding a high initial investment in comparison to its competitive solar technologies. Their solar up-draft towers are huge structures of high initial investment cost that can not be split into small units. That is possible for the relatively also expensive PV solar technology. Also the solar updraft technology is far more expensive compared to the conventional fossil fueled power plants of similar electricity generation. That is why the solar chimney technology has not yet been applied although it is a solar technology of many advantages.

The **Floating Solar Chimney (FSC)** is a fabric low cost alternative of the concrete solar chimney up-draft towers that can make the Floating Solar Chimney technology cost competitive in comparison not only with the renewable electricity generation technologies but also with the conventional fossil fueled electricity generation technologies. Also the FSC technology is cost effective to be split into small units of several MW each.

The Floating Solar Chimney Power Plant, named by the author as **Solar Aero-Electric Power Plant (SAEP)** due to its similarity to the Hydro-Electric power plant, is a set of three major components:

- **The Solar Collector**. It is a large greenhouse open around its periphery with a transparent roof supported a few meters above the ground.
- **The Floating Solar Chimney (FSC)**. It is a tall fabric cylinder placed at the centre of the solar collector through which the warm air of the greenhouse, due to its relative buoyancy to the ambient air, is up-drafting.
- **The Turbo-Generators**. It is a set of air turbines geared to appropriate electric generators in the path of up-drafting warm air flow that are forced to rotate generating electricity. The gear boxes are adjusting the rotation speed of the air turbines to the generator rotation speed defined by the grid frequency and their pole pairs.

Source: Solar Energy, Book edited by: Radu D. Rugescu,
ISBN 978-953-307-052-0, pp. 432, February 2010, INTECH, Croatia, downloaded from SCIYO.COM

An indicative figure of a solar chimney Power Plant with a circular solar collector and a Floating Solar Chimney inclined due to external winds is shown in next figure(1).

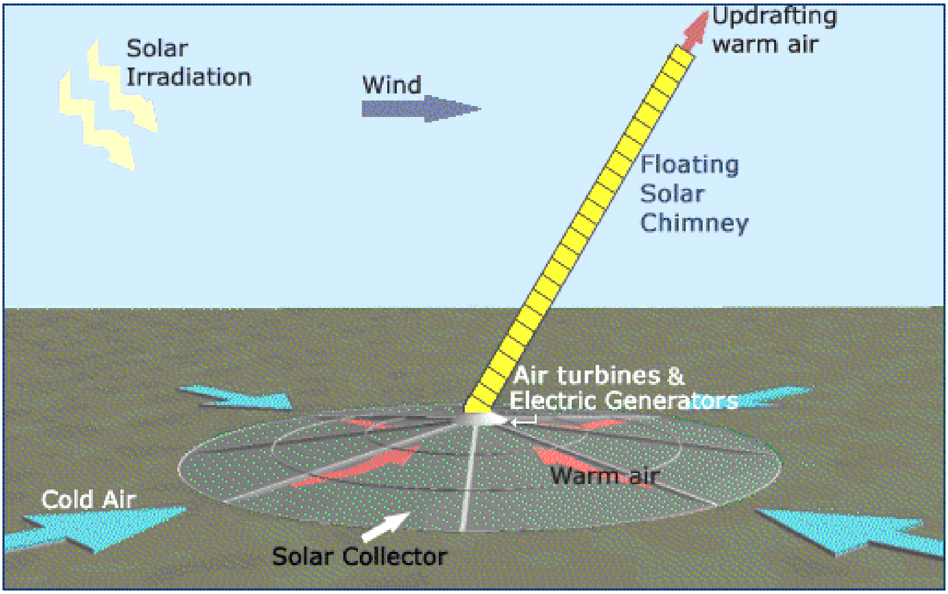

Fig. 1. Floating Solar Chimney Power Plant in operation

Because of its patented construction the FSC is a free standing lighter than air structure that can tilt when external winds appear. Low cost Floating Solar Chimneys up to 1000 m with internal diameters 25 m ÷ 40 m, can be constructed with an existing polyester fabric, giving to their respective Solar Aero-Electric Power Plants, low investment costs.
By this innovating Floating Solar Chimney Technology of heights of the FSCs up to1000m, up to 1.2 % of the arriving horizontal solar radiation on the solar collector surface, can be converted to electricity

1.2 Similarity to hydro-electric power plants

The Floating Solar Chimney power plants, due to their similarity to hydro-electric power plants, are named by the author Solar Aero Electric Power Plants (SAEPs).
Their similarity is due to the following facts:

- The hydro-electric PPs operate due to falling water gravity, while the solar aero-electric PPs operate due to the up-drafting warm air buoyancy.
- The electricity generation units of hydro-electric PPs are water turbines engaged to electric generators while the generation units of solar aero-electric PPs are air turbines engaged to electric generators.
- The energy produced by the hydro-electric PPs is proportional to the falling water height, while the energy produced by the solar aero-electric PPs is proportional to up-drafting height of warm air, which is equal to the height of the solar chimneys.
- That is why Prof J. Sclaigh in his book named the solar chimney technology power plants as the hydro-electric power plants of deserts.

1.3 Continuous operation

As it will be shown later the SAEPs operate continuously due to the ground thermal storage. The minimum electric power is generated when the sun is just starting rising, while the maximum electric power is achieved about 2 hours after the sun's maximum irradiation on ground. The power generation profile can become smoother if we increase the solar collector thermal capacity. This can be done by putting on its ground area closed tubes filled with water (as happens already in conventional greenhouses).

