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Preface

The area of adaptive control has grown to be one of the richest in terms of algorithms, design techniques, analytical tools, and modifications. Several books and research monographs already exist on the topics of parameter estimation and adaptive control.

Despite this rich literature, the field of adaptive control may easily appear to an outsider as a collection of unrelated tricks and modifications. Students are often overwhelmed and sometimes confused by the vast number of what appear to be unrelated designs and analytical methods achieving similar results. Researchers concentrating on different approaches in adaptive control often find it difficult to relate their techniques with others without additional research efforts.

The purpose of this book is to alleviate some of the confusion and difficulty in understanding the design, analysis, and robustness of a wide class of adaptive control for continuous-time plants. The book is the outcome of several years of research, whose main purpose was not to generate new results, but rather unify, simplify, and present in a tutorial manner most of the existing techniques for designing and analyzing adaptive control systems.

The book is written in a self-contained fashion to be used as a textbook on adaptive systems at the senior undergraduate, or first and second graduate level. It is assumed that the reader is familiar with the materials taught in undergraduate courses on linear systems, differential equations, and automatic control. The book is also useful for an industrial audience where the interest is to implement adaptive control rather than analyze its stability properties. Tables with descriptions of adaptive control schemes presented in the book are meant to serve this audience. The personal computer floppy disk, included with the book, provides several examples of simple adaptive control systems that will help the reader understand some of the implementation aspects of adaptive systems.

A significant part of the book, devoted to parameter estimation and learning in general, provides techniques and algorithms for on-line fitting of dynamic or static models to data generated by real systems. The tools for design and analysis presented in the book are very valuable in understanding and analyzing similar parameter estimation problems that appear in neural networks, fuzzy systems, and other universal approximators. The book will be of great interest to the neural and fuzzy logic audience who will benefit from the strong similarity that exists between adaptive systems, whose stability properties are well established, and neural networks, fuzzy logic systems where stability and convergence issues are yet to be resolved.

The book is organized as follows: Chapter 1 is used to introduce adaptive control as a method for controlling plants with parametric uncertainty. It also provides some background and a brief history of the development of adaptive control. Chapter 2 presents a review of various plant model representations that are useful for parameter identification and control. A considerable number of stability results that are useful in analyzing and understanding the properties of adaptive and nonlinear systems in general are presented in Chapter 3. Chapter 4 deals with the design and analysis of on-line parameter estimators or adaptive laws that form the backbone of every adaptive control scheme presented in the chapters to follow. The design of parameter identifiers and adaptive observers for stable plants is presented in Chapter 5. Chapter 6 is devoted to the design and analysis of a wide class of model reference adaptive controllers for minimum phase plants. The design of adaptive control for plants that are not necessarily minimum phase is presented in Chapter 7. These schemes are based on pole placement control strategies and are referred to as adaptive pole placement control. While Chapters 4 through 7 deal with plant models that are free of disturbances, unmodeled dynamics and noise, Chapters 8 and 9 deal with the robustness issues in adaptive control when plant model uncertainties, such as bounded disturbances and unmodeled dynamics, are present.

The book can be used in various ways. The reader who is familiar with stability and linear systems may start from Chapter 4. An introductory course in adaptive control could be covered in Chapters 1, 2, and 4 to 9, by excluding the more elaborate and difficult proofs of theorems that are presented either in the last section of chapters or in the appendices. Chapter 3 could be used for reference and for covering relevant stability results that arise during the course. A higher-level course intended for graduate students that are interested in a deeper understanding of adaptive control could cover all chapters with more emphasis on the design and stability proofs. A course for an industrial audience could contain Chapters 1, 2, and 4 to 9 with emphasis on the design of adaptive control algorithms rather than stability proofs and convergence.

Acknowledgments

The writing of this book has been surprisingly difficult and took a long time to evolve to its present form. Several versions of the book were completed only to be put aside after realizing that new results and techniques would lead to a better version. In the meantime, both of us started our families that soon enough expanded. If it were not for our families, we probably could have finished the book a year or two earlier. Their love and company, however, served as an insurance that we would finish it one day.

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We would especially like to express our deepest appreciation to Laurent Praly and Kostas Tsakalis. Laurent was the first researcher to recognize and publicize the beneficial effects of dynamic normalization on robustness that opened the way to a wide class of robust adaptive control algorithms addressed in the book. His interactions with us and our students is highly appreciated. Kostas, a former student of the first author, is responsible for many mathematical tools and stability arguments used in Chapters 6 and

9. His continuous interactions helped us to decipher many of the cryptic concepts and robustness properties of model reference adaptive control.

