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Stochastic improvement of structural design

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1. Introduction

It is well understood nowadays that design is not an one-step process, but that it evolves along many phases which, starting from an initial idea, include drafting, preliminary evaluations, trial and error procedures, verifications and so on. All those steps can include considerations that come from different areas, when functional requirements have to be met which pertain to fields not directly related to the structural one, as it happens for noise, environmental prescriptions and so on; but even when that it's not the case, it is very frequent the need to match against opposing demands, for example when the required strength or stiffness is to be coupled with lightness, not to mention the frequently encountered problems related to the available production means.

All the previous cases, and the many others which can be taken into account, justify the introduction of particular design methods, obviously made easier by the ever-increasing use of numerical methods, and first of all of those techniques which are related to the field of mono- or multi-objective or even multidisciplinary optimization, but they are usually confined in the area of deterministic design, where all variables and parameters are considered as fixed in value. As we discuss below, the random, or stochastic, character of one or more parameters and variables can be taken into account, thus adding a deeper insight into the real nature of the problem in hand and consequently providing a more sound and improved design.

Many reasons can induce designers to study a structural project by probabilistic methods, for example because of uncertainties about loads, constraints and environmental conditions, damage propagation and so on; the basic methods used to perform such analyses are well assessed, at least for what refers to the most common cases, where structures can be assumed to be characterized by a linear behaviour and when their complexity is not very great.

Another field where probabilistic analysis is increasingly being used is that related to the requirement to obtain a product which is 'robust' against the possible variations of manufacturing parameters, with this meaning both production tolerances and the settings of machines and equipments; in that case one is looking for the 'best' setting, i.e. that which minimizes the variance of the product against those of design or control variables.

A very usual case – but also a very difficult to be dealt – is that where it is required to take into account also the time variable, which happens when dealing with a structure which degrades because of corrosion, thermal stresses, fatigue, or others; for example, when studying very light structures, such as those of aircrafts, the designer aims to ensure an assigned life to them, which are subjected to random fatigue loads; in advanced age the aircraft is interested by a WFD (Widespread Fatigue Damage) state, with the presence of many cracks which can grow, ultimately causing failure. This case, which is usually studied by analyzing the behaviour of significant details, is a very complex one, as one has to take into account a large number of cracks or defects, whose sizes and locations can't be predicted, aiming to delay their growth and to limit the probability of failure in the operational life of the aircraft within very small limits (about $10^{-7}\pm10^{-9}$).

The most widespread technique is a 'decoupled' one, in the sense that a forecast is introduced by one of the available methods about the amount of damage which will probably take place at a prescribed instant and then an analysis in carried out about the residual strength of the structure; that is because the more general study which makes use of the stochastic analysis of the structure is a very complex one and still far away for the actual solution methods; the most used techniques, as the first passage theory, which claim to be the solution, are just a way to move around the real problems.

In any case, the probabilistic analysis of the structure is usually a final step of the design process and it always starts on the basis of a deterministic study which is considered as completed when the other starts. That is also the state that will be considered in the present chapter, where we shall recall the techniques usually adopted and we shall illustrate them by recalling some case studies, based on our experience.

For example, the first case which will be illustrated is that of a riveted sheet structure of the kind most common in the aeronautical field and we shall show how its study can be carried out on the basis of the considerations we introduced above.

The other cases which will be presented in this paper refer to the probabilistic analysis and optimization of structural details of aeronautical as well as of automotive interest; thus, we shall discuss the study of an aeronautical panel, whose residual strength in presence of propagating cracks has to be increased, and with the study of an absorber, of the type used in cars to reduce the accelerations which act on the passengers during an impact or road accident, and whose design has to be improved. In both cases the final behaviour is influenced by design, manufacturing process and operational conditions.

2. General methods for the probabilistic analysis of structures

If we consider the n-dimensional space defined by the random variables which govern a generic problem ("design variables") and which consist of geometrical, material, load, environmental and human factors, we can observe that those sets of coordinates ($\mathbf{x}$) that correspond to failure define a domain (the 'failure domain' Ω_f) in opposition to the remainder of the same space, that is known as the 'safety domain' (Ω_s) as it corresponds to survival conditions.

In general terms, the probability of failure can be expressed by the following integral:

$$P_f = \int_{\Omega_f} f(\mathbf{x}) \cdot d\mathbf{x} = \int_{\Omega_f} f_{x_1 x_2 \cdots x_n}(x_1, x_2, \cdots x_n) \cdot dx_1 dx_2 \cdots dx_n \quad (1)$$

where f_i represents the joint density function of all variables, which, in turn, may happen to be also functions of time. Unfortunately that integral cannot be solved in a closed form in most cases and therefore one has to use approximate methods, which can be included in one of the following typologies:

1) methods that use the limit state surface (LSS, the surface that constitutes the boundary of the failure region) concept: they belong to a group of techniques that model variously the

LSS in both shape and order and use it to obtain an approximate probability of failure; among these, for instance, particularly used are FORM (First Order Reliability Method) and SORM (Second Order Reliability Method), that represent the LSS respectively through the hyper-plane tangent to the same LSS at the point of the largest probability of occurrence or through an hyper-paraboloid of rotation with the vertex at the same point.

2) Simulation methodologies, which are of particular importance when dealing with complex problems: basically, they use Monte-Carlo (MC) technique for the numerical evaluation of the integral above and therefore they define the probability of failure on a frequency basis.

As pointed above, it is necessary to use a simulation technique to study complex structures, but in the same cases each trial has to be carried out through a numerical analysis (for example by FEM); if we couple that circumstance with the need to perform a very large number of trials, which is the case when dealing with very small probabilities of failure, very large runtimes are obtained, which are really impossible to bear. Therefore different means have been introduced in recent years to reduce the number of trials and to make acceptable the simulation procedures.