2. History

The Solar Chimney technology for electricity generation was inspired by several engineering pioneers early in the first decade of the 20th century.
In 1926 Prof Engineer Bernard Dubos proposed to the French Academy of Sciences the construction of a Solar Aero-Electric Power Plant in North Africa with its solar chimney on the slope of a sufficient height mountain. His proposal is shown in the following figure(2), found in a book of 1954 ("Engineer's Dream" Willy Ley, Viking Press 1954)

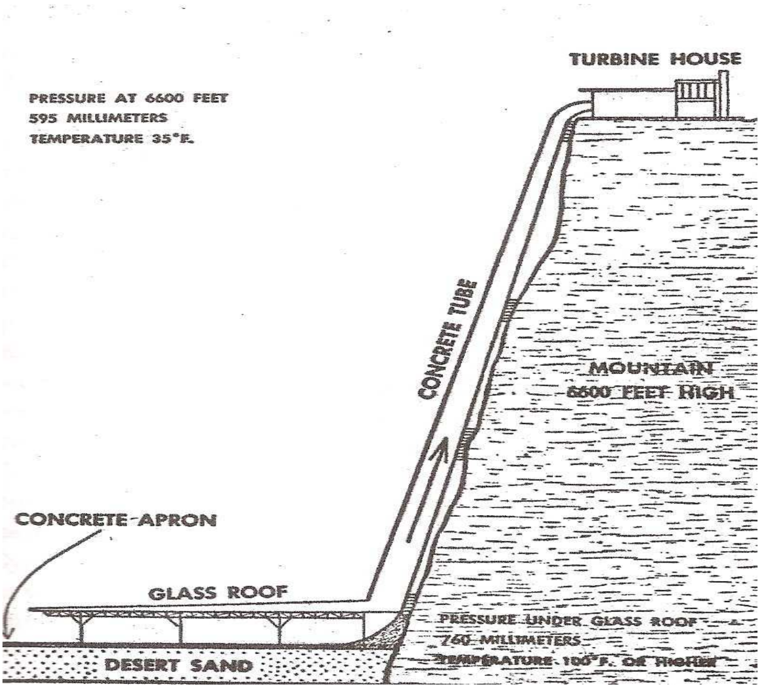

Fig. 2. (from the book: "Engineer's Dream"By: Willy Ley, Viking Press 1954)

Lately Schaich, Bergerman and Partners, under the direction of Prof. Dr. Ing. Jorg Schlaigh, built an operating model of a SAEPP in 1982 in Manzaranes (Spain), which was funded by the German Government.
This solar chimney power plant, shown in next figure (3) was of rating power 50 KW. Its greenhouse had a surface area of 46000 m^2 and its solar chimney was made out of steel tubes of 10 m diameter and had a height of 195 m.

This demo SAEP was operating successfully for approximately 6 years. During its operation, optimization data were taken.
The collected operational data were in accordance with the theoretical results of the scientific team of Prof Jorg Schlaigh.

Fig. 3. A view of the Manzanares Solar Chimney Power Plant

Prof. Jorg Schlaigh in 1996 published a book (Schlaigh 1995) presenting the solar chimney technology. He proposed in his book the huge reinforced concrete solar chimneys of heights of 500m-1000m.
The proposed concrete solar chimneys are huge and very expensive. Therefore the investment cost per produced KWh on the solar chimney technology with concrete chimneys is in the same cost range with the competitive solar thermal technologies. The generated KWh, by the CSP Parabolic Through for example, it has almost the same direct production cost, but the CSP power plants can be split into small units and developed using reasonable recourses.
However the proposed solar chimney technology had an important benefit in comparison with the major renewable technologies (Wind, SCP, PV).
That is its ability, equipping its solar collectors, with thermal storage facilities of negligible cost, to generate uninterrupted electricity of a controlled smooth profile for 24h/day, 365days/year.
The last decade several business plans and a series of scientific research papers have focused on the solar chimney technology, whereby the author with a series of patents and papers has introduced and scientifically supported the floating solar technology (Papageorgiou 2004, 2009).

3. Principles of operation of the solar chimney technology and its annual efficiency Information

3.1 Short description and principles of operation

A floating solar chimney power plant (SAEP) is made of three major components:

- A large solar collector, usually circular, which is made of a transparent roof supported a few meters above the ground (the greenhouse). The transparent roof can be made of glass or crystal clear plastic. A second cover made of thin crystal clear plastic is suggested to be hanged just underneath the roof in order to increase its thermal efficiency. The periphery of the solar collector is open in order that the ambient air can move freely into it.
- A tall fabric free standing lighter than air cylinder (the floating solar chimney) placed in the center of the greenhouse which is up drafting the warm air of the greenhouse, due to its buoyancy, to the upper atmospheric layers.
- A set of air turbines geared to appropriate electric generators (the turbo generators), placed with a horizontal axis in a circular path around the base of the FSC or with a vertical axis inside the entrance of the solar chimney. The air turbines are caged and can be just a rotor with several blades or a two stage machine (i.e. with a set of inlet guiding vanes and a rotor of several blades). The gear boxes are adjusting the rotation frequency of the air turbines to the electric generator rotation frequency defined by the grid frequency and the electric generator pole pairs.

The horizontal solar irradiation passing through the transparent roof of the solar collector is heating the ground beneath it. The air beneath the solar collector is becoming warm through a heat transfer process from the ground area to the air. This heat transfer is increased due to the greenhouse effect of the transparent roof.

This warm air becomes lighter than the ambient air. The buoyancy of the warm air is forcing the warm air to escape through the solar chimney. As the warm air is up drafting through the chimney, fresh ambient air is entering from the open periphery of the greenhouse. This fresh air becomes gradually warm, while moving towards the bottom of the solar chimney, and it is also up-drafting.

Thus a large quantity of air mass is continuously circulating from ground to the upper layers of the atmosphere. This circulating air mass flow is offering a part of its thermodynamic energy to the air turbines which rotate and force the geared electric generators also to rotate. Thus the rotational mechanical power of the air turbines is transformed to electrical power. An indicative diagram of the SAEP operation is shown in the next figure(4).

Thus the first two parts of the SAEPs form a huge thermodynamic device up drafting the ground ambient air to the upper atmosphere layers and the third part of the SAEP is the electricity generating unit.

The solar energy arriving on the horizontal surface area A_c of the greenhouse of the SAEP is given by $E_{IR}=A_c \cdot W_y$, where Wy is the annual horizontal solar irradiation in KWh/m^2, at the place of installation of the SAEP and is given by the meteorological data nearly everywhere.

The average annual horizontal solar irradiance is given by $G_{av}=W_y/A_c$.

The horizontal solar irradiation is offering thermal power $P_{Th}= \dot{m} \cdot c_p \cdot (T_{03}-T_{02})$ to the up drafting air mass flow $\dot{m}$ of the ambient air, $c_p \approx 1005$ and T_{02} is equal to the average ambient temperature T_0 plus ~ 0.5 ^0K, in order that it is taken into account the outer air stream increased inlet temperature due to its proximity to the ground on its entrance inside the solar collector.

