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On the Adaptive Tracking Control of 3-D Overhead Crane Systems

Yang, Jung Hua
National Pingtung University of Science and Technology
Pingtung, Taiwan

1. Introduction

For low cost, easy assembly and less maintenance, overhead crane systems have been widely used for material transportation in many industrial applications. Due to the requirements of high positioning accuracy, small swing angle, short transportation time, and high safety, both motion and stabilization control for an overhead crane system becomes an interesting issue in the field of control technology development. Since the overhead crane system is underactuated with respect to the sway motion, it is very difficult to operate an overhead traveling crane automatically in a desired manner. In general, human drivers, often assisted by automatic anti-sway system, are always involved in the operation of overhead crane systems, and the resulting performance, in terms of swiftness and safety, heavily depends on their experience and capability. For this reason, a growing interest is arising about the design of automatic control systems for overhead cranes. However, severely nonlinear dynamic properties as well as lack of actual control input for the sway motion might bring about undesired significant sway oscillations, especially at take-off and arrival phases. In addition, these undesirable phenomena would also make the conventional control strategies fail to achieve the goal. Hence, the overhead crane systems belong to the category of incomplete control system, which only allow a limited number of inputs to control more outputs. In such a case, the uncontrollable oscillations might cause severe stability and safety problems, and would strongly constrain the operation efficiency as well as the application domain. Furthermore, an overhead crane system may experience a range of parameter variations under different loading condition. Therefore, a robust and delicate controller, which is able to diminish these unfavorable sway and uncertainties, needs to be developed not only to enhance both efficiency and safety, but to make the system more applicable to other engineering scopes.

The overhead crane system is non-minimum phase (or has unstable zeros in linear case) if a nonlinear state feedback can hold the system output identically zero while the internal dynamics become unstable. Output tracking control of non-minimum phase systems is a highly challenging problem encountered in many practical engineering applications such as aircraft control [1], marine vehicle control [2], flexible link manipulator control [3], inverted pendulum system control [4]. The non-minimum phase property has long been recognized to be a major obstacle in many control problems. It is well known that unstable zeros cannot be moved with state feedback while the poles can be arbitrarily placed (if completely controllable). In most standard adaptive control as well as in nonlinear adaptive control, all algorithms require that the plant to be minimum phase. This chapter presents a new procedure for designing output tracking controller for non-minimum phase systems (The overhead crane systems).

Several researchers have dealt with the modeling and control problems of overhead crane system. In [5], a simple proportional derivative (PD) controller is designed to asymptotically regulate the overhead crane system to the desired position with natural damping of sway oscillation. In [6], the authors propose an output feedback proportional derivative controller that stabilizes a nonlinear crane system. In [7], the authors proposed an indirect adaptive scheme, based on dynamic feedback linearization techniques, which was applied to overhead crane systems with two control input. In [8], Li *et al* attacked the under-actuated problem by blending four local controllers into one overall control strategy; moreover, experimental results delineating the performance of the controller were also provided. In [9], a nonlinear controller is proposed for the trolley crane systems using Lyapunov functions and a modified version of sliding-surface control is then utilized to achieve the objective of cart position control. However, the sway angle dynamics has not been considered for stability analysis. In [10], the authors proposed a saturation control law based on a guaranteed cost control method for a linearized version of 2-DOF crane system dynamics. In [11], the authors designed a nonlinear controller for regulating the swinging energy of the payload. In [12], a fuzzy logic control system with sliding mode Control concept is developed for an overhead crane system. Y. Fang *et al*. [13] develop a nonlinear coupling control law to stabilize a 3-DOF overhead crane system by using LaSalle invariance theorem. However, the system parameters must be known in advance. Ishide *et al.* [14] train a fuzzy neural network control architecture for an overhead traveling crane by using back-propagation method. However, the trolley speed is still large even when the destination is arrived, which would result in significant residual sway motion, low safety, and poor positioning accuracy. In the paper [15], a nonlinear tracking controller for the load position and velocity is designed with two loops: an outer loop for position tracking, and an inner loop for stabilizing the internally oscillatory dynamics using a singular perturbation design. But the result is available only when the sway angle dynamics is much faster than the cart motion dynamics. In the paper [16], a simple control scheme, based on second-order sliding modes, guarantees a fast precise load transfer and swing suppression during the load movement, despite of model uncertainties. In the paper [17], it proposes a stabilizing nonlinear control law for a crane system having constrained trolley stroke and pendulum length using the Lyapunov's second method and performs some numerical experiments to examine the validity of the control law. In the paper [18], the variable structure control scheme is used to regulate the trolley position and the hoisting motion towards their desired values. However the input torques exhibit a lot of chattering. This chattering is not desirable as it might shorten the lifetime of the motors used to drive the crane. In the paper [19], a new fuzzy controller for anti-swing and position control of an overhead traveling crane is proposed based on the Single Input Rule Modules (SIRMs). Computer simulation results show that, by using the fuzzy controller, the crane can be smoothly driven to the destination in a short time with low swing angle and almost no overshoot. D. Liu *et al.* [20] present a practical solution to analyze and control the overhead crane. A sliding mode fuzzy control algorithm is designed for both X-direction and Y-direction transports of the overhead crane. Incorporating the robustness characteristics of SMC and FLC, the proposed control law can guarantee a swing-free transportation. J.A. Mendez *et al.* [21] deal with the design and implementation of a self-tuning controller for an overhead crane. The proposed neurocontroller is a self-tuning system consisting of a conventional controller combined with a **NN** to calculate the coefficients of the controller *on-line.* The aim of the proposed scheme is to reduce the training-time of the controller in order to make the real-time application of this algorithm possible. Ho-Hoon Lee *et al.* [22] proposes a new approach for the anti-swing control of overhead cranes, where a model-based control scheme is designed based on a V-shaped Lyapunov function. The proposed control is free from the conventional constraints of small load mass, small load swing, slow hoisting speed, and small hoisting distance, but only guarantees asymptotic stability with all internal signals bounded. This paper also proposes a practical trajectory generation method for a near minimum-time control, which is independent of hoisting speed and distance. In this paper [23], robustness of the proposed intelligent gantry crane system is evaluated. The evaluation result showed that the intelligent gantry crane system not only has produced good performances compared with the automatic crane system controlled by classical PID controllers but also is more robust to parameter variation than the automatic crane system controlled by classical PID controllers. In this paper [24], the I-PD and PD controllers designed by using the CRA method for the trolley position and load swing angle of overhead crane system have been proposed. The advantage of CRA method for designing the control system so that the system performances are satisfied not only in the transient responses but also in the steady-state responses, have also been confirmed by the simulation results.