In this section, therefore, we resume briefly the different methods which are available to carry out analytic or simulation procedures, pointing out the difficulties and/or advantages which characterize them and the particular problems which can arise in their use.

2.1 LSS-based analytical methods

Those methods come from an idea by Cornell (1969), as modified by Hasofer and Lind (1974) who, taking into account only those cases where the design variables could be considered to be normally distributed and uncorrelated, each defined by their mean value μ_I and standard deviation σ_I, modeled the LSS in the standard space, where each variable is represented through the corresponding standard variable, i.e.

$$u_i = \frac{x_i - \mu_i}{\sigma_i} \tag{2}$$

If the LSS can be represented by a hyperplane (fig. 1), it can be shown that the probability of failure is related to the distance β of LSS from the origin in the standard space and therefore is given by

$$P_{fFORM} = 1 - \Phi(\beta) = \Phi(-\beta) \tag{3}$$

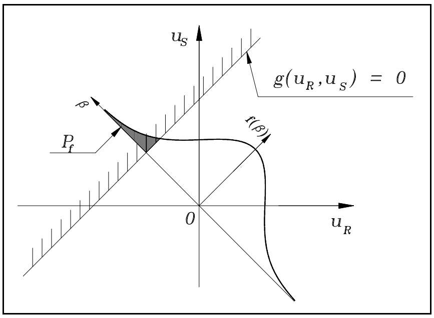

Fig. 1. Probability of failure for a hyperplane LSS

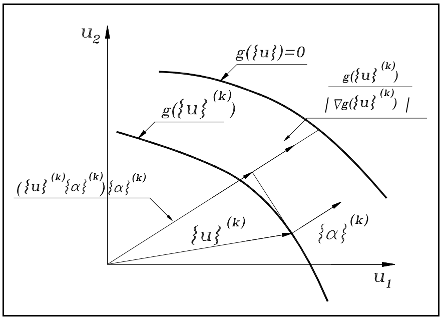

Fig. 2. The search for the design point according to RF's method

It can be also shown that the point of LSS which is located at the least distance β from the origin is the one for which the elementary probability of failure is the largest and for that reason it is called the maximum probability point (MPP) or the design point (DP).

Those concepts have been applied also to the study of problems where the LSS cannot be modeled as an hyperplane; in those cases the basic methods try to approximate the LSS by means of some polynomial, mostly of the first or the second degree; broadly speaking, in both cases the technique adopted uses a Taylor expansion of the real function around some suitably chosen point to obtain the polynomial representation of the LSS and it is quite obvious to use the design point to build the expansion, as thereafter the previous Hasofer and Lind's method can be used.

It is then clear that the solution of such problems requires two distinct steps, i.e. the research of the design point and the evaluation of the probability integral; for example, in the case of FORM (First Order Reliability Method) the most widely applied method, those two steps are coupled in a recursive form of the gradient method (fig. 2), according to a technique introduced by Rackwitz and Fiessler (RF's method). If we represent the LSS through the function $g(\mathbf{x}) = 0$ and indicate with α_I the direction cosines of the inward-pointing normal to the LSS at a point x_0, given by

$$\alpha_i = -\frac{1}{|\nabla g|_0}\left(\frac{\partial g}{\partial u_i}\right)_0 \tag{4}$$

starting from a first trial value of $\mathbf{u}$, the kth n-uple is given by

$$\{\mathbf{u}\}_k = \left[\{\mathbf{u}\}_{k-1}^T \cdot \{\alpha\}_k + \frac{g(\{\mathbf{u}\}_{k-1})}{\nabla g(\{\mathbf{u}\}_{k-1})}\right] \cdot \{\alpha\}_k \tag{5}$$

thus obtaining the required design point within an assigned approximation; its distance from the origin is just β and then the probability of failure can be obtained through eq. 3 above.

One of the most evident errors which follow from that technique is that the probability of failure is usually over-estimated and that error grows as curvatures of the real LSS increase; to overcome that inconvenience in presence of highly non-linear surfaces, the SORM

(Second Order Reliability Method) was introduced, but, even with Tved's and Der Kiureghian's developments, its use implies great difficulties. The most relevant result, due to Breitung, appears to be the formulation of the probability of failure in presence of a quadratic LSS via FORM result, expressed by the following expression:

$$P_{fSORM} = \Phi(-\beta) \cdot \prod_{i=1}^{n-1} (1 - \beta \cdot \kappa_i)^{-1/2} = P_{fFORM} \cdot \prod_{i=1}^{n-1} (1 - \beta \cdot \kappa_i)^{-1/2} \tag{6}$$

where κ_I is the i-th curvature of the LSS; if the connection with FORM is a very convenient one, the evaluation of curvatures usually requires difficult and long computations; it is true that different simplifying assumptions are often introduced to make solution easier, but a complete analysis usually requires a great effort. Moreover, it is often disregarded that the above formulation comes from an asymptotic development and that consequently its result is so more approximate as β values are larger.

As we recalled above, the main hypotheses of those procedures are that the random variables are uncorrelated and normally distributed, but that is not the case in many problems; therefore, some methods have been introduced to overcome those difficulties.

For example, the usually adopted technique deals with correlated variables via an orthogonal transformation such as to build a new set of variables which are uncorrelated, using the well known properties of matrices. For what refers to the second problem, the current procedure is to approximate the behaviour of the real variables by considering dummy gaussian variables which have the same values of the distribution and density functions; that assumption leads to an iterative procedure, which can be stopped when the required approximation has been obtained: that is the original version of the technique, which was devised by Ditlevsen and which is called Normal Tail Approximation; other versions exist, for example the one introduced by Chen and Lind, which is more complex and which, nevertheless, doesn't bring any deeper knowledge on the subject.