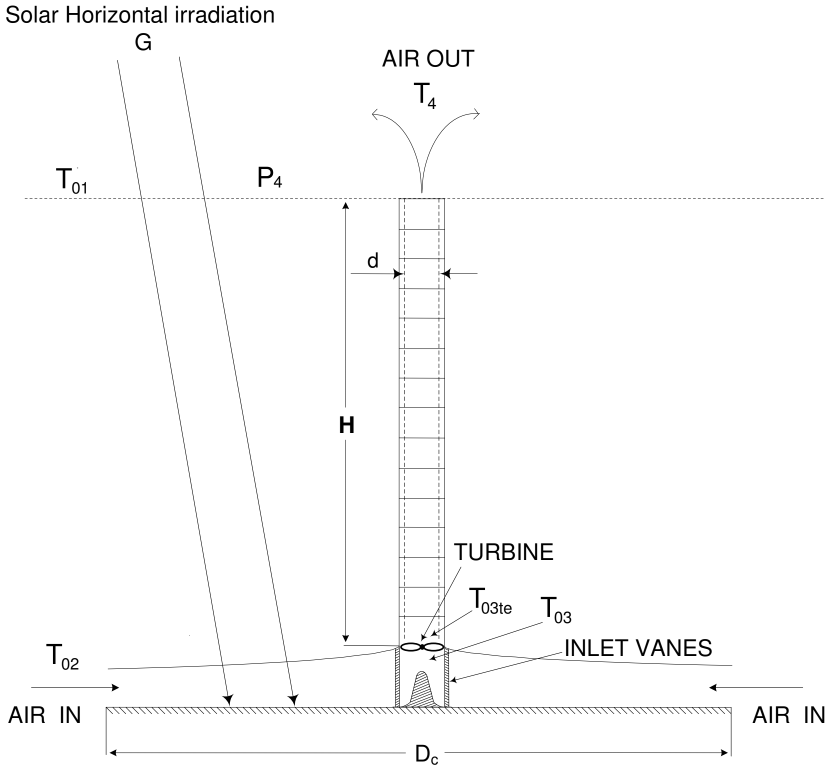

Fig. 4. Schematic diagram of the SAEP in operation

3.2 Annual average efficiency of SAEPs

The annual efficiency of the solar collector η_{sc} is defined as the average ratio of the thermal power P_{Th} absorbed by the air mass flow to the horizontal solar irradiation arriving on the greenhouse roof $G_{av}\cdot A_c$, where G_{av} is the average horizontal irradiance and A_c the greenhouse surface area.

The annual average double glazing solar collector efficiency η_{sc} is theoretically estimated to ~50%, while the annual efficiency for the single glazing solar collector is estimated to 2/3 of the previous figure i.e. ~33%.

Thus the average exit temperature T_{03} from the solar collector can be calculated by the equation $\dot{m}\cdot c_p\cdot (T_{03}-T_{02})= \eta_{sc}\cdot G_{av}\cdot A_c$ where T_{02} is the average inlet air temperature.

The exit thermal power P_{Th} from the solar collector is transformed to electric power P, plus power thermal losses P_L (to the air turbines, gear boxes and electric generators), plus warm air kinetic power at the top exit of the solar chimney P_{KIN} and friction thermal losses inside the solar chimney P_{FR}.

The maximum efficiency of the solar chimney is the Carnot efficiency defined as the ratio of the temperature difference between the incoming and outcoming air temperatures of the up-drafting air divided by the ambient air temperature.

This maximum efficiency has been proven (Gannon & Backstrom 2000) to be equal to:

$$\eta_{FSC,max}=g\cdot H/(c_p\cdot T_0) \quad (1)$$

Due to friction and kinetic losses in the solar chimney the actual solar chimney efficiency η_{FSC} is for a properly designed SAEP approximately 90% of its maximum Carnot efficiency (close to the optimum point of operation of the SAEP).

The combined efficiency η_T of the air turbines, gear boxes and electric generators is within the range of 80%.

The average annual efficiency of the SAEP is the product of the average efficiencies of its three major components i.e. the solar collector, the floating solar chimney and the turbo-generators i.e. $\eta_{av}=\eta_{sc}\cdot\eta_{FSC}\cdot\eta_T$.

Thus the annual average efficiency of a SAEP of proper design, with a double glazing solar collector should be approximately:

$$\eta_{av}=(1.2\cdot H/1000)\ \% \quad (2)$$

While for the SAEP with a single cover collector it is approximately:

$$\eta_{av}=(0.79\cdot H/1000)\ \%. \quad (3)$$

The formulae have been calculated for $g=9.81$, $c_p=1005$ and $T_0\approx 293.2^0K(20^0C)$.

This means that if the annual horizontal irradiation arriving on the place of installation of the SAEP is 2000 KWh/m^2, the solar collector surface area is 10^6 m^2 (one square Km) and the solar chimney height is 750 m the SAEP can generate approximately 18 million KWh. The same SAEP with a single glazing roof will generate approximately only 12 million KWh.

Following approximate analysis, for a SAEP with a double cover roof of given dimensions (Ac=Greenhouse area in m^2 and d=internal diameter of the Floating Solar Chimney in m) to be installed in a place of annual horizontal solar irradiation Wy in KWh/m^2 the diagram showing the relation between the annual efficiency of the SAEP and its FSC height H can be calculated.

The following figure (5) shows the annual efficiency as a function of FSC's height for a SAEP of $Ac=10^6m^2$, d=40m and Wy=1700 KWh/m^2 (Cyprus, South Spain).

The calculated efficiency curve is practically independent of the annual horizontal solar irradiation W_y. However it depends on the FSC internal diameter d. The reason is that a smaller diameter will increase the warm air speed at the top exit of the FSC and consequently will increase the kinetic power losses and decrease the average annual efficiency. If we vary the solar collector diameter of the SAEP its FSC internal diameter should vary proportionally in order to keep almost constant the air speed at the top exit of the FSC and consequently the annual efficiency of the SAEP.

Hence we should notice that in order to receive the efficiency diagram as shown in the following figure (5) figure the kinetic and friction losses of the Floating Solar Chimney should be approximately 10% of the total chimney power. This can be achieved if the internal diameter of the FSC is appropriate in order to keep the average air speed in the range of 7÷8 m/sec, and the FSC internal surface has a low friction loss coefficient.