Although most of the control schemes mentioned above have claimed an adaptive stabilizing tracking/regulation for the crane motion, the stability of the sway angle dynamics is hardly taken into account. Hence, in this chapter, a nonlinear control scheme which incorporates both the cart motion dynamics and sway angle dynamics is devised to ensure the overall closed-loop system stability. Stability proof of the overall system is guaranteed via Lyapunov analysis. To demonstrate the effectiveness of the proposed control schemes, the overhead crane system is set up and satisfactory experimental results are also given.

2. Dynamic Model of Overhead Crane

The aim of this section is to drive the dynamic model of the overhead crane system. The model is dived using Lagrangian method. The schematic plotted in Figure 1 represents a three degree of freedom overhead crane system. To facilitate the control development, the following assumptions with regard to the dynamic model used to describe the motion of overhead crane system will be made. The dynamic model for a three degree of freedom (3-DOF) overhead crane system (see Figure 1) is assumed to have the following postulates.

A1: The payload and the gantry are connected by a mass-less, rigid link.

A2: The angular position and velocity of the payload and the rectilinear position and velocity of the gantry are measurable.

A3: The payload mass is concentrated at a point and the value of this mass is exactly known; moreover, the gantry mass and the length of the connecting rod are exactly known.

A4: The hinged joint that connects the payload link to the gantry is frictionless.

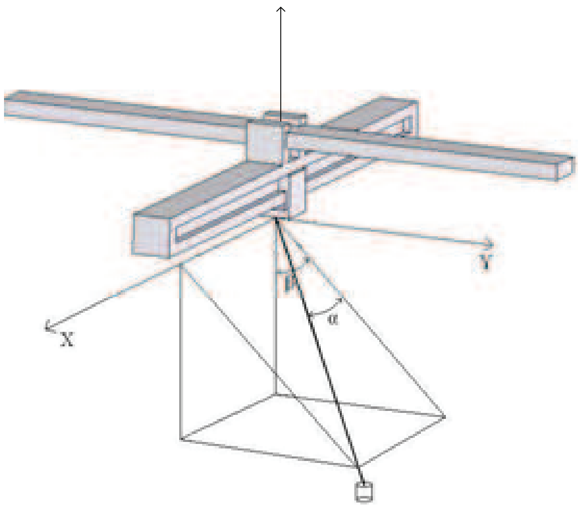

Fig. 1. 3-D Overhead Crane System

The 3-D crane system will be derived based on Lagrange-Euler approach. Consider the 3-dimensional overhead crane system as shown in Figure 1. The cart can move horizontally in x-y plane, in which the moving distance of the cart along the X-rail is denoted as *x(t)* and the distance on the Y-rail measured from the initial point of the construction frame is denoted as *y(t)*. The length of the lift line is denoted as *l*. Define the angle between the lift line and its projection on the y-z plane as $\alpha(t)$ and the angle between the projection line and the negative axis as $\beta(t)$. Then the kinetic energy and potential energy of the system can be found in Equation (1.1) and (1.2), respectively and be expressed as the following equations.

