

ECE 454 and ECE 554 Supplemental reading

ECE 454 and ECE 554 Supplemental reading

Collection edited by: Thad Welch

Content authors: Don Johnson, Minh Do, C. Burrus, Roy Ha, Michael Haag, Nguyen Huu Phuong, Richard Baraniuk, Melissa Selik, Nasser Kehtarnavaz, Philipos Loizou, Mohammad Rahman, Anders Gjendemsjø, Robert Nowak, Justin Romberg, Stephen Kruzick, Mark Davenport, Tuan Do-Hong, Ricardo Radaelli-Sanchez, Catherine Elder, Ivan Selesnick, Benjamin Fite, Douglas Jones, Swaroop Appadwedula, Matthew Berry, Mark Haun, Jake Janevitz, Michael Kramer, Dima Moussa, Daniel Sachs, and Brian Wade

Online: <<http://cnx.org/content/col11416/1.1>>

This selection and arrangement of content as a collection is copyrighted by Thad Welch.

It is licensed under the Creative Commons Attribution License: <http://creativecommons.org/licenses/by/3.0/>

Collection structure revised: 2012/04/02

For copyright and attribution information for the modules contained in this collection, see the "[Attributions](#)" section at the end of the collection.

ECE 454 and ECE 554 Supplemental reading

Table of Contents

- [Chapter 1. ECE 454/ECE 554 Supplemental Reading for Chapter 1](#)
 - [1.1. Introduction to Digital Signal Processing](#)
 - [1.2. Introduction to Fundamentals of Signal Processing](#)
 - [What is Digital Signal Processing?](#)
 - [Overview of Key Concepts in Digital Signal Processing](#)
 - [1.3. m09 - An Overview of Discrete-Time Signals](#)
 - [\[Discrete-Time Signals\]Discrete-Time Signals](#)
 - [References](#)
- [Chapter 2. ECE 454/ECE 554 Supplemental Reading for Chapter 2](#)
 - [2.1. DSP notation](#)
 - [Introduction](#)
 - [XSL Transformations and Macros](#)
 - [Signal Processing Notation](#)
 - [Time](#)
 - [Period](#)
 - [Frequency](#)
 - [Signal](#)
 - [Special Signals](#)
 - [Impulse](#)
 - [Unit Step](#)
 - [Sinc](#)
 - [Norm](#)
 - [Convolution](#)
 - [Circular Convolution](#)
 - [Twiddle Factor](#)
 - [Fourier Transform](#)
 - [CTFT](#)
 - [Laplace Transform](#)
 - [z-transform](#)
 - [Complex Numbers](#)
 - [Inner Product](#)
 - [2.2. Discrete-Time Systems in the Time-Domain](#)
 - [2.3. Signals Represent Information](#)
 - [Analog Signals](#)
 - [Digital Signals](#)
 - [2.4. Introduction to Systems](#)
 - [Cascade Interconnection](#)

- [Parallel Interconnection](#)
 - [Feedback Interconnection](#)
- [2.5. Discrete-Time Signals and Systems](#)
 - [Real- and Complex-valued Signals](#)
 - [Complex Exponentials](#)
 - [Sinusoids](#)
 - [Unit Sample](#)
 - [Unit Step](#)
 - [Symbolic Signals](#)
 - [Discrete-Time Systems](#)
- [2.6. Systems in the Time-Domain](#)
- [2.7. Autocorrelation of Random Processes](#)
 - [Autocorrelation Function](#)
 - [Properties of Autocorrelation](#)
 - [Estimating the Autocorrelation with Time-Averaging](#)
 - [Examples](#)
- [2.8. DIGITAL CORRELATION](#)
 - [Cross-correlation and auto-correlation](#)
 - [Auto-correlation](#)
 - [Correlation and data communication](#)
 - [Correlation of periodic signals](#)
- [Chapter 3. ECE 454/ECE 554 Supplemental Reading for Chapter 3](#)
 - [3.1. DTFT Examples](#)
 - [3.2. Discrete-Time Fourier Transform \(DTFT\)](#)
 - [3.3. Continuous Time Fourier Transform \(CTFT\)](#)
 - [Introduction](#)
 - [Fourier Transform Synthesis](#)
 - [Equations](#)
 - [CTFT Definition Demonstration](#)
 - [Example Problems](#)
 - [Fourier Transform Summary](#)
 - [3.4. Discrete Time Fourier Transform \(DTFT\)](#)
 - [Introduction](#)
 - [DTFT synthesis](#)
 - [Equations](#)
 - [DTFT Definition demonstration](#)
 - [DTFT Summary](#)
 - [3.5. Continuous-Time Fourier Transform](#)
 - [Properties of CTFT](#)
 - [Numerical Approximations to CTFT](#)
 - [3.6. Introduction](#)
 - [Why sample?](#)
 - [Claude E. Shannon](#)