The following figure (6) shows the variation of the annual efficiency of a SAEP of a FSC 500m high, installed in a place of annual horizontal solar irradiation 1700KWh/m^2 as function of the internal diameter of its FSC.

The annual electricity generated by the SAEP, E_y can be calculated as a product of the annual efficiency and the arriving horizontal solar irradiation on its greenhouse surface $A_c\cdot W_y$. Thus taking into consideration that the annual efficiency is proportional to the FSC

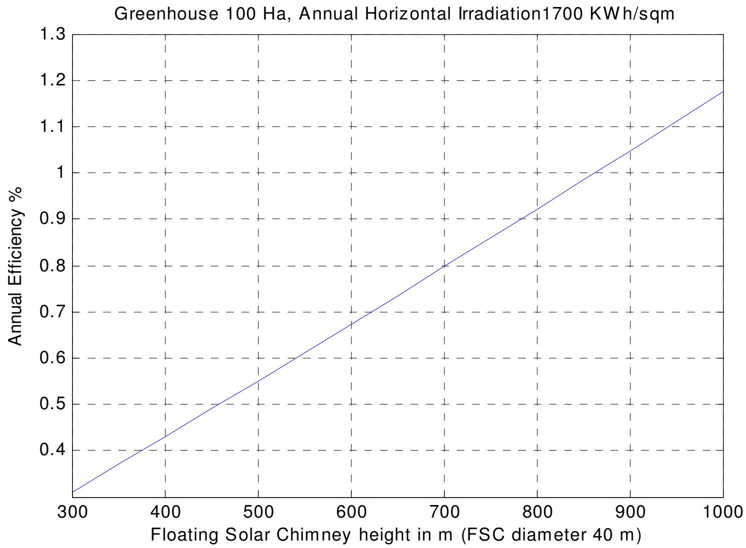

Fig. 5. Annual efficiency of a SAEP as function of its FSC height

height H, the annual generated electricity by the SAEPs is also proportional to the Floating Solar Chimney height H, is as follows:

$$E_y = c \cdot H \cdot A_c \cdot W_y \quad (4)$$

The constant c is mainly depending on the FSC's internal diameter d.

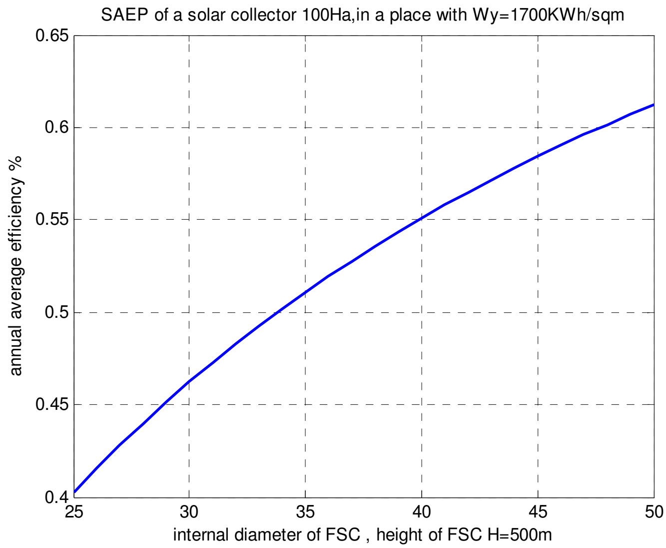

Fig. 6. variation of the annual efficiency of a SAEP with internal FSC diameter

4. Theoretical analysis of the Floating Solar Chimney technology

4.1 Annual average efficiency of SAEPs

The ground thermal storage effect and the daily electricity generation profile, have been studied by several authors (Bernades et.al 2003, Pretorius & Kroger 2006, Pretorious 2007).
The author has used an equivalent approach on the daily power profile study of the floating solar chimney SAEPs using the thermodynamic model see (Backstrom & Gannon 2000) and Fourier series analysis on the time varying temperatures and varying solar irradiance during the 24 hours daily cycle.
Following the code of the author analysis an evaluation of the sensitivity of the various parameters has been made leading to useful results for the initial engineering dimensioning and design of the SAEPs.
The important results of these studies are that the solar chimney power plant annual power production can be increased by using a second glazing below the outer glazing and its output power production can be affected by the ground roughness and ground solar irradiation absorption coefficients.
The thermodynamic cycle analysis proposed in ref. (Gannon Backstrom 2000) is an excellent way of engineering analysis and thermodynamic presentation of the solar chimney power plant operation.
The thermodynamic cycle of the solar chimney operation power plant using the same symbols of the study of ref (Backstrom & Gannon 2000) is shown in the following figure.

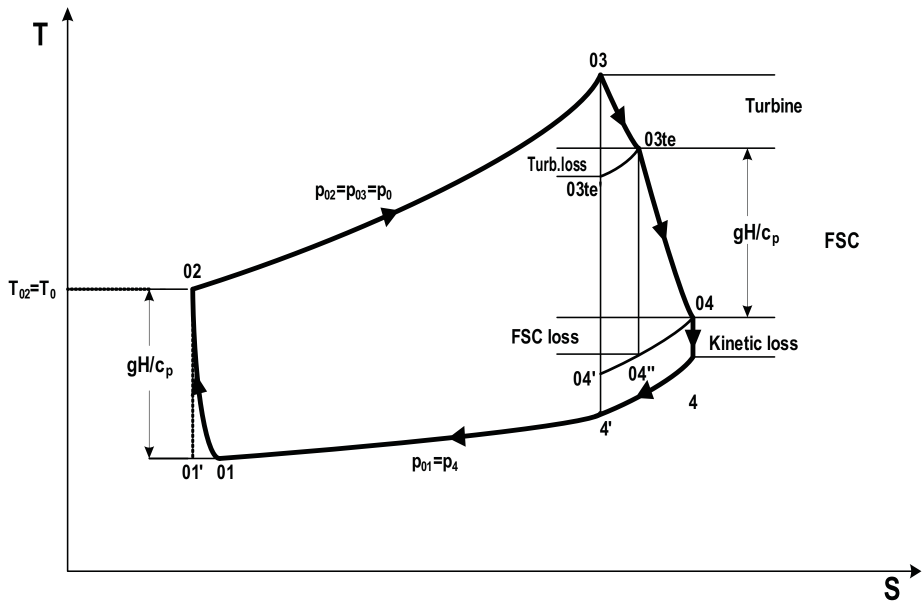

Fig. 7. The thermodynamic diagram of the SAEP

Temperatures, stagnation temperatures (marked with $_0$) and equivalent isentropic temperatures (marked with ') are shown in the indicative diagram on the previous figure.
The main thermodynamic cycle temperatures are defined in the following table:

T_{01}	Isendropic temperature of ambient air in height H (exit of solar chimney)
T_{02}	Ambient temperature in the ground around the solar collector
T_{03}	Inlet temperature in the air turbines
T_{03te}	Exit air temperature from the turbo generators
T_{04}	Stagnation temperature at the top of the solar collector
T_4	Exit temperature of the air mixed with the ambient air at the top of the exit layers

Table 1.Tthermodynamic cycle temperatures

The process {T_{02} to T_{o3}} is assumed as approximately isobaric. This assumption is very reasonable taking into consideration that the heat and expansion of moving air is taking place inside the solar collector.