At last, it is not possible to disregard the advantages connected with the use of the Response Surface Method, which is quite useful when dealing with rather large problems, for which it is not possible to forecast *a priori* the shape of the LSS and, therefore, the degree of the approximation required. That method, which comes from previous applications in other fields, approximate the LSS by a polynomial, usually of second degree, whose coefficients are obtained by Least Square Approximation or by DOE techniques; the procedure, for example according to Bucher and Burgund, evolves along a series of convergent trials, where one has to establish a center point for the i-th approximation, to find the required coefficients, to determine the design point and then to evaluate the new approximating center point for a new trial.

Beside those here recalled, other methods are available today, such as the Advanced Mean Value or the Correction Factor Method, and so on, and it is often difficult to distinguish their own advantages, but in any case the techniques which we outlined here are the most general and known ones; broadly speaking, all those methods correspond to different degree of approximation, so that their use is not advisable when the number of variables is large or when the expected probabilities of failure is very small, as it is often the case, because of the overlapping of the errors, which can bring results which are very far from the real one.

2.2 Simulation-based reliability assessment

In all those cases where the analytical methods are not to be relied on, for example in presence of many, maybe even not gaussian, variables, one has to use simulation methods to assess the reliability of a structure: about all those methods come from variations or developments of an 'original' method, whose name is Monte-Carlo method and which corresponds to the frequential (or *a posteriori*) definition of probability.

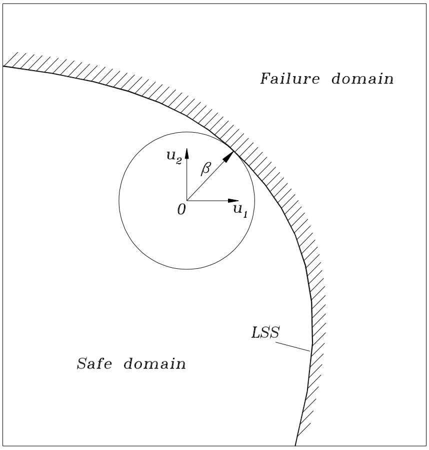

Fig. 3. Domain Restricted Sampling

For a problem with k random variables, of whatever distribution, the method requires the extraction of k random numbers, each of them being associated with the value of one of the variables via the corresponding distribution function; then, the problem is run with the found values and its result (failure of safety) recorded; if that procedure is carried out N times, the required probability, for example that corresponding to failure, is given by $P_f = n/N$, if the desired result has been obtained n times.

Unfortunately, broadly speaking, the procedure, which can be shown to lead to the 'exact' evaluation of the required probability if $N = \infty$, is very slow to reach convergence and therefore a large number of trials have to be performed; that is a real problem if one has to deal with complex cases where each solution is to be obtained by numerical methods, for example by FEM or others. That problem is so more evident as the largest part of the results are grouped around the mode of the result distribution, while one usually looks for probability which lie in the tails of the same distribution, i.e. one deals with very small probabilities, for example those corresponding to the failure of an aircraft or of an ocean platform and so on.

It can be shown, by using Bernouilli distribution, that if p is the 'exact' value of the required probability and if one wants to evaluate it with an assigned e_{max} error at a given confidence level defined by the bilateral protection factor k, the minimum number of trials to be carried out is given by

$$N_{min} = \left(\frac{2 \cdot k}{e_{max}}\right)^2 \frac{1-p}{p} \tag{7}$$

for example, if p = 10^{-5} and we want to evaluate it with a 10% error at the 95% confidence level, we have to carry out at least N_{min} = $1.537 \cdot 10^8$ trials, which is such a large number that usually larger errors are accepted, being often satisfied to get at least the order of magnitude of the probability.

It is quite obvious that various methods have been introduced to decrease the number of trials; for example, as we know that no failure point is to be found at a distance smaller than β from the origin of the axis in the standard space, Harbitz introduced the Domain Restricted Sampling (fig. 3), which requires the design point to be found first and then the trials are carried out only at distances from the origin larger than β; the Importance Sampling Method is also very useful, as each of the results obtained from the trials is weighted according to a function, which is given by the analyst and which is usually centered at the design point, with the aim to limit the number of trials corresponding to results which don't lie in the failure region.

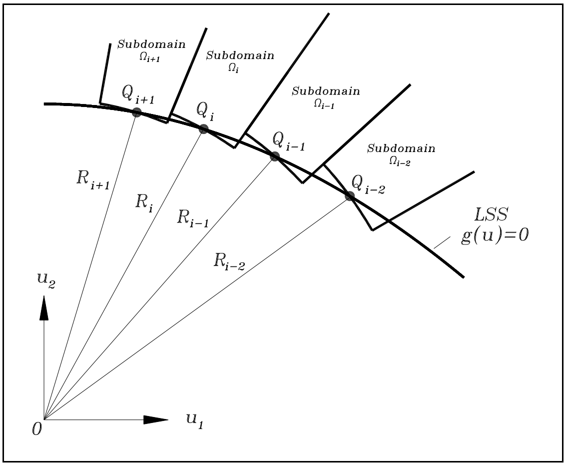

Fig. 4. The method of Directional Simulation

One of the most relevant technique which have been introduced in the recent past is the one known as Directional Simulation; in the version published by Nie and Ellingwood, the sample space is subdivided in an assigned number of sectors through radial hyperplanes (fig. 4); for each sector the mean distance of the LSF is found and the corresponding probability of failure is evaluated, the total probability being given by the simple sum of all results; in this case, not only the number of trials is severely decreased, but a better approximation of the frontier of the failure domain is achieved, with the consequence that the final probability is found with a good approximation.

