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Static behaviour of natural gas and its flow in pipes

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Abstract

A general differential equation that governs static and flow behavior of a compressible fluid in horizontal, uphill and downhill inclined pipes is developed. The equation is developed by the combination of Euler equation for the steady flow of any fluid, the Darcy–Weisbach formula for lost head during fluid flow in pipes, the equation of continuity and the Colebrook friction factor equation. The classical fourth order Runge-Kutta numerical algorithm is used to solve to the new differential equation. The numerical algorithm is first programmed and applied to a problem of uphill gas flow in a vertical well. The program calculates the flowing bottom hole pressure as 2544.8 psia while the Cullender and Smith method obtains 2544 psia for the 5700 ft (above perforations) deep well

Next, the Runge-Kutta solution is transformed to a formula that is suitable for hand calculation of the static or flowing bottom hole pressure of a gas well. The new formula gives close result to that from the computer program, in the case of a flowing gas well. In the static case, the new formula predicts a bottom hole pressure of 2640 psia for the 5790 ft (including perforations) deep well. Ikoku average temperature and deviation factor method obtains 2639 psia while the Cullender and Smith method obtaines 2641 psia for the same well.. The Runge-Kutta algorithm is also used to provide a formula for the direct calculation of the pressure drop during downhill gas flow in a pipe. Comparison of results from the formula with values from a fluid mechanics text book confirmed its accuracy. The direct computation formulas of this work are faster and less tedious than the current methods. They also permit large temperature gradients just as the Cullender and Smith method.

Finally, the direct pressure transverse formulas developed in this work are combined wit the Reynolds number and the Colebrook friction factor equation to provide formulas for the direct calculation of the gas volumetric rate

Introduction

The main tasks that face Engineers and Scientists that deal with fluid behavior in pipes can be divided into two broad categories – the computation of flow rate and prediction of pressure at some section of the pipe. Whether in computation of flow rate, or in pressure transverse, the method employed is to solve the energy equation (Bernoulli equation for liquid and Euler equation for compressible fluid), simultaneously with the equation of lost head during fluid flow, the Colebrook (1938) friction factor equation for fluid flow in pipes and the equation of continuity (conservation of mass / weight). For the case of a gas the equation of state for gases is also included to account for the variation of gas volume with pressure and temperature.

In the first part of this work, the Euler equation for the steady flow of any fluid in a pipe/ conduit is combined with the Darcy – Weisbach equation for the lost head during fluid flow in pipes and the Colebrook friction factor equation. The combination yields a general differential equation applicable to any compressible fluid; in a static column, or flowing through a pipe. The pipe may be horizontal, inclined uphill or down hill.

The accuracy of the differential equation was ascertained by applying it to a problem of uphill gas flow in a vertical well. The problem came from the book of Ikoku (1984), "Natural Gas Production Engineering". The classical fourth order Runge-Kutta method was first of all programmed in FORTRAN to solve the differential equation. By use of the average temperature and gas deviation factor method, Ikoku obtained the flowing bottom hole pressure (P_{wf}) as 2543 psia for the 5700 ft well. The Cullender and Smith (1956) method that allows wide variation of temperature gave a P_{wf} of 2544 psia. The computer program obtaines the flowing bottom hole pressure (P_{wf}) as 2544.8 psia. Ouyang and Aziz (1996) developed another average temperature and deviation method for the calculation of flow rate and pressure transverse in gas wells. The average temperature and gas deviation formulas cannot be used directly to obtain pressure transverse in gas wells. The Cullender and Smith method involves numerical integration and is long and tedious to use.

The next thing in this work was to use the Runge-Kutta method to generate formulas suitable for the direct calculation of the pressure transverse in a static gas column, and in uphill and downhill dipping pipes. The accuracy of the formula is tested by application to two problems from the book of Ikoku. The first problem was prediction of static bottom hole pressure (P_{ws}). The new formula gives a P_{ws} of 2640 psia for the 5790ft deep gas well. Ikoku average pressure and gas deviation factor method gives the P_{ws} as 2639 psia, while the Cullender and Smith method gives the P_{ws} as 2641 psia. The second problem involves the calculation of flowing bottom hole pressure (P_{wf}). The new formula gives the P_{wf} as 2545 psia while the average temperature and gas deviation factor of Ikoku gives the P_{wf} as 2543 psia. The Cullender and Smith method obtains a P_{wf} of 2544 psia. The downhill formula was first tested by its application to a slight modification of a problem from the book of Giles et al.(2009). There was a close agreement between exit pressure calculated by the formula and that from the text book. The formula is also used to calculate bottom hole pressure in a gas injection well.