$$K = \frac{1}{2}m_1\dot{x}^2 + \frac{1}{2}(m_1 + m_2)\dot{y}^2 + \frac{1}{2}m_c(\dot{x}_c^2 + \dot{y}_c^2 + \dot{z}_c^2) \tag{1}$$

$$V = -mgl\cos\alpha\cos\beta \tag{2}$$

where x_c, y_c are the related positions of the load described in the Cartesian coordinate, which can be mathematically written as

$$x_c = x + l\,sin\,\alpha \tag{3}$$

$$y_c = y + l\cos\alpha\sin\beta \tag{4}$$

$$z_c = -l\cos\alpha\cos\beta \tag{5}$$

The following equations express the velocities by taking the time derivative of above equations

$$\dot{x}_c = \dot{x} + l\dot{\alpha}\cos\alpha \tag{6}$$

$$\dot{y}_c = \dot{y} - l\dot{\alpha}\sin\alpha\sin\beta + l\dot{\beta}\cos\alpha\cos\beta \tag{7}$$

$$\dot{z}_c = -l\dot{\alpha}\sin\alpha\cos\beta - l\dot{\beta}\cos\alpha\sin\beta \tag{8}$$

By using the Lagrange-Euler formulation,

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_i}\right) - \frac{\partial L}{\partial q_i} = \tau_i, i = 1,2,3,4. \tag{9}$$

where $L = K - V$, q_i is the element of vector $q = [x \quad y \quad \alpha \quad \beta]^T$ and τ_i is the corresponding external input to the system, we have the following mathematical representation which formulates the system motion

$$M(q)\ddot{q} + C(q,\dot{q}) + G(q) = \tau \tag{10}$$

where $M(q) \in R^{4\times4}$ is inertia matrix of the crane system, $C(q,\dot{q}) \in R^{4\times1}$ is the nonlinear terms coming from the coupling of linear and rotational motion, $G(q) \in R^{4\times1}$ is the terms due to gravity, and $\tau = [u_x \quad u_y \quad 0 \quad 0]^T$ is the input vector.

As mentioned previously, the dynamic equation of motion described the overhead crane system also have the same properties as follows

P1: The inertia matrix $M(q)$ is symmetric and positive definite for all $q \in R^n$.

P2: There exists a matrix $\dot{B}(q,\dot{q})$ such that $C(q,\dot{q}) = B(q,\dot{q})\dot{q}$, and $\forall x \in R^4$ $x^T(\dot{M} - 2B)x = 0$, i.e., $\dot{M} - 2B$ is skew-symmetric. $B(q,\dot{q}_x)\dot{q}_y = B(q,\dot{q}_y)\dot{q}_x$.

P3: The parameters of the system can be linearly extracted as

$$M(q)\ddot{q} + C(q,\dot{q}) + G(q) = W_f(q,\dot{q},\ddot{q})\Phi_f \tag{11}$$

where $W_f(q,\dot{q},\ddot{q})$ is the regressor matrix and Φ_f is a vector containing the system parameters.

Dynamic Model of Overhead Crane

In this section, an adaptive control scheme will be developed for the position tracking of an overhead crane system.

2.1 Model formulation

For design convenience, a general coordinate is defined as follows

$q^T = [q_p^T \quad q_\theta^T]$

where

$q_p^T = [x \quad y]$, $q_\theta^T = [\alpha \quad \beta]$

and using the relations in **P2,** the dynamic equation of an overhead crane (10) is partitioned in the following form

$$\begin{bmatrix} M_{pp} & M_{p\theta} \\ M_{p\theta}^T & M_{\theta\theta} \end{bmatrix} \begin{bmatrix} \ddot{q}_p \\ \ddot{q}_\theta \end{bmatrix} + \begin{bmatrix} B_{pp} & B_{p\theta} \\ B_{\theta p} & B_{\theta\theta} \end{bmatrix} \begin{bmatrix} \dot{q}_p \\ \dot{q}_\theta \end{bmatrix} + \begin{bmatrix} G_p(q) \\ G_\theta(q) \end{bmatrix} = \begin{bmatrix} u_p \\ 0 \end{bmatrix} \tag{12}$$