Other recently appeared variations are related to the extraction of random numbers; those are, in fact, uniformly distributed in the 0-1 range and therefore give results which are rather clustered around the mode of the final distribution. That problem can be avoided if one resorts to use not really random distributions, as those coming from k-discrepancy theory, obtaining points which are better distributed in the sample space.

A new family of techniques have been introduced in the last years, all pertaining to the general family of *genetic algorithms*; that evocative name is usually coupled with an imaginative interpretation which recalls the evolution of animal settlements, with all its content of selection, marriage, breeding and mutations, but it really covers in a systematic and reasoned way all the steps required to find the design point of an LSS in a given region of space. In fact, one has to define at first the size of the population, i.e. the number of sample points to be used when evaluating the required function; if that function is the distance of the design point from the origin, which is to be minimized, a selection is made such as to exclude from the following steps all points where the value assumed by the function is too large. After that, it is highly probable that the location of the minimum is between two points where the same function shows a small value: that coupling is what corresponds to marriage in the population and the resulting intermediate point represents the breed of the couple. Summing up the previous population, without the excluded points, with the breed, gives a new population which represents a new generation; in order to look around to observe if the minimum point is somehow displaced from the easy connection between parents, some mutation can be introduced, which corresponds to looking around the new-found positions.

It is quite clear that, besides all poetry related to the algorithm, it can be very useful but it is quite difficult to be used, as it is sensitive to all different choices one has to introduce in order to get a final solution: the size of the population, the mating criteria, the measure and the way of the introduction in breed of the parents' characters, the percentage and the amplitude of mutations, are all aspects which are to be the objects of single choices by the analyst and which can have severe consequences on the results, for example in terms of the number of generations required to attain convergence and of the accuracy of the method.

That's why it can be said that a general genetic code which can deal with all reliability problems is not to be expected, at least in the near future, as each problem requires specific cares that only the dedicated attentions of the programmer can guarantee.

3. Examples of analysis of structural details

An example is here introduced to show a particular case of stochastic analysis as applied to the study of structural details, taken from the authors' experience in research in the aeronautical field.

Because of their widespread use, the analysis of the behaviour of riveted sheets is quite common in aerospace applications; at the same time the interest which induced the authors to investigate the problems below is focused on the last stages of the operational life of aircraft, when a large number of fatigue-induced cracks appear at the same time in the sheets, before at least one of them propagates up to induce the failure of the riveted joint: the requirement to increase that life, even in presence of such a population of defects (when we say that a stage of Widespread Fatigue Damage, WFD, is taking place) compelled the authors to investigate such a scenario of a damaged structure.

3.1 Probabilistic behaviour of riveted joints

One of the main scopes of the present activity was devoted to the evaluation of the behaviour of a riveted joint in presence of damage, defined for example as a crack which, stemming from the edge of one of the holes of the joint, propagates toward the nearest one, therefore introducing a higher stress level, at least in the zone adjacent to crack tip.

It would be very appealing to use such easy procedures as compounding to evaluate SIF's for that case, which, as it is now well known, gives an estimate of the stress level which is built by reducing the problem at hand to the combination of simpler cases, for which the solution is known; that procedure is entirely reliable, but for those cases where singularities are so near to each other to develop an interaction effect which the method is not able to take into account.

Unfortunately, even if a huge literature is now available about edge cracks of many geometry, the effect of a loaded hole is not usually treated with the extent it deserves, may be for the particular complexity of the problem; for example, the two well known papers by Tweed and Rooke (1979; 1980) deal with the evaluation of SIF for a crack stemming from a loaded hole, but nothing is said about the effect of the presence of other loaded holes toward which the crack propagates.

Therefore, the problem of the increase of the stress level induced from a propagating crack between loaded holes could be approached only by means of numerical methods and the best idea was, of course, to use the results of FEM to investigate the case. Nevertheless, because of the presence of the external loads, which can alter or even mask the effects of loaded holes, we decided to carry out first an investigation about the behaviour of SIF in presence of two loaded holes.

The first step of the analysis was to choose which among the different parameters of the problem were to be treated as random variables.

Therefore a sort of sensitivity analysis was to be carried out; in our case, we considered a very specific detail, i.e. the space around the hole of a single rivet, to analyze the influence of the various parameters.

By using a Monte-Carlo procedure, some probability parameters were introduced according to experimental evidence for each of the variables in order to assess the required influence on the mean value and the coefficient of variation of the number of cycles before failure of the detail.

In any case, as pitch and diameter of the riveted holes are rather standardized in size, their influence was disregarded, while the sheet thickness was assumed as a deterministic parameter, varying between 1.2 and 4.8 mm; therefore, the investigated parameters were the stress level distribution, the size of the initial defect and the parameters of the propagation law, which was assumed to be of Paris' type.

For what refers to the load, it was supposed to be in presence of traction load cycles with $R = 0$ and with a mean value which followed a Gaussian probability density function around 60, 90 and 120 MPa, with a coefficient of variation varying according assigned steps; initial crack sizes were considered as normally distributed from 0.2 mm up to limits depending on the examined case, while for what concerns the two parameters of Paris' law, they were considered as characterized by a normal joint pdf between the exponent n and the logarithm of the other one.

Initially, an extensive exploration was carried out, considering each variable in turn as random, while keeping the others as constant and using the code NASGRO® to evaluate the number of cycles to failure; an external routine was written in order to insert the crack code in a M-C procedure. CC04 and TC03 models of NASGRO® library were adopted in order to take into account corner- as well as through-cracks. For all analyses 1,000 trials/point were carried out, as it was assumed as a convenient figure to be accepted to obtain rather stabilized results, while preventing the total runtimes from growing unacceptably long; the said M-C procedure was performed for an assigned statistics of one input variable at the time.