The direct pressure transverse formulas developed in this work are also combined wit the Reynolds number and the Colebrook friction factor equation to provide formulas for the direct calculation of the gas volumetric rate in uphill and down hill dipping pipes.

A differntial equation for static behaviour of a compressible fluid and its flow in pipes

The Euler equation is generally accepted for the flow of a compressible fluid in a pipe. The equation from Giles et al. (2009) is:

$$\frac{dp}{\gamma} + \frac{vdv}{g} \pm d\ell \sin\theta + dh_l = 0 \tag{1}$$

In equation (1), the plus sign (+) before $d\ell \sin\theta$ corresponds to the upward direction of the positive z coordinate and the minus sign (-) to the downward direction of the positive z coordinate.
The generally accepted equation for the loss of head in a pipe transporting a fluid is that of Darcy-Weisbach. The equation is:

$$H_L = \frac{f L v^2}{2gd} \tag{2}$$

The equation of continuity for compressible flow in a pipe is:

$$W = A\gamma v \tag{3}$$

Taking the first derivation of equation (3) and solving simultaneously with equation (1) and (2) we have after some simplifications,

$$\frac{dp}{d\ell} = -\frac{\left[\frac{fW^2}{2\gamma A^2 dg} \mp \gamma \sin\theta\right].}{\left[1 - \frac{W^2}{\gamma^2 A^2 g}\frac{d\gamma}{dp}\right]} \tag{4}$$

All equations used to derive equation (4) are generally accepted equations No limiting assumptions were made during the combination of these equations. Thus, equation (4) is a general differential equation that governs static behavior compressible fluid flow in a pipe. The compressible fluid can be a liquid of constant compressibility, gas or combination of gas and liquid (multiphase flow).
By noting that the compressibility of a fluid (C_f) is:

$$C_f = \frac{1}{\gamma}\frac{d\gamma}{dp} \tag{5}$$

Equation (4) can be written as:

$$\frac{dp}{d\ell} = -\frac{\left[\dfrac{fW^2}{2\gamma A^2 dg} \mp \gamma \sin\theta\right]}{\left[1 - \dfrac{W^2 C_f}{\gamma A^2 g}\right]} \tag{6}$$

Equation (6) can be simplified further for a gas.
Multiply through equation (6) by γ, then

$$\frac{\gamma dp}{d\ell} = -\frac{\left[\dfrac{fW^2}{2gA^2 dg} \mp \gamma^2 \sin\theta\right]}{\left[1 - \dfrac{W^2 C_f}{\gamma A^2 g}\right]} \tag{7}$$

The equation of state for a non-ideal gas can be written as

$$\gamma = \frac{p\ M}{zRT} \tag{8}$$

Substitution of equation (8) into equation (7) and using the fact that

$$\frac{pdp}{d\ell} = \frac{1}{2}\frac{dp^2}{d\ell}, \text{ gives}$$

$$\frac{dp^2}{d\ell} = -\frac{\left[\dfrac{fW^2\ zRT}{A^2 dMg} \mp \dfrac{2p^2 M \sin\theta}{zRT}\right]}{\left[1 - \dfrac{W^2\ zRTC_f}{MA^2 g\ p}\right]} \tag{9}$$

The cross-sectional area (A) of a pipe is

$$A^2 = \left(\frac{\pi d^2}{4}\right)^2 = \frac{\pi^2 d^4}{16} \tag{10}$$

Then equation (9) becomes:

$$\frac{dP^2}{d\ell} = -\left[\frac{1.621139\dfrac{fW^2\ zRT}{d^5\ M\ g} \mp \dfrac{2\ M\sin\theta P^2}{zRT}}{1-\dfrac{1.621139W^2\ zRTC_f}{M\ g\ d^4\ P}}\right]. \tag{11}$$

The denominator of equation (11) accounts for the effect of the change in kinetic energy during fluid flow in pipes. The kinetic effect is small and can be neglected as pointed out by previous researchers such as Ikoku (1984) and Uoyang and Aziz(1996). Where the kinetic effect is to be evaluated, the compressibility of the gas (C_f) can be calculated as follows:
For an ideal gas such as air,

$$C_f = \frac{1}{p}. \text{ For a non ideal gas, } C_f = \frac{1}{p} - \frac{1}{z}\frac{\partial z}{\partial p}.$$