- [Notation](#)
- [The Sampling Theorem](#)
-
- [3.7. Proof](#)
 - [Introduction](#)
 - [Proof part 1 - Spectral considerations](#)
 - [Proof part II - Signal reconstruction](#)
 - [Summary](#)
 -
- [3.8. Illustrations](#)
 - [Basic examples](#)
 - [The process of sampling](#)
 - [Sampling fast enough](#)
 - [Sampling too slowly](#)
 - [Reconstruction](#)
 - [Conclusions](#)
 -
- [3.9. Sampling and reconstruction with Matlab](#)
 - [Matlab files](#)
 -
- [3.10. Systems view of sampling and reconstruction](#)
 - [Ideal reconstruction system](#)
 - [Ideal system including anti-aliasing](#)
 - [Reconstruction with hold operation](#)
 -
- [3.11. Sampling CT Signals: A Frequency Domain Perspective](#)
 - [Understanding Sampling in the Frequency Domain](#)
 - [Sampling](#)
 - [Relating \$x\[n\]\$ to sampled \$x\(t\)\$](#)
- [3.12. SIGNAL SAMPLING](#)
 - [Sampling of continuous-time signals](#)
 - [The sampling theorem](#)
 - [Aliasing](#)
- [3.13. The DFT: Frequency Domain with a Computer Analysis](#)
 - [Introduction](#)
 - [Sampling DTFT](#)
 - [Choosing M](#)
 - [Case 1](#)
 - [Case 2](#)
 - [Discrete Fourier Transform \(DFT\)](#)
 - [Interpretation](#)
 - [Remark 1](#)
 - [Remark 2](#)

- [Periodicity of the DFT](#)
- [A Sampling Perspective](#)
 - [Inverse DTFT of \$S\(\omega\)\$](#)
- [Connections](#)
- [3.14. Sampling Theorem](#)
 - [Introduction](#)
 - [Nyquist-Shannon Sampling Theorem](#)
 - [Statement of the Sampling Theorem](#)
 - [Proof of the Sampling Theorem](#)
 - [Perfect Reconstruction](#)
 - [Practical Implications](#)
 - [Discrete Time Processing of Continuous Time Signals](#)
 - [Psychoacoustics](#)
 - [Sampling Theorem Summary](#)
- [Chapter 4. ECE 454/ECE 554 Supplemental Reading for Chapter 4](#)
 - [4.1. Examples for Systems in the Time Domain](#)
 - [4.2. Discrete-Time Processing of CT Signals](#)
 - [DT Processing of CT Signals](#)
 - [Analysis](#)
 - [Summary](#)
 - [Note](#)
 - [Application: 60Hz Noise Removal](#)
 - [DSP Solution](#)
 - [Sampling Period/Rate](#)
 - [Digital Filter](#)
 - [4.3. Discrete-Time Systems](#)
 - [Classifications](#)
 - [Convolution](#)
 - [Derivation of the Convolution Sum](#)
 - [The Matrix Formulation of Convolution](#)
 - [The Z-Transform Transfer Function](#)
 - [Frequency Response of Discrete-Time Systems](#)
 - [Fundamental Theorem of Linear, Time-Invariant Systems](#)
 - [Pole-Zero Plots](#)
 - [Relation of PZ Plots, FR Plots, Impulse R](#)
 - [State Variable Formulation](#)
 - [Difference Equations](#)
 - [Flow Graph Representation](#)
 - [Standard Structures](#)
 - [FIR and IIR Structures](#)
 - [Quantization Effects](#)
 - [Multidimensional Systems](#)
 - [References](#)