The results obtained can be illustrated by means of the following pictures and first of all of the fig. 5 where the dependence of the mean value of life from the mean amplitude of remote stress is recorded for different cases where the CV (coefficient of variation) of stress pdf was considered as being constant. The figure assesses the increase of the said mean life to failure in presence of higher CV of stress, as in this case rather low stresses are possible with a relatively high probability and they influence the rate of propagation in a higher measure than large ones.

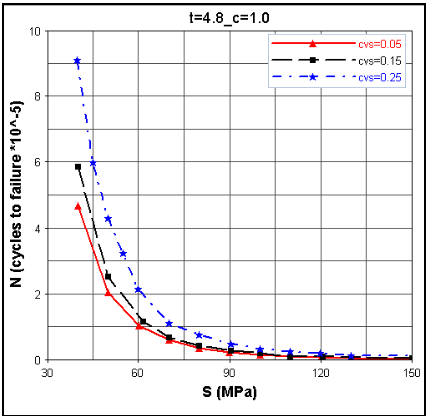

Fig. 5. Influence of the remote stress on the cycles to failure

In fig. 6 the influence of the initial geometry is examined for the case of a corner crack, considered to be elliptical in shape, with length c and depth a; a very interesting aspect of the consequences of a given shape is that for some cases the life for a through crack is longer than the one recorded for some deep corner ones; that case can be explained with the help of the plot of Fig. 7 where the growth of a through crack is compared with those of quarter corner cracks, recording times when a corner crack becomes a through one: as it is clarified in the boxes in the same picture, each point of the dashed curve references to a particular value of the initial depth.

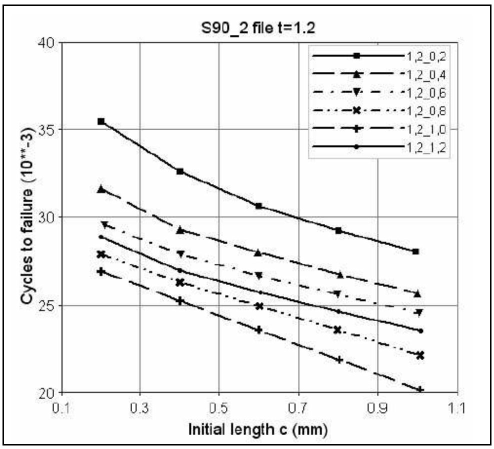

Fig. 6. Influence of the initial length of the crack on cycles to failure

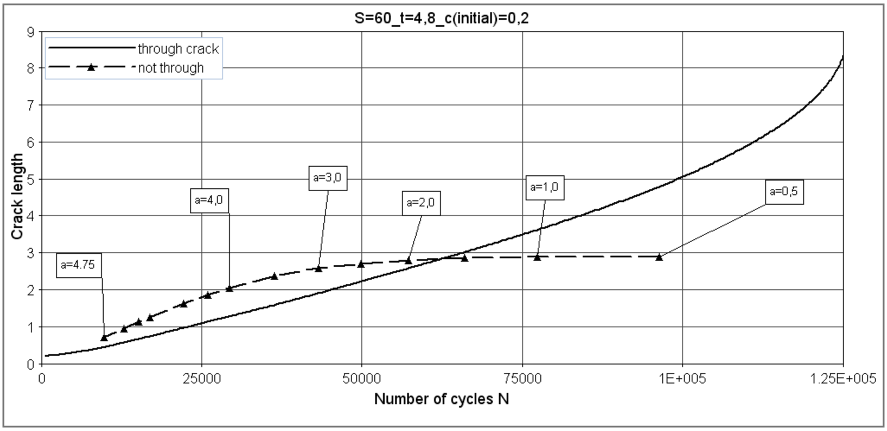

Fig. 7. Propagation behaviour of a corner and a through crack

It can be observed that beyond a certain value of the initial crack depth, depending on the sheet thickness, the length reached when the corner crack becomes a through one is larger than that obtained after the same number of cycles when starting with a through crack, and this effect is presumably connected to the bending effect of corner cracks.
For what concerns the influence exerted by the growth parameters, C and n according to the well known Paris' law, a first analysis was carried out in order to evaluate the influence of spatial randomness of propagation parameters; therefore the analysis was carried out considering that for each stage of propagation the current values of C and n were randomly extracted on the basis of a joint normal pdf between lnC and n. The results, illustrated in Fig. 8, show a strong resemblance with the well known experimental results by Wirkler.
Then an investigation was carried out about the influence of the same ruling parameters on the variance of cycles to failure. It could be shown that the mean value of the initial length has a little influence on the CV of cycles to failure, while on the contrary is largely affected by the CV of the said geometry. On the other hand, both statistical parameters of the distribution of remote stress have a deep influence on the CV of fatigue life.