Matter et al. (1975) and Ohirhian (2008) have proposed equations for the calculation of the compressibility of hydrocarbon gases. For a sweet natural gas (natural gas that contains CO_2 as major contaminant), Ohirhian (2008) has expressed the compressibility of the real gas (C_f) as:

$$C_f = \frac{K}{p}$$

For Nigerian (sweet) natural gas K = 1.0328 when p is in psia
The denominator of equation (11) can then be written as

$$1 - \frac{KW^2zRT}{M\ g\ d^4P^2} \text{ , where K = constant.}$$

Then equation (11) can be written as

$$\frac{dy}{d\ell} = \frac{(A \pm By)}{(1 - \dfrac{G}{y})} \tag{12}$$

where

$$y = p^2,\ A = \frac{1.621139fW^2zRT}{gd^5M},\ B = \frac{2M\sin\theta}{zRT},\ G = \frac{KW^2zRT}{gMd^4}.$$

The plus (+) sign in numerator of equation (12) is used for compressible uphill flow and the negative sign (-) is used for the compressible downhill flow. In both cases the z coordinate is taken positive upward. In equation (12) the pressure drop is $\sqrt{y_1 - y_2}$, with $y_1 > y_2$ and incremental length is $l_2 - l_1$. Flow occurs from point (1) to point (2). Uphill flow of gas occurs in gas transmission lines and flow from the foot of a gas well to the surface. The pressure at

the surface is usually known. Downhill flow of gas occurs in gas injection wells and gas transmission lines.
We shall illustrate the solution to the compressible flow equation by taking a problem involving an uphill flow of gas in a vertical gas well.

Computation of the variables in the gas differential equation
We need to discuss the computation of the variables that occur in the differential equation for gas before finding a suitable solution to it.. The gas deviation factor (z) can be obtained from the chart of Standing and Katz (1942). The Standing and Katz chart has been curve fitted by many researchers. The version that was used in this section of the work that of Gopal(1977). The dimensionless friction factor in the compressible flow equation is a function of relative roughness ($\in / d$) and the Reynolds number (R_N). The Reynolds number is defined as:

$$R_N = \frac{\rho v d}{\mu} = \frac{W d}{A g \mu} \tag{13}$$

The Reynolds number can also be written in terms of the gas volumetric flow rate. Then

$$W = \gamma_b Q_b$$

Since the specific weight at base condition is:

$$\gamma_b = \frac{p_b M}{z_b T_b R} = \frac{28.97 G_g p_b}{z_b T_b R} \tag{14}$$

The Reynolds number can be written as:

$$R_N = \frac{36.88575 G_g P_b Q_b}{R g d \mu_g z_b T_b} \tag{15}$$

By use of a base pressure (p_b) = 14.7psia, base temperature (T_b) = 520°R and R = 1545

$$R_N = \frac{20071 Q_b G_g}{\mu_g d} \tag{16}$$

Where d is expressed in inches, Q_b = MMSCF / Day and μ_g is in centipoises.
Ohirhian and Abu (2008) have presented a formula for the calculation of the viscosity of natural gas. The natural gas can contain impurities of CO_2 and H_2S. The formula is:

$$\mu_g = \frac{0.0109388 - 0.0088234xx - 0.00757210xx^2}{1.0 - 1.3633077xx - 0.0461989xx^2} \quad (17)$$

Where

$$xx = \frac{0.0059723p}{z\left(16.393443 - T/p\right)}$$

In equation (17) μ_g is expressed in centipoises(c p) , p in (psia) and Tin (°R)

The generally accepted equation for the calculation of the dimensionless friction factor (f) is that of Colebrook (1938). The equation is:

$$\frac{1}{\sqrt{f}} = -2\log\left(\frac{\in}{3.7d} + \frac{2.51}{R_N\sqrt{f}}\right) \quad (18)$$

The equation is non-linear and requires iterative solution. Several researchers have proposed equations for the direct calculation of f. The equation used in this work is that proposed by Ohirhian (2005). The equation is

$$f = \left[-2\log\left(a - 2b\ \log\ \left(a + bx_1\right)\right)\right]^{-2} \quad (19)$$

Where

$$a = \frac{\in}{3.7d}\ ,\ b = \frac{2.51}{R_N}.$$

$$x_1 = -1.14\log\left(\frac{\in}{d} + 0.30558\right) + 0.57\log R_N\left(0.01772\log R_N + 1.0693\right)$$

After evaluating the variables in the gas differential equation, a suitable numerical scheme can be used to it.

Solution to the gas differential equation for direct calculation of pressure transverse in static and uphill gas flow in pipes.