The processes {T_4 to T_{01}}, {T_{03te} to T'_{03te}} and {T_{04} to T'_{04}} are definitely isobaric by nature.

By the analysis on the relations between the temperatures the following relationships can be derived:

$$\begin{cases} T_{04}=T_4+\dfrac{\alpha\cdot\upsilon_{ex}^2}{2\cdot c_p}=T_4+C_2\cdot T_4^2,\, T_{03te}=T_{04}+\dfrac{g\cdot H}{c_p} \\ T'_{03te}=T_{03}-\dfrac{T_{03}-T_{03te}}{n_T},\, T''_{04}=T_{04}-k\cdot\dfrac{\alpha\cdot\upsilon_{ex}^2}{2\cdot c_p} \\ T'_{04}=\dfrac{T'_4\cdot T_{04}}{T_4} \text{ and } T''_{04}=T_{03}\cdot\dfrac{T'_{04}}{T'_{03te}} \end{cases} \tag{5}$$

Whereby the parameters participating in the relations are defined as follows:

H =solar chimney height

d=internal solar chimney diameter

$A_{ch}=\pi\cdot d^2/4$, is the solar chimney internal cut area

$\dot{m}$ =moving mass flow

α=kinetic energy correction coefficient, of a usual value of 1.058 calculated in (White 1999).

k=friction loss coefficient inside the solar chimney

$k= k_{in}+ 4\cdot C_d\cdot H/d$ where, for the operation range of Reynolds numbers inside the solar chimney, the drag friction factor C_d is approximately equal to 0.003, see (White 1999) and for no available data k_{in}it is estimated to 0.15.

η_{T}= turbo generators overall efficiency, if not available data estimated to 0.8.

T_0= ambient air temperature

$T_{02}= T_0+0.5$

p_0= ambient atmospheric pressure on ground level at the place of installation of the SAEP, if not available data it is assumed as equal to 101300 Pa.

p_4= ambient atmospheric pressure on top exit at height H, estimated by the formula:

$$p_4=p_0\cdot(1-\frac{g\cdot H}{c_p\cdot T_0})^{3.5} \tag{6}$$

g= gravity constant 9.81

c_p= specific heat of air approximately equal to 1005
R= air constant approximately equal to 287
υ_{ex}= average air speed at the top exit of the solar chimney $\upsilon_{ex}=\dfrac{R\cdot T_4\cdot \dot{m}}{P_4\cdot A_{ch}}$

and: $T_4' = T_{03}\cdot\dfrac{T_0\text{-}C_1}{T_0}, C_1=\dfrac{g\cdot H}{c_p}, C_2=\dfrac{a}{2\cdot c_p}\cdot\left(\dfrac{R\cdot\dot{m}}{A_{ch}\cdot p_4}\right)^2, C_3=T_{03}\cdot(\eta_T\text{-}1)+\dfrac{g\cdot H}{c_p}$

The system of the previous equations can been simplified (see Papageorgiou, 2004),leading to a forth order polynomial equation for T_4 given by:

$$w_1\cdot T_4^4+w_2\cdot T_4^3+w_3\cdot T_4^2+w_4\cdot T_4+w_5=0 \quad (7)$$

Where the coefficients w_1, w_2, w_3, w_4 and w_5 are given by the relations:

$$w_1=C_2^2\cdot(1\text{-}k), w_2=C_2\cdot(2\text{-}k\text{-}\eta_T\cdot C_2\cdot T_4'), w_3=(1\text{-}k)\cdot C_2\cdot C_3+1\text{-}2\cdot\eta_T\cdot C_2\cdot T_4'$$
$$w_4=C_3\text{-}\eta_T\cdot T_4'\cdot(1\text{-}C_1\cdot C_2), w_5=\text{-}\eta_T\cdot T_4'\cdot C_1$$

The proper root of the previous polynomial equation is the temperature T_4.
It is easy using the previous relations to calculate T_{03te} by the formula:

$$T_{03te}=T_4+C_2\cdot T_4^2+\frac{g\cdot H}{c_p} \quad (8)$$

Thus the overall electrical power of the generators is given by the relation:

$$P=\dot{m}\cdot c_p\cdot(T_{03}-T_{03te})=\dot{m}\cdot c_p\cdot(T_{03}\text{-}T_4\text{-}C_2\cdot T_4^2-\frac{g\cdot H}{c_p}) \quad (9)$$

As a final result we can say that the air mass flow $\dot{m}$ and the exit temperature T_{03} of the moving air mass through solar collector can define, through the previous analytical procedure, based on the thermodynamic cycle analysis, the electrical power output P of the SAEP.
The proposed thermodynamic analysis, though it looks more complicated than the analysis based on the buoyancy of warm air inside the chimney and the relevant pressure drop to the air turbine used by Bernades M.A. dos S., Vob A., Weinrebe G. and Pretorius J.P., Kroger D.G., it is an equivalent thermodynamic analysis that takes into consideration all necessary and non negligible effects and parameters of the process in the SAEP.
An approximate procedure for T_{03} calculation is given by Shlaigh in his relative book.
The approximate average equation relating the average exit solar collector air temperature T_{03} to its input air temperature T_{02} near the point of optimal operation of the SAEP can be written as follows:

$$ta\cdot G_{av}\cdot A_c=\dot{m}\cdot C_p\cdot(T_{03}-T_{02})+\beta\cdot A_c\cdot(T_{03}\text{-}T_{02}) \quad (10)$$

where:
- β is the approximate thermal power losses coefficient of the Solar Collector (to the ambient and ground) per m² of its surface area and °C of the temperature difference (T_{03}-T_{02}) . An average value of β for double glazing solar collectors is ~3.8÷4 W/m²/°C.