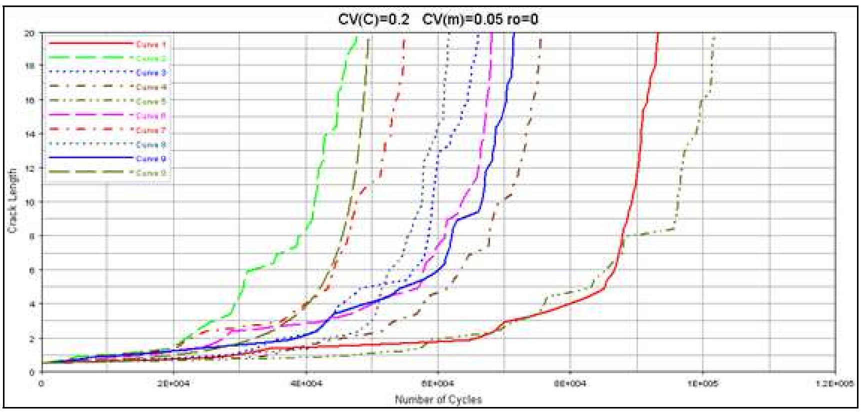

Fig. 8. Crack propagation histories with random parameters

Once the design variables were identified, the attention had to be focused on the type of structure that one wants to use as a reference; in the present case, a simple riveted lap joint for aeronautical application was chosen (fig. 9), composed by two 2024-T3 aluminium sheets, each 1 mm thick, with 3 rows of 10 columns of 5 mm rivets and a pitch of 25 mm. Several reasons suggest to analyze such a structure before beginning a really probabilistic study; for example, the state of stress induced into the component by external loads has to be evaluated and then it is important to know the interactions between existing singularities when a MSD (Multi-Site Damage) or even a WFD (Widespread Fatigue Damage) takes place. Several studies were carried out, in fact (for example, Horst, 2005), considering a probabilistic initiation of cracks followed by a deterministic propagation, on the basis that such a procedure can use very simple techniques, such as compounding (Rooke, 1986). Even if such a possibility is a very appealing one, as it is very fast, at least once the appropriate fundamental solutions have been found and recorded, some doubts arise when one comes to its feasibility.
The fundamental equation of compounding method is indeed as follows:

$$K = K^{*} + \sum\left(K_i - K^{*}\right) + K_e \tag{8}$$

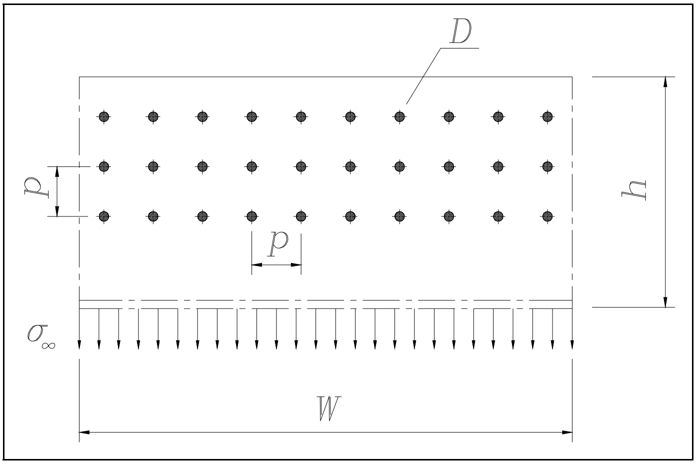

Fig. 9. The model used to study the aeronautical panel in WFD conditions

where the SIF at the crack tip of the crack we want to investigate is expressed by means of the SIF at the same location for the fundamental solution, K^*, plus the increase, with respect to the same 'fundamental' SIF, $(K_i - K^*)$, induced by each other singularity, taken one at a time, plus the effect of interactions between existing singularities, still expressed as a SIF, K_e. As the largest part of literature is related to the case of a few cracks, the K_e term is usually neglected, but that assumption appears to be too weak when dealing with WFD studies, where the singularities approach each other; therefore one of the main reasons to carry out such deterministic analysis is to verify the extent of this approximation. It must be stressed that no widely known result is available for the case of rivet-loaded holes, at least for cases matching with the object of the present analysis; even the most known papers, which we quoted above deal with the evaluation of SIF for cracks which initiate on the edge of a

loaded hole, but it is important to know the consequence of rivet load on cracks which arise elsewhere.
Another aspect, related to the previous one, is the analysis of the load carried by each pitch as damage propagates; as the compliance of partially cracked pitches increases with damage, one is inclined to guess that the mean load carried by those zones decreases, but the nonlinearity of stresses induced by geometrical singularities makes the quantitative measure of such a variation difficult to evaluate; what's more, the usual expression adopted for SIF comes from fundamental cases where just one singularity is present and it is given as a linear function of remote stress. One has to guess if such a reference variable as the stress at infinity is still meaningful in WFD cases.
Furthermore, starting to study the reference structure, an appealing idea to get a fast solution can be to decompose the structure in simple and similar details, each including one pitch, to be analyzed separately and then added together, considering each of them as a finite element or better as a finite strip; that idea induces to consider the problem of the interactions between adjacent details.
In fact, even if the structure is considered to be a two-dimensional one, the propagation of damage in different places brings the consequence of varying interactions, for both normal and shearing stresses. For all reasons above, an extensive analysis of the reference structure is to be carried out in presence of different MSD scenarios; in order to get fast solutions, use can be made of the well known BEASY® commercial code, but different cases are to be verified by means of more complex models.
On the basis of the said controls, a wide set of scenarios could be explored, with two, three and also four cracks existing at a time, using a two-dimensional DBEM model; in the present case, a 100 MPa remote stress was considered, which was transferred to the sheet through the rivets according to a 37%, 26% and 37% distribution of load, as it is usually accepted in literature; that load was applied through an opportune pressure distribution on the edge of each hole. This model, however, cannot take into account two effects, i.e. the limited compliance of holes, due to the presence of rivets and the variations of the load carried by rivets mounted in cracked holes; both those aspects, however, were considered as not very relevant, following the control runs carried out by FEM.

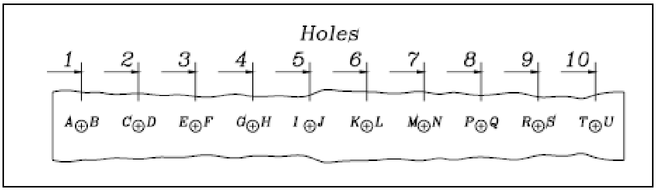

Fig. 10. The code used to represent WFD scenarios

For a better understanding of the following illustrations, one has to refer to fig. 10, where we show the code adopted to identify the cracks; each hole is numbered and each hole side is indicated by a capital letter, followed, if it is the case, by the crack length in mm; therefore, for example, E5J7P3 identifies the case when three cracks are present, the first, 5 mm long, being at the left side of the third hole (third pitch, considering sheet edges), another, 7 mm long, at the right side of the fifth hole (sixth pitch), and the last, 3 mm long, at the left side of the eight hole (eighth pitch).