The classical fourth order Range Kutta method that allows large increment in the independent variable when used to solve a differential equation is used in this work. The solution by use of the Runge-Kutta method allows direct calculation of pressure transverse.. The Runge-Kutta approximate solution to the differential equation

$$\frac{dy}{dx} = f(x, y) \qquad \text{at } x = x_n \tag{20}$$

given that $y = y_o$ when $x = x_o$ is

$$y = y_o + \frac{1}{6}(k_1 + 2(k_2 + k_3) + k_4) \tag{21}$$

where

$$k_1 = Hf(x_o, y_o)$$

$$k_2 = Hf(x + \frac{1}{2}H, y_o + \frac{1}{2}k_1)$$

$$k_3 = Hf(x_o + \frac{1}{2}H,\ y_o + \frac{1}{2}k_1)$$

$$k_4 = Hf(x_o + H,\ y + k_3)$$

$$H = \frac{x_n - x_o}{n}$$

n = number of applications

The Runge-Kutta algorithm can obtain an accurate solution with a large value of H. The Runge-Kutta Algorithm can solve equation (6) or (12). The test problem used in this work is from the book of Ikoku (1984), "Natural Gas Production Engineering". Ikoku has solved this problem with some of the available methods in the literature.

Example 1

Calculate the sand face pressure (p_{wf}) of a flowing gas well from the following surface measurements.

Flow rate (Q) = 5.153 MMSCF / Day
Tubing internal diameter (d) = 1.9956in
Gas gravity (G_g) = 0.6
Depth = 5790ft (bottom of casing)

Temperature at foot of tubing (T_{wf}) = 160 ^oF

Surface temperature (T_{sf}) = 83 ^oF

Tubing head pressure (p_{tf}) = 2122 psia
Absolute roughness of tubing ($\in$) = 0.0006 in
Length of tubing (l) = 5700ft (well is vertical)

Solution

When length (ℓ) is zero, p = 2122 psia
That is $(x_o, y_o) = (0, 2122)$
By use of 1 step Runge-Kutta.

$$H = \frac{5700 - 0}{1} = 5700\text{ft.}$$

The Runge-Kutta algorithm is programmed in Fortran 77 and used to solve this problem. The program is also used to study the size of depth(length) increment needed to obtain an accurate solution by use of the Runge-Kutta method. The first output shows result for one-step Runge-Kutta (Depth increment = 5700ft). The program obtaines 2544.823 psia as the flowing bottom hole pressure (P_{wf}).

```
TUBING HEAD PRESSURE =      2122.0000000 PSIA
SURFACE TEMPERATURE =       543.0000000 DEGREE RANKINE
TEMPERATURE AT TOTAL DEPTH =    620.0000000 DEGREE RANKINE
GAS GRAVITY =   6.000000E-001
GAS FLOW RATE =          5.1530000 MMSCFD
DEPTH AT SURFACE =          .0000000 FT
TOTAL DEPTH =      5700.0000000 FT
INTERNAL TUBING DIAMETER =         1.9956000 INCHES
ROUGHNESS OF TUBING =  6.000000E-004 INCHES
INCREMENTAL DEPTH =      5700.0000000 FT

    PRESSURE PSIA        DEPTH FT

       2122.000              .000
       2544.823          5700.000
```

To check the accuracy of the Runge-Kutta algorithm for the depth increment of 5700 ft another run is made with a smaller length increment of 1000 ft. The output gives a p_{wf} of 2544.823 psia. as it is with a depth increment of 5700 ft. This confirmes that the Runge-Kutta solution can be accurate for a length increment of 5700 ft.

```
TUBING HEAD PRESSURE =      2122.0000000 PSIA
SURFACE TEMPERATURE =       543.0000000 DEGREE RANKINE
TEMPERATURE AT TOTAL DEPTH =     620.0000000 DEGREE RANKINE
GAS GRAVITY =   6.000000E-001
GAS FLOW RATE =          5.1530000 MMSCFD
DEPTH AT SURFACE =          .0000000 FT
TOTAL DEPTH =      5700.0000000 FT
INTERNAL TUBING DIAMETER =         1.9956000 INCHES
ROUGHNESS OF TUBING =  6.000000E-004 INCHES
INCREMENTAL DEPTH =      1000.0000000 FT

    PRESSURE PSIA        DEPTH FT
     2122.000                .000
     2206.614            1140.000
     2291.203            2280.000
     2375.767            3420.000
     2460.306            4560.000
     2544.823            5700.000
```

In order to determine the maximum length of pipe (depth) for which the computed P_{wf} can be considered as accurate, the depth of the test well is arbitrarily increased to 10,000ft and the program run with one step (length increment = 10,000ft). The program produces the P_{wf} as 2861.060 psia..

```
TUBING HEAD PRESSURE =      2122.0000000 PSIA
SURFACE TEMPERATURE =       543.0000000 DEGREE RANKINE
TEMPERATURE AT TOTAL DEPTH =    687.0000000 DEGREE RANKINE
GAS GRAVITY =   6.000000E-001
GAS FLOW RATE =          5.1530000 MMSCFD
DEPTH AT SURFACE =          .0000000 FT
TOTAL DEPTH =     10000.0000000 FT
INTERNAL TUBING DIAMETER =         1.9956000 INCHES
ROUGHNESS OF TUBING =  6.000000E-004 INCHES
INCREMENTAL DEPTH =     10000.0000000 FT

    PRESSURE PSIA        DEPTH FT

     2122.000                .000
     2861.060           10000.000
```