- 4.4. Eigenvectors of LSI Systems
- 4.5. LSI/LTI Systems
 - Characterizing LSI Systems
 - 1.
 - 2.
 - Understanding Conditions on Matrix \mathbf{I} for Shift Invariance
 - Upshot for LSI Systems
 - Summary: LSI Systems and Impulse Response
- 4.6. System Classifications and Properties
 - Introduction
 - Classification of Systems
 - Continuous vs. Discrete
 - Linear vs. Nonlinear
 - Time Invariant vs. Time Variant
 - Causal vs. Noncausal
 - Stable vs. Unstable
- 4.7. Discrete Time Systems
 - Introduction
 - Discrete Time Systems
 - Linearity and Time Invariance
 - Difference Equation Representation
 - Discrete Time Systems Summary
- 4.8. PROPERTIES OF THE DIGITAL CONVOLUTION
 - Commutativity
 - Associativity
 - Distributivity
 - Impulse response for causal system and signal
 - System identification
- 4.9. Discrete Time Convolution
 - Introduction
 - Convolution and Circular Convolution
 - Convolution
 - Operation Definition
 - Definition Motivation
 - Graphical Intuition
 - Circular Convolution
 - Definition Motivation
 - Graphical Intuition
 - Interactive Element
 - Convolution Summary
- 4.10. Linear-Phase FIR Filters
 - THE AMPLITUDE RESPONSE
 - WHY LINEAR-PHASE?

- [WHY LINEAR-PHASE: EXAMPLE](#)
 - [WHY LINEAR-PHASE: EXAMPLE \(2\)](#)
 - [WHY LINEAR-PHASE: MORE](#)
- [4.11. m21 - Convolution of Discrete-Time Signals](#)
 - [Convolution](#)
 - [Derivation of the Convolution Sum](#)
 - [The Matrix Formulation of Convolution](#)
 - [References](#)
- [Chapter 5. ECE 454/ECE 554 Supplemental Reading for Chapter 5](#)
 - [5.1. DFT as a Matrix Operation](#)
 - [Matrix Review](#)
 - [Representing DFT as Matrix Operation](#)
 - [5.2. Filtering with the DFT](#)
 - [Introduction](#)
 - [Compute IDFT of \$Y\[k\]\$](#)
 - [DFT Pair](#)
 - [Regular Convolution from Periodic Convolution](#)
 - [DSP System](#)
 - [5.3. m10 - The Discrete Fourier Transform](#)
 - [The Discrete Fourier Transform](#)
 - [Definition of the DFT](#)
 - [Matrix Formulation of the DFT](#)
 - [Extensions of \$x\(n\)\$](#)
 - [Convolution](#)
 - [Examples of the DFT](#)
 - [References](#)
 - [5.4. DFT Basis](#)
 - [DFT Properties](#)
 - [Complex Sinusoids in a nutshell](#)
 - [Summary: Frequency](#)
 - [Impact on DFT](#)
 - [DFT Notation](#)
 - [5.5. INTRODUCTORY DISCRETE FOURIER TRANSFORM \(DFT\)](#)
 - [INTRODUCTORY DISCRETE FOURIER TRANSFORM \(DFT\)](#)
 - [From the DTFT to the DFT](#)
 - [Properties of the DFT](#)
 - [5.6. Discrete-Time Signals](#)
 - [The Discrete Fourier Transform](#)
 - [Definition of the DFT](#)
 - [Matrix Formulation of the DFT](#)
 - [Extensions of \$x\(n\)\$](#)
 - [Convolution](#)
 - [Properties of the DFT](#)