We are thankful to our former and current students and visitors who collaborated with us in research and contributed to this work: Farid Ahmed-Zaid, C. C. Chien, Aniruddha Datta, Marios Polycarpou, Houmair Raza, Alex Stotsky, Tim Sun, Hualin Tan, Gang Tao, Hui Wang, Tom Xu, and Youping Zhang. We are grateful to many colleagues for stimulating discussions at conferences, workshops, and meetings. They have helped us broaden our understanding of the field. In particular, we would like to mention Anu Annaswamy, Erwei Bai, Bob Bitmead, Marc Bodson, Stephen Boyd, Sara Dasgupta, the late Howard Elliot, Li-chen Fu, Fouad Giri, David Hill, Ioannis Kanellakopoulos, Pramod Khargonekar, Hassan Khalil, Bob Kosut, Jim Krause, Miroslav Krstić, Rogelio Lozano-Leal, Iven Mareels, Rick Middleton, David Mudget, Romeo Ortega, Brad Riedle, Charles Rohrs, Ali Saberi, Shankar Sastry, Lena Valavani, Jim Winkelman, and Erik Ydstie. We would also like to extend our thanks to our colleagues at the University of Southern California, Wayne State University, and Ford Research Laboratory for their friendship, support, and technical interactions. Special thanks, on behalf of the second author, go to the members of the Control Systems Department of Ford Research Laboratory, and Jessy Grizzle and Anna Stefanopoulou of the University of Michigan.

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Petros A. Ioannou
Jing Sun

List of Acronyms

ALQC	Adaptive linear quadratic control
APPC	Adaptive pole placement control
B-G	Bellman Gronwall (lemma)
BIBO	Bounded-input bounded-output
CEC	Certainty equivalence control
I/O	Input/output
LKY	Lefschetz-Kalman-Yakubovich (lemma)
LQ	Linear quadratic
LTI	Linear time invariant
LTV	Linear time varying
MIMO	Multi-input multi-output
MKY	Meyer-Kalman-Yakubovich (lemma)
MRAC	Model reference adaptive control
MRC	Model reference control
PE	Persistently exciting
PI	Proportional plus integral
PPC	Pole placement control
PR	Positive real
SISO	Single input single output
SPR	Strictly positive real
TV	Time varying
UCO	Uniformly completely observable
a.s.	Asymptotically stable
e.s.	Exponentially stable
m.s.s.	(In the) mean square sense
u.a.s.	Uniformly asymptotically stable
u.b.	Uniformly bounded
u.s.	Uniformly stable
u.u.b.	Uniformly ultimately bounded
w.r.t.	With respect to

Chapter 1

Introduction

1.1 Control System Design Steps

The design of a controller that can alter or modify the behavior and response of an unknown plant to meet certain performance requirements can be a tedious and challenging problem in many control applications. By plant, we mean any process characterized by a certain number of inputs u and outputs y, as shown in Figure 1.1.

The plant inputs u are processed to produce several plant outputs y that represent the measured output response of the plant. The control design task is to choose the input u so that the output response $y(t)$ satisfies certain given performance requirements. Because the plant process is usually complex, i.e., it may consist of various mechanical, electronic, hydraulic parts, etc., the appropriate choice of u is in general not straightforward. The control design steps often followed by most control engineers in choosing the input u are shown in Figure 1.2 and are explained below.

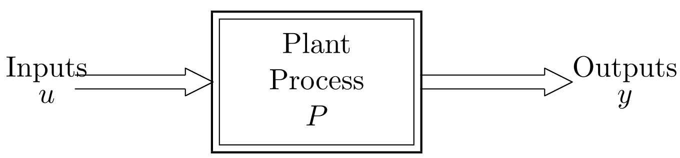

Figure 1.1 Plant representation.