where M_{pp}, $M_{p\theta}$, $M_{\theta\theta}$, B_{pp}, $B_{p\theta}$, $B_{\theta p}$, $B_{\theta\theta}$ are 2×2 matrices partitioned from the inertia matrix $M(q)$ and the matrix $B(q,\dot{q})$, respectively, G_p, G_θ are 2×1 vectors, and $u_p^T = [u_x \quad u_y]$. Before investigating the controller design, let the error signals be defined as

$$e = q - q_d = [e_p^T \quad e_\theta^T]^T \tag{13}$$

and the stable hypersurface plane is defined as

$$s = \dot{e} + Ke = \begin{bmatrix} \dot{e}_p + K_p e_p \\ \dot{e}_\theta + K_\theta e_\theta \end{bmatrix} = \begin{bmatrix} s_p \\ s_\theta \end{bmatrix} \tag{14}$$

where

$$e_p = q_p - q_{pd} = [x - x_d \quad y - y_d]^T \equiv [e_x \quad e_y]^T,$$

$$e_\theta = q_\theta - q_{\theta d} = [\alpha - \alpha_d \quad \beta - \beta_d]^T \equiv [e_\alpha \quad e_\beta]^T,$$

$$K_p = \begin{bmatrix} k_1 & 0 \\ 0 & k_2 \end{bmatrix}, \ K_\theta = \begin{bmatrix} k_3 & 0 \\ 0 & k_4 \end{bmatrix}$$

and x_d, y_d, α_d and β_d are defined trajectories of x, y, α and β respectively, and K_p, K_θ are some arbitrary positive definite matrices.

Then, after a lot of mathematical arrangements, the dynamics of the newly defined signal vectors s_p, s_θ can be derived as

$$\begin{bmatrix} M_{PP} & M_{P\theta} \\ M_{P\theta}^{T} & M_{\theta\theta} \end{bmatrix} \begin{bmatrix} \dot{s}_p \\ \dot{s}_\vartheta \end{bmatrix} + \begin{bmatrix} B_{PP} & B_{P\theta} \\ B_{\theta P} & B_{\theta\theta} \end{bmatrix} \begin{bmatrix} s_p \\ s_\theta \end{bmatrix} = \begin{bmatrix} \tau_p + u_P \\ \tau_\theta \end{bmatrix} \quad (15)$$

where

$$\tau_p = M_{pp}(-\ddot{q}_{pd} + k_p\dot{e}_p) + M_{p\theta}(-\ddot{q}_\theta + k_\theta\dot{e}_\theta) + B_{pp}(-\dot{q}_{pd} + k_p e_p) + B_{p\theta}(-\dot{q}_\theta + k_p e_\theta) \quad (16)$$

$$\tau_\theta = M_{\theta p}(-\ddot{q}_{pd} + k_p\dot{e}_p) + M_{\theta\theta}(-\ddot{q}_\theta + k_\theta\dot{e}_\theta) + B_{\theta p}(-\dot{q}_{pd} + k_p e_p) + B_{\theta\theta}(-\dot{q}_\theta + k_\theta e_\theta) \quad (17)$$

Remark 1: The desired trajectories x_d, y_d, α_d and β_d should be carefully chosen so as to satisfy the internal dynamics, as shown in the lower part of equation (15), when the control objective is achieved, i.e.,

$$M_{P\theta}{}^{T}(q_d)\begin{bmatrix} \ddot{x}_d \\ \ddot{y}_d \end{bmatrix} + M_{\theta\theta}(q_d)\begin{bmatrix} \ddot{\alpha}_d \\ \ddot{\beta}_d \end{bmatrix} + B_{\theta P}(q,\dot{q})\begin{bmatrix} \dot{x}_d \\ \dot{y}_d \end{bmatrix} + B_{\theta\theta}(q,\dot{q})\begin{bmatrix} \dot{\alpha}_d \\ \dot{\beta}_d \end{bmatrix} + G_\theta(q) = 0 \quad (18)$$

Without loss of generality, we always choose an exponentially-convergent trajectories with final constant values for x_d, y_d and zero for α_d, β_d.

2.2 Adaptive Controller Design

In this subsection, an adaptive nonlinear control will be presented to solve the tracking control problem.