- [Examples of the DFT](#)
- [The Discrete-Time Fourier Transform](#)
 - [Definition of the DTFT](#)
 - [Properties](#)
 - [Evaluation of the DTFT by the DFT](#)
 - [Examples of DTFT](#)
- [The Z-Transform](#)
 - [Definition of the Z-Transform](#)
 - [Properties](#)
 - [Examples of the Z-Transform](#)
 - [Inversion of the Z-Transform](#)
 - [Solution of Difference Equations using the Z-Transform](#)
 - [Region of Convergence for the Z-Transform](#)
 - [Relation of the Z-Transform to the DTFT and the DFT](#)
- [Relationships Among Fourier Transforms](#)
- [Wavelet-Based Signal Analysis](#)
 - [The Basic Wavelet Theory](#)
 - [Generalization of the Basic Wavelet System](#)
- [References](#)
- [5.7. Efficiency of Frequency-Domain Filtering](#)
- [5.8. Conclusions: Fast Fourier Transforms](#)
 - [References](#)
- [5.9. The Cooley-Tukey Fast Fourier Transform Algorithm](#)
 - [Modifications to the Basic Cooley-Tukey FFT](#)
 - [The Split-Radix FFT Algorithm](#)
 - [Evaluation of the Cooley-Tukey FFT Algorithms](#)
 - [The Quick Fourier Transform, An FFT based on Symmetries](#)
 - [Input and Output Symmetries](#)
 - [Further Reductions if the Length is Even](#)
 - [Arithmetic Complexity and Timings](#)
 - [Conclusions](#)
- [References](#)
- [5.10. DTFT and Convolution](#)
- [5.11. Short Time Fourier Transform](#)
 - [Short Time Fourier Transform](#)
 - [Sampled STFT](#)
 - [Spectrogram Example](#)
 - [Effect of window length R](#)
 - [Effect of L and N](#)
 - [Effect of R and L](#)
- [5.12. Spectrograms](#)
- [5.13. m19 - Wavelet-Based Signal Analysis](#)
 - [Wavelet-Based Signal Analysis](#)

- [The Basic Wavelet Theory](#)
 - [Generalization of the Basic Wavelet System](#)
 - [References](#)
- [Chapter 6. ECE 454/ECE 554 Supplemental Reading for Chapter 6](#)
 - [6.1. Discrete-Time Signals](#)
 - [Real- and Complex-valued Signals](#)
 - [Complex Exponentials](#)
 - [Sinusoids](#)
 - [Unit Sample](#)
 - [Symbolic-valued Signals](#)
 - [6.2. Difference Equation](#)
 - [Introduction](#)
 - [General Formulas for the Difference Equation](#)
 - [Difference Equation](#)
 - [Conversion to Z-Transform](#)
 - [Conversion to Frequency Response](#)
 - [Example](#)
 - [Solving a LCCDE](#)
 - [Direct Method](#)
 - [Homogeneous Solution](#)
 - [Particular Solution](#)
 - [Indirect Method](#)
 - [6.3. The Z Transform: Definition](#)
 - [Basic Definition of the Z-Transform](#)
 - [The Complex Plane](#)
 - [Region of Convergence](#)
 - [6.4. Table of Common z-Transforms](#)
 - [6.5. Understanding Pole/Zero Plots on the Z-Plane](#)
 - [Introduction to Poles and Zeros of the Z-Transform](#)
 - [The Z-Plane](#)
 - [Examples of Pole/Zero Plots](#)
 - [Interactive Demonstration of Poles and Zeros](#)
 - [Applications for pole-zero plots](#)
 - [Stability and Control theory](#)
 - [Pole/Zero Plots and the Region of Convergence](#)
 - [Frequency Response and Pole/Zero Plots](#)
 - [6.6. m15 - The Z-Transform](#)
 - [The Z-Transform](#)
 - [Definition of the Z-Transform](#)
 - [Examples of the Z-Transform](#)
 - [Inversion of the Z-Transform](#)
 - [Relation of the Z-Transform to the DTFT and the DFT](#)
 - [References](#)

- 6.7. Differential Equations
 - Differential Equations
 - General Formulas for the Differential Equation
 - Conversion to Laplace-Transform
 - Conversion to Frequency Response
 - Solving a LCCDE
 - Direct Method
 - Homogeneous Solution
 - Particular Solution
 - Indirect Method
 - Summary
- Chapter 7. ECE 454/ECE 554 Supplemental Reading for Chapter 7
 - 7.1. Filtering in the Frequency Domain
 - 7.2. FREQUENCY RESPONSE OF LTI (LSI) SYSTEMS
 - Frequency response
 - Magnitude frequency response on decibel scale
 - Eigen-function and eigen-value in DSP systems
 - Frequency response of systems in cascade or in parallel
 - Frequency response in terms of filter coefficients
 - 7.3. FREQUENCY RESPONSE OF LTI (LSI) SYSTEMS
 - FREQUENCY RESPONSE OF LTI (LSI) SYSTEMS
 - Frequency response
 - Magnitude frequency response on decibel scale
 - Eigen-function and eigen-value in DSP systems
 - Frequency response of systems in cascade or in parallel
 - Frequency response in terms of filter coefficients
 - 7.4. IIR Filtering: Introduction
 - Introduction
 - 7.5. Discrete-Time Filtering Example
 - 7.6. FIR Filter Example
 - 7.7. Overview of Digital Filter Design
 - Perspective on FIR filtering
 - 7.8. Window Design Method
 - L2 optimization criterion
 - Window Design Method
 - 7.9. Frequency Sampling Design Method for FIR filters
 - Important Special Case
 - Important Special Case #2
 - Special Case 2a
 - Comments on frequency-sampled design
 - Extended frequency sample design
 - 7.10. Parks-McClellan FIR Filter Design
 - Formal Statement of the $L-\infty$ (Minimax) Design Problem