Step 1. Modeling

The task of the control engineer in this step is to understand the processing mechanism of the plant, which takes a given input signal $u(t)$ and produces the output response $y(t)$, to the point that he or she can describe it in the form of some mathematical equations. These equations constitute the mathematical model of the plant. An exact plant model should produce the same output response as the plant, provided the input to the model and initial conditions are exactly the same as those of the plant. The complexity of most physical plants, however, makes the development of such an exact model unwarranted or even impossible. But even if the exact plant model becomes available, its dimension is likely to be infinite, and its description nonlinear or time varying to the point that its usefulness from the control design viewpoint is minimal or none. This makes the task of modeling even more difficult and challenging, because the control engineer has to come up with a mathematical model that describes accurately the input/output behavior of the plant and yet is simple enough to be used for control design purposes. A simple model usually leads to a simple controller that is easier to understand and implement, and often more reliable for practical purposes.

A plant model may be developed by using physical laws or by processing the plant input/output (I/O) data obtained by performing various experiments. Such a model, however, may still be complicated enough from the control design viewpoint and further simplifications may be necessary. Some of the approaches often used to obtain a simplified model are

(i) Linearization around operating points
(ii) Model order reduction techniques

In approach (i) the plant is approximated by a linear model that is valid around a given operating point. Different operating points may lead to several different linear models that are used as plant models. Linearization is achieved by using Taylor's series expansion and approximation, fitting of experimental data to a linear model, etc.

In approach (ii) small effects and phenomena outside the frequency range of interest are neglected leading to a lower order and simpler plant model. The reader is referred to references [67, 106] for more details on model reduction techniques and approximations.

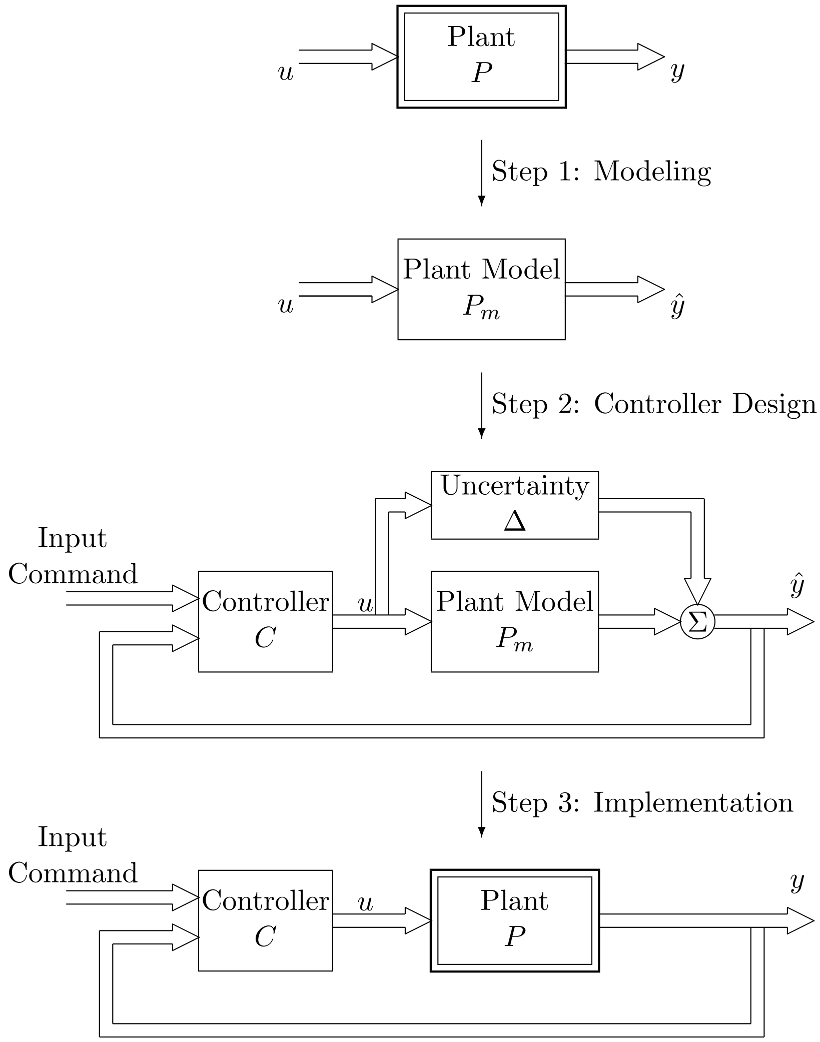

Figure 1.2 Control system design steps.

In general, the task of modeling involves a good understanding of the plant process and performance requirements, and may require some experience from the part of the control engineer.

Step 2. Controller Design

Once a model of the plant is available, one can proceed with the controller design. The controller is designed to meet the performance requirements for