Next the total depth of 10000ft is subdivided into ten steps (length increment = 1,000ft). The program gives the P_{wf} as 2861.057 psia for the length increment of 1000ft.

```
TUBING HEAD PRESSURE =      2122.0000000 PSIA
SURFACE TEMPERATURE =       543.0000000 DEGREE RANKINE
TEMPERATURE AT TOTAL DEPTH =     687.0000000 DEGREE RANKINE
GAS GRAVITY =   6.000000E-001
GAS FLOW RATE =          5.1530000 MMSCFD
DEPTH AT SURFACE =          .0000000 FT
TOTAL DEPTH =     10000.0000000 FT
INTERNAL TUBING DIAMETER =         1.9956000 INCHES
ROUGHNESS OF TUBING =  6.000000E-004 INCHES
INCREMENTAL DEPTH =      1000.0000000 FT

    PRESSURE PSIA        DEPTH FT
     2122.000                .000
     2197.863            1000.000
     2273.246            2000.000
     2348.165            3000.000
     2422.638            4000.000
     2496.680            5000.000
     2570.311            6000.000
     2643.547            7000.000
     2716.406            8000.000
     2788.903            9000.000
     2861.057           10000.000
```

The computed values of P_{wf} for the depth increment of 10,000ft and 1000ft differ only in the third decimal place. This suggests that the depth increment for the Range - Kutta solution to the differential equation generated in this work could be a large as 10,000ft. By neglecting the denominator of equation (6) that accounts for the kinetic effect, the
result can be compared with Ikoku's average temperature and gas deviation method that uses an average value of the gas deviation factor (z) and negligible kinetic effects. In the program z is allowed to vary with pressure and temperature. The temperature in the program also varies with depth (length of tubing) as

$$T = GTG \times \text{current length} + T_{sf}, \text{ where, } GTG = \frac{(T_{wf} - T_{sf})}{\text{Total Depth}}$$

The program obtains the P_{wf} as 2544.737 psia when the kinetic effect is ignored. The output is as follows:

```
TUBING HEAD PRESSURE =      2122.0000000 PSIA
SURFACE TEMPERATURE =       543.0000000 DEGREE RANKINE
TEMPERATURE AT TOTAL DEPTH =     620.0000000 DEGREE RANKINE
GAS GRAVITY =   6.000000E-001
GAS FLOW RATE =          5.1530000 MMSCFD
DEPTH AT SURFACE =          .0000000 FT
TOTAL DEPTH =      5700.0000000 FT
INTERNAL TUBING DIAMETER =        1.9956000 INCHES
ROUGHNESS OF TUBING =  6.000000E-004 INCHES
INCREMENTAL DEPTH =      5700.0000000 FT

    PRESSURE PSIA        DEPTH FT

    2122.000                .000
    2544.737            5700.000
```

Comparing the P_{wf} of 2544.737 psia with the P_{wf} of 2544.823 psia when the kinetic effect is considered, the kinetic contribution to the pressure drop is 2544.823 psia – 2544.737psia = 0.086 psia.The kinetic effect during calculation of pressure transverse in uphill dipping pipes is small and can be neglected as pointed out by previous researchers such as Ikoku (1984) and Uoyang and Aziz(1996)
Ikoku obtained 2543 psia by use of the the average temperature and gas deviation method. The average temperature and gas deviation method goes through trial and error calculations in order to obtain an accurate solution. Ikoku also used the Cullendar and Smith method to solve the problem under consideration. The Cullendar and Smith method does not consider the kinetic effect but allows a wide variation of the temperature. The Cullendar and Smith method involves the use of Simpson rule to carry out an integration of a cumbersome function. The solution to the given problem by the Cullendar and Smith method is p_{wf} = 2544 psia.
If we neglect the denominator of equation (12), then the differential equation for pressure transverse in a flowing gas well becomes

$$\frac{dy}{dl} = A + By \tag{22}$$

where

$$A = \frac{1.621139 f W^2 zRT}{g\, d^5 M} \tag{23}$$

$$B = \frac{2M \sin\theta}{zRT} = \frac{2 \times 28.79 G_g \sin\theta}{RTz} \tag{24}$$