- [Outline of L- \$\infty\$ Filter Design](#)
 - [Conditions for L- \$\infty\$ Optimality of a Linear-phase FIR Filter](#)
 - [Alternation Theorem](#)
 - [Optimality Conditions for Even-length Symmetric Linear-phase Filters](#)
 - [L- \$\infty\$ Optimal Lowpass Filter Design Lemma](#)
 - [Computational Cost](#)
- [7.11. Overview of IIR Filter Design](#)
 - [IIR Filter](#)
 - [IIR Filter Design Problem](#)
 - [Outline of IIR Filter Design Material](#)
 - [Comments on IIR Filter Design Methods](#)
- [7.12. Properties of IIR Filters](#)
 - [Frequency-Domain Formulation of IIR Filters](#)
 - [Calculation of the IIR Filter Frequency Response](#)
 - [Pole-Zero Locations for IIR Filters](#)
 - [Summary](#)
 - [References](#)
- [7.13. FIR Digital Filters](#)
 - [Frequency-Domain Description of FIR Filters](#)
 - [Linear-Phase FIR Filters](#)
 - [The Discrete Time Fourier Transform with Normalization](#)
 - [Four Types of Linear-Phase FIR Filters](#)
 - [Calculation of FIR Filter Frequency Response](#)
 - [Zero Locations for Linear-Phase FIR Filters](#)
 - [Section Summary](#)
 - [FIR Digital Filter Design](#)
 - [References](#)
- [Chapter 8. ECE 454/ECE 554 Supplemental Reading for Chapter 8](#)
 - [8.1. Filter Structures](#)
- [Chapter 9. ECE 454/ECE554 Supplemental reading for Chapter 11](#)
 - [9.1. Fast fourier transform \(FFT\)](#)
 - [Decimation in time FFT](#)
 - [Derivation](#)
 - [The final picture](#)
 - [9.2. The Fast Fourier Transform \(FFT\)](#)
 - [Introduction](#)
 - [Deriving the FFT](#)
 - [FFT and the DFT](#)
 - [Speed Comparison](#)
 - [Conclusion](#)
 - [9.3. The FFT Algorithm](#)
 - [The Fast Fourier Transform FFT](#)
 - [How does the FFT work?](#)

- [Decimation in Time FFT](#)
 - [Derivation](#)
- [9.4. Overview of Fast Fourier Transform \(FFT\) Algorithms](#)
 - [History of the FFT](#)
 - [Summary of FFT algorithms](#)
- [9.5. Convolution Algorithms](#)
 - [Fast Convolution by the FFT](#)
 - [Fast Convolution by Overlap-Add](#)
 - [Fast Convolution by Overlap-Save](#)
 - [Block Processing, a Generalization of Overlap Methods](#)
 - [Introduction](#)
 - [Block Signal Processing](#)
 - [Block Convolution](#)
 - [Block Recursion](#)
 - [Block State Formulation](#)
 - [Block Implementations of Digital Filters](#)
 - [Multidimensional Formulation](#)
 - [Periodically Time-Varying Discrete-Time Systems](#)
 - [Multirate Filters, Filter Banks, and Wavelets](#)
 - [Distributed Arithmetic](#)
 - [Direct Fast Convolution and Rectangular Transforms](#)
 - [Number Theoretic Transforms for Convolution](#)
 - [Results from Number Theory](#)
 - [Number Theoretic Transforms](#)
 - [References](#)
- [Index](#)

Chapter 1. ECE 454/ECE 554 Supplemental Reading for Chapter 1

1.1. Introduction to Digital Signal Processing*

Not only do we have analog signals --- signals that are real- or complex-valued functions of a continuous variable such as time or space --- we can define **digital** ones as well. Digital signals are **sequences**, functions defined only for the integers. We thus use the notation $s(n)$ to denote a discrete-time one-dimensional signal such as a digital music recording and $s(m, n)$ for a discrete-"time" two-dimensional signal like a photo taken with a digital camera. Sequences are fundamentally different than continuous-time signals. For example, continuity has no meaning for sequences.