- Gav is the annual average horizontal irradiance on the surface of the solar collector.
- The annual average solar horizontal irradiance G_{av} is given by the formula: Wy/8760hours, where W_y is the annual horizontal irradiation of the place of installation of the SAEPP, (in KWh/m^2)
- ta is the average value of the product: {roof transmission coefficient for solar irradiation X soil absorption coefficient for solar irradiation}.An average value of the coefficient ta for a double glazing roof is ~ 0.70 .
- and Ac is the Solar Collector's surface area.

Using in the equation an approximation for the function $T_{03}(\dot{m})$, it gives as:

$$T_{03}(\dot{m}) = [\, ta \cdot G / (\beta + \dot{m} \cdot C_p/A_c) \,] - T_{02} \tag{11}$$

Where T_{02} is, approximately, equal to the ambient temperature (T_0 in ^0K), plus 0.5 degrees of Celsius. The increase is due mainly to ground thermal storage around the Solar Collector. The inlet ambient air temperature as passing above it is increasing entering to the solar collector.

The proper value of β, giving the average solar collector thermal losses, has been calculated by the heat transfer analysis of the solar collector. An introduction on this analysis is given on the next paragraph. The heat transfer analysis uses time Fourier series in order to take into account the ground thermal storage phenomena during a daily cycle of operation.

The instantaneous efficiency of the SAEP is given by the formula:

$$\eta = P / (A_C \cdot G) \tag{12}$$

where $A_C \cdot G$ is the solar irradiation power arriving on the horizontal solar collector surface area A_c and P is the maximum generated electric power. This efficiency is for a given value of horizontal solar irradiance G. However we can prove that for an almost constant mass flow near the point of maximum power output, the maximum electric power P and the horizontal irradiance G are almost proportional, thus the previous formula is giving also the annual efficiency of the SAEP defined as the annual generated electricity in KWh divided by the annual horizontal irradiation arriving on top of the roof of the greenhouse of the SAEP i.e

$$\eta = P_{av} / (A_C \cdot G_{av}) = E_y(KWh) / W_Y \tag{13}$$

As an example let us consider that a SAEP has the following dimensions and constants: $A_c = 10^6 m^2$ (DD=1000m), H = 800 m, d=40 m, k = 0.49, α = 1.1058, $\eta_T = 0.8$, the average ambient temperature is $T_o = 296.2\ ^oK$ and the ambient pressure is $P_o = 101300$ Pa. Let us assume that the horizontal solar irradiance G is varying between 100 W/m^2 to 500 W/m^2 ($G_{av} \approx 240 W/m^2$). In following figure the effect of the G on the power output as function of mass flow of this SAEP is shown.

If the maximum (daily average during summer operation) G_{av} is 500 W/m^2 the maximum power output of this SAEP, achieved for $\dot{m}_M$ = ~10000 Kg/sec is 5 MW. Thus its efficiency is approximately 1%. Let us assume that the rated power output P_R of a SAEP is the maximum power output for the maximum average solar irradiance. As we can observe on the above figure, the maximum power output point of operation ($\dot{m}_M$) is approximately the same for any horizontal solar irradiance G.

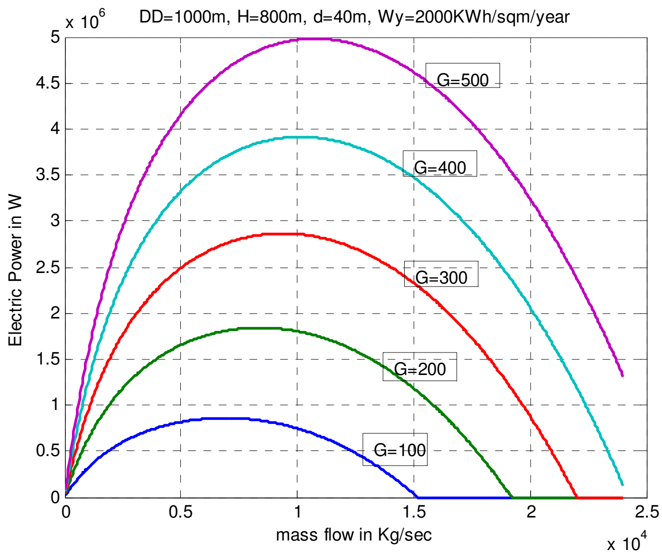

Fig. 9. Electrical Power as function of mass flow for various values of G

Thus if we can control the operation of the SAEP to operate with the proper constant mass flow, close to $\dot{m}_M$, we should achieve almost the possible maximum electric power output by the SAEP for any horizontal solar irradiance. This is referred to as an optimal operation of the SAEP.
As we see later this can be achieved by using induction generators and gear boxes of proper transmission rate.
As a rule of thumb we can state that $\dot{m}_M$ for optimal operation of the SAEP can be calculated approximately by the formula $\dot{m}_M = \rho \cdot \upsilon \cdot (\pi \cdot d^2/4)$, where air speed is υ it is estimated to 7-8 m/sec, the air density is given by $\rho = p_0/(287 \cdot 307.15)$ and d is the internal solar chimney diameter.
A more accurate calculation can be done if we work out on the mass flow for maximum electric power output per annual average horizontal solar irradiance $G_{av,annual} = Wy/8760$. This can be done using the thermodynamic cycle analysis for variable mass flow $\dot{m}$ and G_{av}. The calculated efficiency for the annual average horizontal solar irradiance $G_{av} = 2100000/8760 \approx 240 W/m^2$, of the previously defined SAEP, is 0.94 % (i.e. 6% lower than the calculated efficiency of 1% for the maximum summer average horizontal solar irradiance of $500 W/m^2$).