The equation is valid in any consistent set of units. If we assume that the pressure and temperature in the tubing are held constant from the mid section of the pipe to the foot of the tubing, the Runge-Kutta method can be used to obtain the pressure transverse in the tubing as follows.

$$\frac{dy}{d\ell} = \frac{\dfrac{46.9643686\,GgQ_b^2 fzRT}{gd^5} + \dfrac{59.940Gg\ \mathrm{Sin}\ \theta y}{zRT}}{1 - \dfrac{46.9643686KzGgQ_b^2}{gRd^4}\left(\dfrac{p_b}{T_b}\right)^2\left(\dfrac{T}{y}\right)} \tag{25}$$

The weight flow rate (W) in equation (12) is related to Q_b (the volumetric rate measurement at a base pressure (P_b) and a base temperature (T_b)) in equation (25) by:

$$W = \gamma_b Q_b \tag{26}$$

Equation (25) is a general differential equation that governs pressure transverse in a gas pipe that conveys gas uphill. When the angle of inclination (θ) is zero, $\sin\theta$ is zero and the differential equation reduces to that of a static gas column. The differential equation (25) is valid in any consistent set of units. The constant K = 1.0328 for Nigerian Natural Gas when the unit of pressure is psia.

The classical 4th order Runge Kutta alogarithm can be used to provide a formula that serves as a general solution to the differential equation (25). To achieve this, the temperature and gas deviation factors are held constant at some average value, starting from the mid section of the pipe to the inlet end of the pipe. The solution to equation (25) by the Runge Katta algorithm can be written as:

$$p_1 = \sqrt{p_2^2 + \bar{y}}. \tag{27}$$

Where

$$\bar{y} = \frac{aa}{6}\left(1 + x + 0.5x^2 + 0.36x^3\right) + \frac{p_2^2}{6}\left(4.96x + 1.48x^2 + 0.72x^3\right) + \frac{u}{6}\left(4.96 + 1.96x + 0.72x^2\right)$$

$$aa = \left(\frac{46.9643686\, G_g Q_b^2 f_2 z_2 R T_2}{gd^5} + \frac{57.94 G_g \sin\theta P_2^2}{z_2 T_z R}\right) L$$

$$u = \frac{46.9643686\, G_g Q_b^2 f_2 z_{av} T_{av} L}{gd^5}$$

$$x = \frac{57.940\, G_g \sin\theta L}{z_{av} T_{av} R}$$

When $Q_b = 0$, equation (27) reduces to the formula for pressure transverse in a static gas column.

In equation (27), the component k_4 in the Runge Kutta method given by $k_4 =$

$H f(x_o + H, y + k_3)$ was given some weighting to compensate for the fact that the temperature and gas deviation factor vary between the mid section and the inlet end of the pipe.

Equation (27) can be converted to oil field units. In oil field units in which L is in feet, R = 1545, temperature is in ^oR, g = 32.2 ft/sec^2, diameter (d) is in inches, pressure (p) is in pound per square inch (psia), flow rate (Q_b) is in MMSCF / Day, P_b = 14.7 psia and T_b = 520^o R.,the variables aa, u and x that occur in equation (25) can be written as:

$$u = \frac{25.130920 G_g Q_b^2 f_2 z_{av} T_{av} L}{d^5}$$

$$x = 0.03749 \times \frac{G_g L \sin\theta}{z_{av} T_{av}}$$

The following steps are taken in order to use equation (27) to solve a problem.

1. Evaluate the gas deviation factor at a given pressure and temperature. When equation (27) is used to calculate pressure transverse in a gas well, the given pressure and temperature are the surface temperature and gas exit pressure (tubing head pressure).
2. Evaluate the viscosity of the gas at surface condition. This step is only necessary when calculating pressure transverse in a flowing gas well. It is omitted when static pressure transverse is calculated.
3. Evaluate the Reynolds number and dimensionless friction factor by use of surface properties. This step is also omitted when considering a static gas column.
4. Evaluate the coefficient aa in the formula. This coefficient depends only on surface properties.

5. Evaluate the average pressure (p_{av}) and average temperature (T_{av}).
6. Evaluate the average gas deviation factor.(z_{av})
7. Evaluate the coefficients x and u in the formula. Note that u = 0 when $Q_b = 0$.
8. Evaluate $\bar{y}$ in the formula.
9. Evaluate the pressure p_1. In a flowing gas well, p_1 is the flowing bottom hole pressure. In a static column, it is the static bottom hole pressure.

Equation (27) is tested by using it to solve two problems from the book of Ikoku(1984), "Natural Gas Production Engineering". The first problem involves calculation of the static bottom hole in a gas well. The second involves the calculation of the flowing bottom hole pressure of a gas well.