Despite such fundamental differences, the theory underlying digital signal processing mirrors that for analog signals: Fourier transforms, linear filtering, and linear systems parallel what previous chapters described. These similarities make it easy to understand the definitions and why we need them, but the similarities should not be construed as "analog wannabes." We will discover that digital signal processing is **not** an approximation to analog processing. We must explicitly worry about the fidelity of converting analog signals into digital ones. The music stored on CDs, the speech sent over digital cellular telephones, and the video carried by digital television all evidence that analog signals can be accurately converted to digital ones and back again.

The key reason why digital signal processing systems have a technological advantage today is the **computer**: computations, like the Fourier transform, can be performed quickly enough to be calculated as the signal is produced, [\[1\]](#) and programmability means that the signal processing system can be easily changed. This flexibility has obvious appeal, and has been widely accepted in the marketplace. Programmability means that we can perform signal processing operations impossible with analog systems (circuits). We will also discover that digital systems enjoy an **algorithmic** advantage that contributes to rapid processing speeds: Computations can be restructured in non-obvious ways to speed the processing. This flexibility comes at a price, a consequence of how computers work. How do computers perform signal processing?

1.2. Introduction to Fundamentals of Signal Processing*

What is Digital Signal Processing?

To understand what is **Digital Signal Processing (DSP)** let's examine what does each of its words mean. “**Signal**” is any physical quantity that carries information. “**Processing**” is a series of steps or operations to achieve a particular end. It is easy to see that **Signal Processing** is used everywhere to extract information from signals or to convert information-carrying signals from one form to another. For example, our brain and ears take input speech signals, and then process and convert them into meaningful words. Finally, the word “**Digital**” in Digital Signal Processing means that the process is done by computers, microprocessors, or logic circuits.

The field DSP has expanded significantly over that last few decades as a result of rapid developments in computer technology and integrated-circuit fabrication. Consequently, DSP has played an increasingly important role in a wide range of disciplines in science and technology. Research and development in DSP are driving advancements in many high-tech areas including telecommunications, multimedia, medical and scientific imaging, and human-computer interaction.

To illustrate the digital revolution and the impact of DSP, consider the development of digital cameras. Traditional film cameras mainly rely on physical properties of the optical lens, where higher quality requires bigger and larger system, to obtain good images. When digital cameras were first introduced, their quality were inferior compared to film cameras. But as microprocessors become more powerful, more sophisticated DSP algorithms have been developed for digital cameras to correct optical defects and improve the final image quality. Thanks to these developments, the quality of consumer-grade digital cameras has now surpassed the equivalence in film cameras. As further developments for digital cameras attached to cell phones (cameraphones), where due to small size requirements of the lenses, these cameras rely on DSP power to provide good images. Essentially, digital camera technology uses computational power to overcome physical limitations. We can find the similar trend happens in many other applications of DSP such as digital communications, digital imaging, digital television, and so on.

In summary, DSP has foundations on Mathematics, Physics, and Computer Science, and can provide the key enabling technology in numerous applications.

Overview of Key Concepts in Digital Signal Processing

The two main characters in DSP are **signals** and **systems**. A **signal** is defined as any physical quantity that varies with one or more independent variables such as time (one-dimensional signal), or space (2-D or 3-D signal). Signals exist in several types. In the real-world, most of signals are **continuous-time** or **analog signals** that have values continuously at every value of time. To be processed by a computer, a continuous-time signal has to be first **sampled** in time into a **discrete-time signal** so that its values at a discrete set of time instants can be stored in computer memory locations. Furthermore, in order to be processed by logic circuits, these signal values

have to be **quantized** in to a set of discrete values, and the final result is called a **digital signal**. When the quantization effect is ignored, the terms discrete-time signal and digital signal can be used interchangeability.

In signal processing, a **system** is defined as a process whose input and output are signals. An important class of systems is the class of **linear time-invariant** (or **shift-invariant**) **systems**. These systems have a remarkable property is that each of them can be completely characterized by an **impulse response function** (sometimes is also called as **point spread function**), and the system is defined by a **convolution** (also referred to as a **filtering**) operation. Thus, a linear time-invariant system is equivalent to a (linear) **filter**. Linear time-invariant systems are classified into two types, those that have **finite-duration impulse response (FIR)** and those that have an **infinite-duration impulse response (IIR)**.