4.2 Maximum exit warm air speed without air turbines

Using the thermodynamic cycle diagram, the maximum top exit warm air speed of the solar collector plus the FSC alone (i.e. without the air turbines) can be calculated.
In the previous set of equations we should assume that $n_T = 0$. Thus:

T_{03}=T_{03te} and $T'_{04}=T''_{04}$. If we consider that the kinetic losses are approximately equal to $T'_{04} - T'_4 \approx \frac{a \cdot \upsilon^2}{2 \cdot c_p}$, the friction losses are equal to $T''_{04} - T'_{04} = k \cdot \frac{a \cdot \upsilon^2}{2 \cdot c_p}$ and taking into consideration that the equations $T'_4 = T_{03} \cdot \frac{T_0\text{-}C_1}{T_0}, C_1 = \frac{g \cdot H}{c_p}$ the following relation is derived:

$$2 \cdot g \cdot H \cdot \frac{\Delta T}{T_0} = (k+1) \cdot a \cdot \upsilon^2 \qquad (14)$$

Where $\Delta T = T_{03} - T_0$ (we can approximately consider that $T_0 \approx T_{02}$).
Thus the maximum exit top air speed in a free passage solar chimney (without air turbines) is given by the formula:

$$\upsilon = \sqrt{2 \cdot g \cdot H \cdot \frac{\Delta T}{T_0} / [(k+1) \cdot a]}\,. \qquad (15)$$

For example the exit top speed of the up-drafting air inside the FSC of H=800m height, with ordinary values for coefficients a=1.1058 and k=0.49 and ambient air temperature T_0=296.2 ^{0}K (23 ^{0}C) as function of ΔT is given in the next figure:

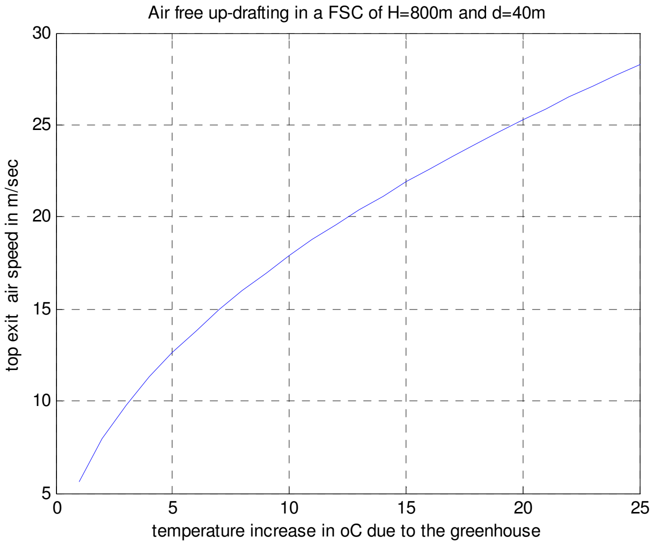

Fig. 10. Free air speed as a function of temperature increase

The temperature increase ΔT as a function of the greenhouse surface area A_c is given by the approximate formula $\Delta T \approx$ ta·G / (β + $\dot{m}$·c_p/A_c) where ta≈0.7, β≈4, c_p=1005, and $\dot{m}_M = \rho \cdot \upsilon \cdot (\pi \cdot d^2/4)$ where ρ≈1.17Kg/m^3, and d=40m. Thus The approximate double glazing solar collector area, generating the free up-drafting air speed υ can be defined by ΔT, υ and G by the equation $A_c \approx \dot{m} \cdot c_p / [(ta \cdot G)/\Delta T - \beta]$.

The approximate solar collector area A_c as a function of the temperature increase ΔT for various values of equivalent horizontal solar irradiance G=250,300,350,400 and 450 W/m^2, is shown in the following figure.

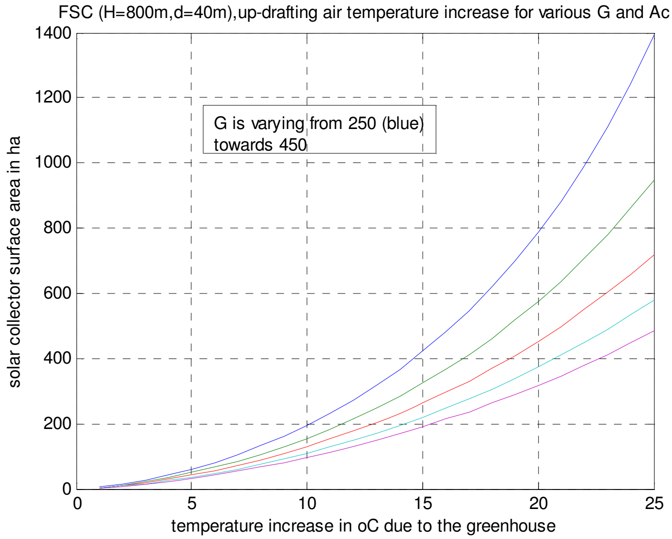

Fig. 11. The solar collector area as a function of its generating temperature increase

Example: for a solar collector of surface area A_c=400Ha (i.e.400000m^2), with a diameter $D_c \approx$715m, for an equivalent horizontal solar irradiation G of 250W/m^2, the created temperature difference ΔT is ~14.5^{0}C and the free up-drafting air speed υ inside the FSC of H=800m height and d=40m internal diameter will be ~21m/sec, while for G=450W/m^2, ΔT is ~22.5^{0}C and υ is ~27m/sec.
For one dimensional analysis $a \approx 1$ and if the friction losses are negligible, i.e. $k \approx 0$, we have:

$$\upsilon \approx \sqrt{2 \cdot g \cdot H \cdot \frac{\Delta T}{T_0}} \tag{17}$$

Therefore free up-drafting warm air top speed formula, in an adiabatic and free friction FSC, due to its buoyancy, is similar to free falling water speed due to gravity given by:

$$\upsilon_{water} \approx \sqrt{2 \cdot g \cdot H}$$

4.3 The thermal heat transfer model of the SAEP

In order to use the previous thermodynamic cycle analysis of the SAEP we should calculate the warm air temperature T_{03} at the entrance of the air turbine or at the exit of the solar collector. The calculation of this average temperature can be done by using the previously proposed approximate analysis. However the temperature T_{03} is varying during the 24 hours daily cycle.
In order for the daily variation to be calculated and consequently the electric power daily variation using the previously proposed thermodynamic cycle analysis, we should make a

heat transfer model and use it for the calculation of the exit temperature as function mainly of daily horizontal irradiance profile and ambient temperature daily profile.
The SAEP heat transfer model with a circular collector is shown in the indicative diagram of the previous figure.
The circular solar collector of this SAEP is divided into a series of M circular sectors of equal width Δr as shown in the next figure.
In this figure the cut of a circular sector of the solar collector of the SAEP is shown with the heat transfer coefficients of the process (radiation and convection) and the temperatures of ground (T_s), moving air (T), inner curtain (T_c), outer glazing (T_w), ambient air (T_0) and sky (T_{sk}). The ground absorbs a part of the transmitted irradiation power due to the horizontal solar irradiance G (ta ·G).
The wind is moving with a speed υ_w and on the ground it is a thin sheet of water inside a dark plastic film. The ground is characterized by its density ρ_{gr}, its specific heat capacity c_{gr} and its thermal conductivity k_{gr}.