Example 2
Calculate the static bottom hole pressure of a gas well having a depth of 5790 ft. The gas gravity is 0.6 and the pressure at the well head is 2300 psia. The surface temperature is 83°F and the average flowing temperature is 117°F.
Solution
Following the steps that were listed for the solution to a problem by use of equation (27) we have:

1. Evaluation of z – factor.
The standing equation for P_c and T_c are:

$$P_c\,(\text{psia}) = 677.0 + 15.0\,G_g - 37.5\,G_g^2$$

$$T_c\,(^\circ R) = 168.0 + 325.0\,G_g - 12.5\,G_g^2$$

Substitution of $G_g = 0.6$ gives, $P_c = 672.5$ psia and $T_c = 358.5$°R. Then $P_r = 2300/672.5 = 3.42$ and $T_r = 543/358.5 = 1.52$

The Standing and Katz chart gives $z_2 = 0.78$.
Steps 2 and 3 omitted in the static case.

4. $$aa = \left(\frac{25.13092\,G_g Q_b^2 f\, z_2 T_2}{d^5} + \frac{0.037417 G_g p_2^2 \sin\theta}{z_2 T_2} \right) L$$

Here, $G_g = 0.6$, $Q_b = 0.0$, $z_2 = 0.78$, d = 1.9956 inches, $p_2 = 2300$ psia,

$T_2 = 543$ °R and L = 5700 ft. Well is vertical, $\theta = 90°$, $\sin\theta = 1$. Substitution of the given values gives:
$aa = 0.0374917 \times 0.6 \times 2300^2 \times 5790 / (0.78 \times 543) = 1626696$

5. $p_{av} = \sqrt{2300^2 + 0.5 \times 1626696} = 2470.5$ psia

Reduced $p_{av} = 2470.5 / 672.5 = 3.68$

$T_{av} = 117\,°F = 577\,°R$

Reduced $T_{av} = 577/358.5 = 1.61$

From the standing and Katz chart, $z_{av} = 0.816$

7. In the static case u = 0, so we only evaluate x

$$x = \frac{0.0374917 \times 0.6 \times 5790 \mathrm{Sin} 90^{o}}{0.816 \times 577} = 0.2766$$

8. $$\overline{y} = \frac{aa}{6}\left(1 + x + 0.5x^2 + 0.36x^3\right) + \frac{p_2^2}{6}\left(4.96x + 1.48x^2 + 0.72x^3\right)$$

Substitution of a = 1626696, x = 0.2766 and P_2 = 2300 gives

$$\overline{y} = 358543 + 1322856 = 1681399$$

9. $$p_1 = \sqrt{p_2^2 + \overline{y}} = \left(2300^2 + 1681399\right)^{0.5} = 2640.34 \text{ psia} \approx 2640 \text{ psia}$$

Ikoku used 3 methods to work this problem. His answers of the static bottom hole pressure are:

Average temperature and deviation factor = 2639 psia
Sukkar and Cornell method = 2634 psia
Cullender and Smith method = 2641 psia

The direct calculation formula of this work is faster.

Example 3

Use equation (27) to solve the problem of example 1 that was previously solved by computer programming.

Solution

1. Obtain the gas deviation factor at the surface. From example 2, the pseudocritical properties for a 0.6 gravity gas are, P_c = 672.5 psia. and T_c = 358.5, then

$P_r = 2122 / 672.5 = 3.16$

$T_r = 543 / 358.5 = 1.52$

From the Standing and Katz chart, Z_2 =0.78

2. Obtain, the viscosity of the gas at surface condition. By use of Ohirhian and Abu equation, $$xx = \frac{0.0059723\,p}{z\left(16.393443 - \frac{T}{p}\right)} = \frac{0.0059723 \times 2122}{0.78\left(16.393443 - \frac{543}{2122}\right)} = 0.9985$$

Then $$\mu_g = \frac{0.0109388 - 0.008823(0.9985) - 0.0075720(0.9985)^2}{1.0 - 1.3633077(0.9985) - 0.0461989(0.9985)^2} = 0.0133 \text{ cp}$$

3. Evaluation of the Reynolds number and dimensionless friction factor

$$R_N \frac{20071 Q_b G_g}{\mu g d} = \frac{20071 \times 5.153 \times 0.6}{0.0133 \times 1.9956} = 2.34 \times 10^6$$