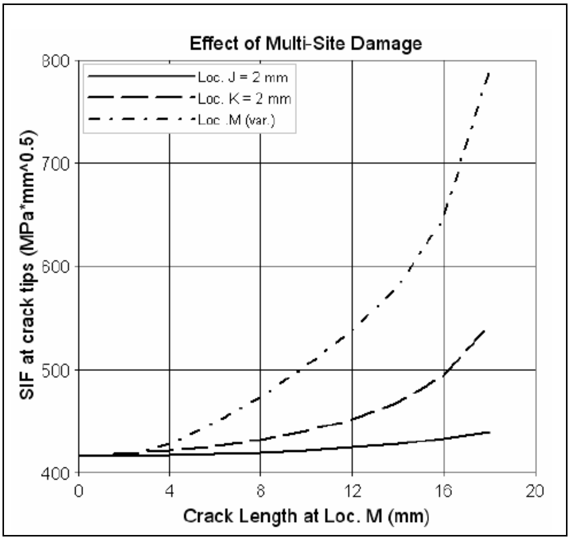

Fig. 11. Behaviour of J2K2Mx scenario

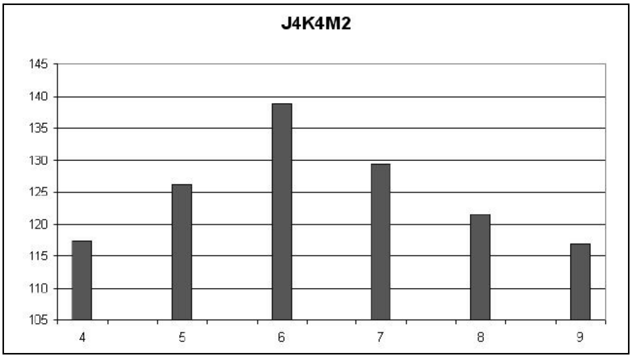

Fig. 12. Mean longitudinal stress loading different pitches for a 2 mm crack in pitch 7

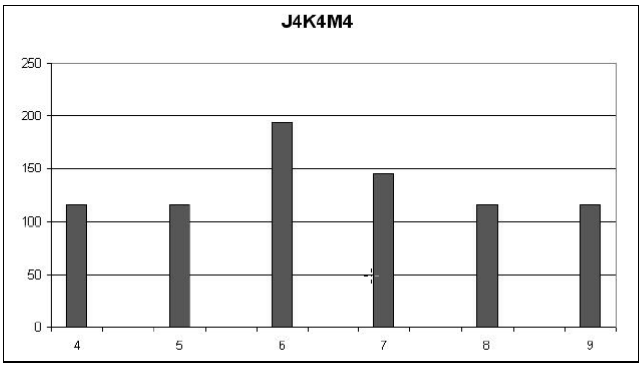

Fig. 13. Mean longitudinal stress loading different pitches for a 4 mm crack in pitch 7

In fig. 11 a three cracks scenario is represented, where in pitch 6 there are two cracks, each 2 mm long and another crack is growing at the right edge of the seventh hole, i.e. in the adjacent seventh pitch; if we consider only LEFM, we can observe that the leftmost crack (at location J) is not much influenced by the presence of the propagating crack at location M, while the central one exhibits an increase in SIF which can reach about 20%.

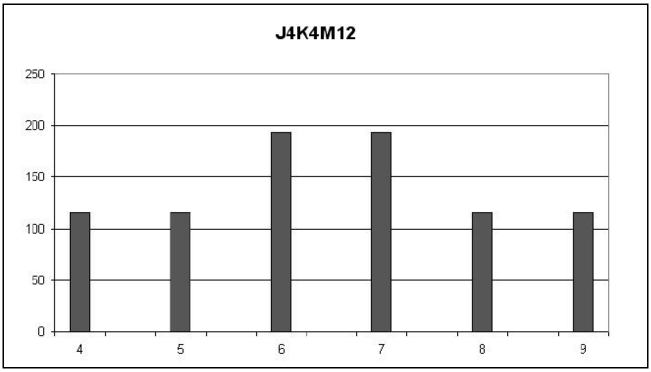

Fig. 14. Mean longitudinal stress loading different pitches for a 12 mm crack in pitch 7

The whole process can be observed by considering the mean longitudinal stress for different scenarios, as illustrated in Fig. 12, 13 and 14; in the first one, we can observe a progressive increase in the mean longitudinal stress around pitch no. 6, which is the most severely reduced and the influence of the small crack at location M is not very high.

As the length of crack in pitch 7 increases, however, the mean longitudinal stresses in both pitches 6 and 7 becomes quite similar and much higher of what is recorded in safe zones, where the same longitudinal stresses are not much increased in respect to what is recorded for a safe structure, because the transfer of load is distributed among many pitches.

The main results obtained through the previously discussed analysis can be summarized by observing that in complex scenarios high interactions exist between singularities and damaged zones, which can prevent the use of simple techniques such as compounding, but that the specific zone to be examined gets up to a single pitch beyond the cracked ones, of course on both sides. At the same time, as expected, we can observe that for WFD conditions, in presence of large cracks, the stress levels become so high that the use of LEFM can be made only from a qualitative standpoint.