A signal can be viewed as a **vector** in a **vector space**. Thus, **linear algebra** provides a powerful framework to study signals and linear systems. In particular, given a vector space, each signal can be represented (or expanded) as a **linear combination of elementary signals**. The most important **signal expansions** are provided by the **Fourier transforms**. The Fourier transforms, as with general transforms, are often used effectively to transform a problem from one domain to another domain where it is much easier to solve or analyze. The two domains of a Fourier transform have physical meaning and are called the **time domain** and the **frequency domain**.

Sampling, or the conversion of **continuous-domain real-life signals** to **discrete numbers** that can be processed by computers, is the essential bridge between the analog and the digital worlds. It is important to understand the connections between signals and systems in the real world and inside a computer. These connections are convenient to analyze in the frequency domain. Moreover, many signals and systems are specified by their **frequency characteristics**.

Because any **linear time-invariant system** can be characterized as a **filter**, the design of such systems boils down to the design the associated filters. Typically, in the **filter design** process, we determine the coefficients of an FIR or IIR filter that closely approximates the desired **frequency response** specifications. Together with Fourier transforms, the **z-transform** provides an effective tool to analyze and design digital filters.

In many applications, signals are conveniently described via **statistical models** as **random signals**. It is remarkable that optimum linear filters (in the sense of **minimum mean-square error**), so called **Wiener filters**, can be determined using only **second-order statistics** (**autocorrelation** and **crosscorrelation** functions) of a **stationary process**. When these statistics cannot be specified beforehand or change over time, we can employ **adaptive filters**, where the filter coefficients are adapted to the signal statistics. The most popular algorithm to adaptively adjust the filter coefficients is the **least-mean square (LMS)** algorithm.

[Discrete-Time Signals]Discrete-Time Signals

Although the discrete-time signal $x(n)$ could be any ordered sequence of numbers, they are usually samples of a continuous-time signal. In this case, the real or imaginary valued mathematical function $x(n)$ of the integer n is not used as an analogy of a physical signal, but as some representation of it (such as samples). In some cases, the term digital signal is used interchangeably with discrete-time signal, or the label digital signal may be use if the function is not real valued but takes values consistent with some hardware system.

Indeed, our very use of the term "discrete-time" indicates the probable origin of the signals when, in fact, the independent variable could be length or any other variable or simply an ordering index. The term "digital" indicates the signal is probably going to be created, processed, or stored using digital hardware. As in the continuous-time case, the Fourier transform will again be our primary tool.

Notation has been an important element in mathematics. In some cases, discrete-time signals are best denoted as a sequence of values, in other cases, a vector is created with elements which are the sequence values. In still other cases, a polynomial is formed with the sequence values as coefficients for a complex variable. The vector formulation allows the use of linear algebra and the polynomial formulation allows the use of complex variable theory.

References

1. Barbara Burke Hubbard. (1996). *The World According to Wavelets*. [Second Edition 1998]. Wellesley, MA: A K Peters.
2. C. Sidney Burrus, Ramesh A. Gopinath and Haitao Guo. (1998). *Introduction to Wavelets and the Wavelet Transform*. Upper Saddle River, NJ: Prentice Hall.
3. Ingrid Daubechies. (1992). *Ten Lectures on Wavelets*. [Notes from the 1990 CBMS-NSF Conference on Wavelets and Applications at Lowell, MA]. Philadelphia, PA: SIAM.
4. Martin Vetterli and Jelena Kova\vcević. (1995). *Wavelets and Subband Coding*. Upper Saddle River, NJ: Prentice-Hall.
5. Gilbert Strang and T. Nguyen. (1996). *Wavelets and Filter Banks*. Wellesley, MA: Wellesley-Cambridge Press.

Solutions

Thank You for previewing this eBook

You can read the full version of this eBook in different formats:

- HTML (Free /Available to everyone)
- PDF / TXT (Available to V.I.P. members. Free Standard members can access up to 5 PDF/TXT eBooks per month each month)
- Epub & Mobipocket (Exclusive to V.I.P. members)

To download this full book, simply select the format you desire below