The dimensionless friction factor by Ohirhian formula is

$$f = \left[-2\log\left(a - 2b\log\left(a + bx_1\right)\right)\right]^{-2}$$

Where

$$a = \in /3.7d,\ b = 2.51/R_N$$

$$x_1 = -1.14\log\left(\frac{\in}{d} + 0.30558\right) + 0.57\ \log R_N\left(0.01772\ \log R_N + 1.0693\right)$$

Substitute of $\in = 0.0006,\ \ d = 1.9956,\ R_N = 2.34\times10^6$ gives $f = 0.01527$

4. Evaluate the coefficient aa in the formula. This coefficient depends only on surface properties.

$$aa = \left(\frac{25.13092\,G_g Q_b^2 f\, z_2 T_2}{d^5} + \frac{0.037417 G_g p_2^2 \sin\theta}{z_2 T_2}\right)L$$

Here, G_g = 0.6, Q_b = 5.153 MMSCF/Day, f = 0.01527, z_2 = 0.78, d = 1.9956 inches, p_2 = 2122 psia, T_2 = 543 °R, z = 5700 ft

Substitution of the given values gives;

aa = (81.817446 + 239.14594) × 5700 = 1829491

5. Evaluate P_{av}

$$p_{av} = \sqrt{p_2^2 + 0.5aa} = \sqrt{2122^2 + 0.5\times1829491} = 2327.6\ psia$$

6. Evaluation of average gas deviation factor.

Reduced average pressure = p_{av} / p_c = 2327.6 / 672.5 = 3.46

$$T_{av} = T_2 + \alpha L/2$$

Where α is the geothermal gradient.

$$\alpha = \left(T_1 - T_2\right)/L = \left(620 - 543\right)/5700 = 0.01351$$

T_{av} at the mid section of the pipe is 2850 ft. Then, T_{av} = 543 + 0.01351 × 2850 = 581.5 °R

Reduced T_{av} = 581.5 / 358.5 = 1.62

Standing and Katz chart gives z_{av} = 0.822

7. Evaluation of the coefficients x and u

$$x = \frac{0.0374917 G_g L}{z_{av} T_{av}} = \frac{0.0374919\times0.6\times5700}{0.822\times581.5} = 0.26824$$

$$u = \frac{25.13092 G_g Q_b^2 f\ z_{av} T_{av}\, L}{d^5}$$

$$= \frac{25.13092\times0.6\times5.153^2\times0.01527\times0.822\times581.5\times5700}{1.9956^5} = 526662$$

8. Evaluate $\bar{y}$

$$\bar{y} = \frac{aa}{6}\left(1 + x + 0.5x^2 + 0.36x^3\right) + \frac{p_2^2}{6}\left(4.96x + 1.48x^2 + 0.72x^3\right) + \frac{u}{6}\left(4.96 + 1.96 + 0.72x^2\right)$$

Where u = 526662, x = 0.26824, p_2 = 2122 psia and aa = 1829491. Then,

$$\bar{y} = 399794 + 1088840 + 485752 = 1974386\, psia^2$$

9. Evaluate p_1 (the flowing bottom hole pressure)

$$p_1 = \sqrt{p_2 + \bar{y}}$$
$$= \sqrt{2122^2 + 1974386} = 2545.05 \text{ psia}$$
$$\approx 2545 \text{ psia}$$

The computer program obtains, the flowing bottom hole pressure as 2544.823 psia. For comparison with other methods of solution, the flowing bottom hole pressure by:

Average Temperature and Deviation Factor, P_1 = 2543 psia

Cullender and Smith, P_1 = 2544

The direct calculating formula of this work is faster. The Cullendar and Smith method is even more cumbersome than that of Ikoku.t involves the use of special tables and charts (Ikoku, 1984) page 338 - 344.

The differential equation for static gas behaviour and its downhill flow in pipes

The problem of calculating pressure transverse during downhill gas flow in pipes is encountered in the transportation of gas to the market and in gas injection operations. In the literature, models for pressure prediction during downhill gas flow are rare and in many instances the same equations for uphill flow are used for downhill flow.

In this section, we present the use of the Runge-Kutta solution to the downhill gas flow differential equation.

During downhill gas flow in pipes, the negative sign in the numerator of differential equation (12) is used..The differential equation also breaks down to a simple differential equation for pressure transverse in static columns when the flow rate is zero. The equation to be solved is:

$$\frac{dy}{d\ell} = \frac{(A - By)}{(1 - \frac{G}{y})} \tag{28}$$

Where $y = p^2$,

$$A = \frac{1.621139\, f\, W^2 zRT}{gd^5 M},\; B = \frac{2M \sin\theta}{zRT},\; G = \frac{KW^2 zRT}{gMd^4}$$

Also, the molecular weight (M) of a gas, can be expressed as M = 28.97Gg.