If some knowledge about what to expect and how the coupled sheets will behave during the accumulation of damage has been obtained at this point of the analysis, we also realize, as pointed above, that no simple method can be used to evaluate the statistics of failure times, as different aspects will oppose and first of all the amount of the interactions between cracked holes; for that reason the only way which appears to be of some value is the direct M-C interaction as applied to the whole component, i.e. the evaluation of the 'true' history foe the sheets, to be performed the opportune number of times to extract reliable statistics; as the first problem the analyst has to overcome in such cases is the one related to the time consumption, it is of uttermost importance to use the most direct and quick techniques to obtain the desired results; for example, the use of DBEM coupled with an in-house developed code can give, if opportunely built, such guarantees.

In the version we are referring to, the structure was considered to be entirely safe at the beginning of each trial; then a damage process followed, which was considered as to be of Markow type. For the sake of brevity we shall not recall here the characters of such a process, which we consider to be widely known today; we simply mention that we have to define the initial scenario, the damage initiation criterion and the transitional probabilities for damage steps. In any case, we have to point out that other hypothesis could be assumed and first that of an initial damage state as related to EIFS (Equivalent Initial Flaw Size) or to the case of a rogue flaw, for example, don't imply any particular difficulty.
Two possible crack locations were considered at each hole, corresponding to the direction normal to the remote stress; the probability distribution of crack appearance in time was considered as lognormal, given by the following function:

$$f(N_i) = \frac{1}{\sigma_{\ln} N_i \sqrt{2\pi}} \exp\left[-\frac{1}{2} \left(\frac{\ln(N_i) - \mu_{\ln}}{\sigma_{\ln}} \right)^2 \right] \tag{10}$$

with an immediate meaning of the different parameters; it has to be noted that in our case the experimental results available in literature were adapted to obtain P-S-N curves, in order to make the statistics dependent on the stress level. At each time of the analysis the extraction of a random number for each of the still safe locations was carried out to represent the probability of damage cumulated locally and compared with the probability coming from eq. (10) above; in the positive case, a new crack was considered as initiated in the opportune location.
In order to save time, the code started to perform the search only at a time where the probability to find at least one cracked location was not less than a quantity p chosen by the user; it is well known that, if p_f is the probability of a given outcome, the probability that the same outcome is found at least for one among n cases happening simultaneously is given by:

$$p = 1 - (1 - p_f)^n ; \tag{11}$$

in our case n is the number of possible locations, thus obtaining the initial analysis time, by inverting the probability function corresponding to eq. (11) above; in our trials it was generally adopted p = 0.005, which revealed to be a conservative choice, but of course other values could also be accepted. A particular choice had also to be made about the kind and the geometry of the initial crack; it is evident that to follow the damage process accurately a defect as small as possible has to be considered, for example a fraction of mm, but in that case some difficulties arise.
For example, such a small crack would fall in the range of *short cracks* and would, therefore, require a different treatment in propagation; in order to limit our analysis to a two-dimensional case we had to consider a crack which was born as a through one and therefore we choose it to be characterized by a length equal to the thickness of the sheet, i.e., 1.0 mm in our case.
Our choice was also justified by the fact that generally the experimental tests used to define the statistics represented in eq. (10) above record the appearance of a crack when the defect reaches a given length or, if carried out on drilled specimens, even match the initiation and the failure times, considering that in such cases the propagation times are very short. Given

an opportune integration step, the same random extraction was performed in correspondence of still safe locations, up to the time (cycle) when all holes were cracked; those already initiated were considered as propagating defects, integrating Paris-Erdogan's law on the basis of SIF values recorded at the previous instant. Therefore, at each step the code looked for still safe locations, where it performed the random extraction to verify the possible initiation of defect, and at the same time, when it met a cracked location, it looked for the SIF value recorded in the previous step and, considering it as constant in the step, carried out the integration of the growth law in order to obtain the new defect length.

The core of the analysis was the coupling of the code with a DBEM module, which in our case was the commercial code BEASY®; a reference input file, representing the safe structure, was prepared by the user and submitted to the code, which analyzed the file, interpreted it and defined the possible crack locations; then, after completing the evaluations needed at the particular step, it would build a new file which contained the same structure, but as damaged as it came from the current analysis and it submitted it to BEASY®; once the DBEM run was carried out, the code read the output files, extracted the SIF values pertaining to each location and performed a new evaluation. For each ligament the analysis ended when the distance between two singularities was smaller than the plastic radius, as given by Irwin

$$r_p = \frac{K_I^2}{\pi \sigma_y^2} \tag{11}$$

where σ_y is the yield stress and K_I the mode-I SIF; that measure is adopted for cracks approaching a hole or an edge, while for the case of two concurrent cracks the limit distance is considered to be given by the sum of the plastic radiuses pertaining to the two defects. Once such limit distance was reached, the ligament was considered as broken, in the sense that no larger cracks could be formed; however, to take into account the capability of the ligament to still carry some load, even in the plastic field, the same net section was still considered in the following steps, thus renouncing to take into account the plastic behaviour of the material. Therefore, the generic M-C trial was considered as ended when one of three conditions are verified, the first being the easiest, i.e. when a limit number of cycles given by the user was reached. The second possibility was that the mean longitudinal stress evaluated in the residual net section reached the yield stress of the material and the third, obviously, was met when all ligaments were broken. Several topics are to be further specified and first of all the probabilistic capabilities of the code, which are not limited to the initiation step. The extent of the probabilistic analysis can be defined by the user, but in the general case, it refers to both loading and propagation parameters.

For the latter, user inputs the statistics of the parameters, considering a joint normal density which couples lnC and n, with a normal marginal distribution for the second parameter; at each propagation step the code extracted at each location new values to be used in the integration of the growth law.

The variation of remote stress was performed in the same way, but it was of greater consequences; first of all we have to mention that a new value of remote stress was extracted at the beginning of each step from the statistical distribution that, for the time being, we considered as a normal one, and then kept constant during the whole step: therefore, variations which occurred for shorter times went unaccounted. The problem which was met when dealing with a variable load concerned the probability of crack initiation, more than

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