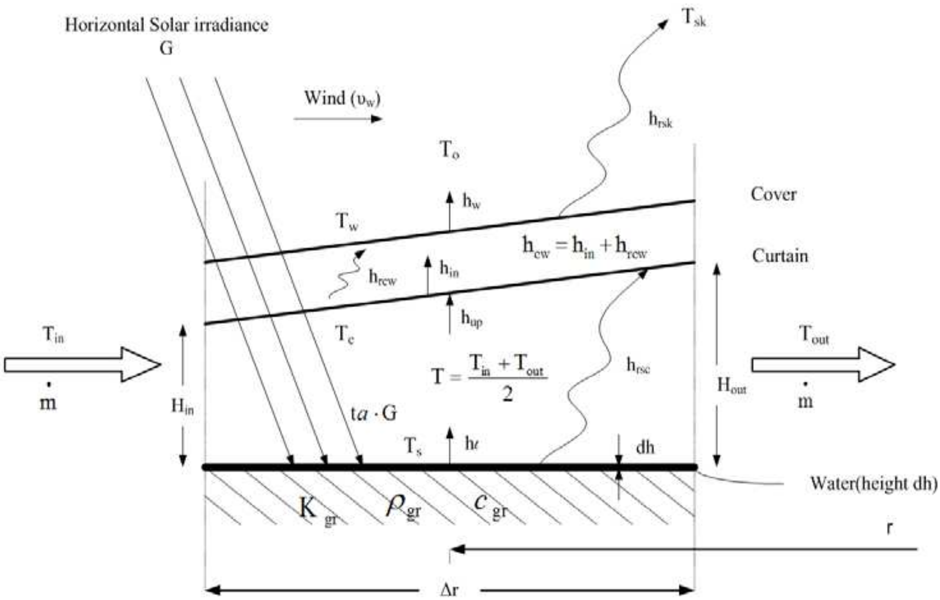

Fig. 12. The cut of a circular sector of a double glazing circular solar collector

The mth circular sector (m=1up to M) will have a width $\Delta r = (D_c - D_{in})/M$, an average radius $r_m = Dc/2 - \Delta r \cdot (m1-1/2)$ and an average height $H_m = (H_{in,m} + H_{ex,m})/2$.
For a linear variation of the roof height H_m= Hin+(Hout-Hin)· (m-1/2)/M, where D_c=solar collector diameter and D_{in}=Final internal diameter of the solar collector.
These consecutive circular sectors, for the moving air stream of mass flow $\dot{m}$, are special tubes of nearly parallel flat surfaces and therefore they have equivalent average diameters $d_{e,m} = 2 * H_m$.
As the ambient air moves towards the entrance of the first circular sector it is assumed that its temperature T_0 increases to T_0+dT due to the ground heat transfer convection to inlet air, around the solar collector. As an approximation dT is estimated to 0.5 °K.

The exit temperature of the first sector is the inlet temperature for the second etc. and finally the exit temperature of the final Mth sector is the T_{03}, i.e. the inlet stagnation temperature to the air turbines.

The solar chimney heat transfer analysis during a daily 24 hours cycle, is too complicated to be presented analytically in this text however we can use the results of this analysis in order to have a clear picture of the operational characteristics of the SAEPs. Using the code of the heat transfer analysis for moving mass flow $\dot{m}_M$, the daily variation of the exit temperature T_{03} can be calculated. Using these calculated daily values of the T_{03} and by the thermodynamic cycle analysis for the optimal mass flow $\dot{m}_M$ the daily power profile of the electricity generation can be calculated.

With this procedure the 24 hour electricity generation power profile of a SAEP with a solar collector of surface area $A_c=10^6 m^2$ and a FSC of H=800m height and d=40m internal diameter for an average day of the year has been calculated. The SAEP is installed in a place with annual horizontal solar irradiation W_y=1700 KWh/m^2.

In the following figure three electric power profiles are shown with or without artificial thermal storage.

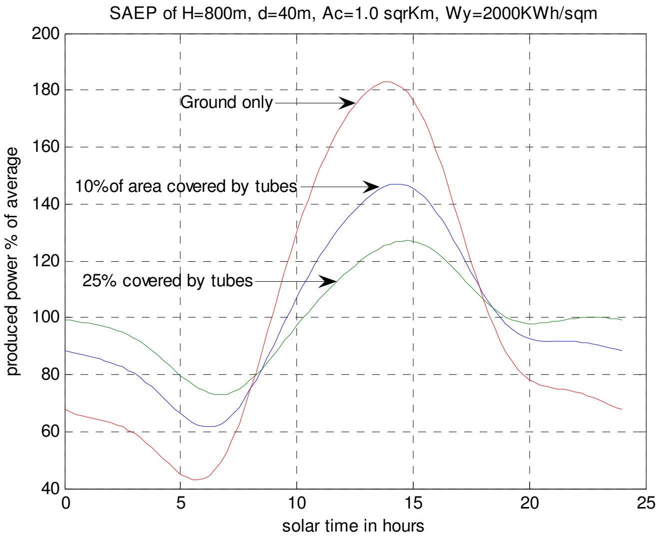

Fig. 13. The average daily SAEP's electricity generating profiles

The relatively smooth profile shows the electric power generation when only the ground acts as a thermal storage means. While the smoother profiles are achieved when the greenhouse is partly covered (~10% or ~25% of its area) by plastic black tubes of 35cm of diameter filled with water, i.e. there is also additional thermal storage of an equivalent water sheet of 35·π/4=27.5 cm on a small part of the solar collector.

The daily profiles show that the SAEP operates 24hours/day, due to the greenhouse ground (and artificial) thermal storage. That is a considerable benefit of the FSC technology compared to the rest solar technologies and the wind technology which if they are not equipped with energy mass storage systems they can not operate continuously.