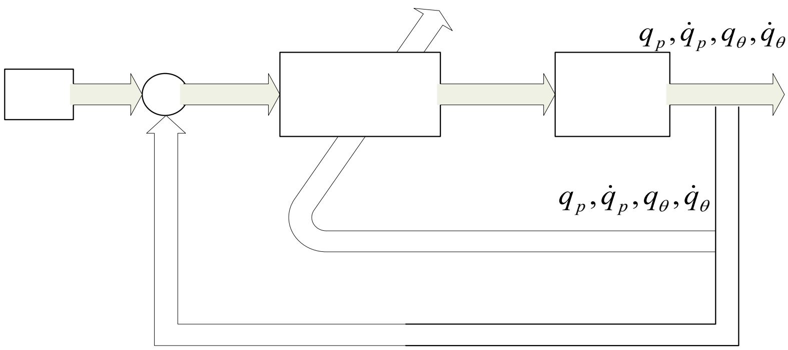

Fig. 2. An Adaptive Self-tuning Controller Block Diagram

As indicated by property **P3** in section 1.2, the dynamic equations of an overhead crane have the well-known linear-in-parameter property. Thus, we define

$$\omega_1\phi_1 = M_{pp}(\ddot{q}_{pd} + k_p\dot{e}_p) + M_{p\theta}(-\ddot{q}_{\theta d}k_\theta\dot{e}_\theta) + B_{pp}(\dot{q}_{pd} + k_p e_p) + B_{p\theta}k_p e_p \tag{19}$$

$$\omega_2\phi_2 = M_{\theta p}(\ddot{q}_{pd} + k_p\dot{e}_p) + M_{\theta\theta}(k_\theta\dot{e}_\theta) + B_{\theta p}(\dot{q}_{pd} + k_p e_p) + B_{\theta\theta}k_\theta e_\theta \tag{20}$$

where ω_1, ω_2 are regressor matrices with appropriate dimensions, and ϕ_1, ϕ_2 are their corresponding vectors of unknown constant parameters, respectively. As a majority of the adaptive controller, the following signal is defined

$$\dot{Z}_x = \begin{cases} 2(\sqrt{Z}_x a_x(t) + b_x(t)), & Z_x(t) > 0 \\ 2b_x(t), Z_x(t) = 0, & b_x(t) > 0 \\ \delta_x, Z_x(t) = 0, & b_x(t) \le 0 \end{cases} \tag{21}$$

where δ_x is some small positive constant and

$$a_x(t) = \frac{\|s_p\|^2}{\|s_p\|^2 + \varepsilon}(-s_\theta{}^T\omega_2\hat{\phi}_2 - s_\theta{}^T K_{v\theta}s_\theta) \tag{22}$$

$$b_x(t) = \frac{\varepsilon}{\|s_p\|^2 + \varepsilon}(-s_\theta{}^T\omega_2\hat{\phi}_2 - s_\theta{}^T K_{v\theta}s_\theta) \tag{23}$$

Remark 2: Note that (21) is simply to define a differential equation of which its variable $Z_x(t)$ remains positive. Let another signal *k(t)* be defined to be its positive root, i.e., $k = \sqrt{Z_x}$, It can be shown that

$$\dot{k}(t) = \frac{1}{k(t)}\left(\frac{k\|s_p\|^2 + \varepsilon}{\|s_p\|^2 + \varepsilon}\right)(-s_\theta^T\omega_2\hat{\phi}_2 - s_\theta^T K_{v\theta}s_\theta) \qquad k \ne 0 \tag{24}$$

In the sequel, we will first assume that there exists a measure zero set of time sequences $\{t_i\}_{i=1}^{\infty}$ such that $Z(t_i) = 0$ or $k(t_i) = 0$, $i = 1,2,3,\ldots\infty$, and then, verify the existence assumption valid.
Now let the adaptive control law be designed as

$$u_P = -\omega_1\hat{\phi}_1 - \tau_v - K_{vp}s_p \tag{25}$$

$$\tau_v = \left[\frac{(k-1)s_p}{\left\| s_p \right\|^2 + \varepsilon} (-s_\theta{}^T \omega_2 \hat{\phi}_2 - s_\theta{}^T K_{v\theta} s_\theta) \right] \tag{26}$$

where

$$\omega_1 \hat{\phi}_1 = \hat{M}_{pp}(\ddot{q}_{pd} + k_p \dot{e}_p) + \hat{M}_{p\theta}(k_\theta \dot{e}_\theta) + \hat{B}_{pp}(\dot{q}_{pd} + k_p e_p) + \hat{B}_{p\theta} k_p e_p \tag{27}$$

$$\omega_2 \hat{\phi}_2 = \hat{M}_{\theta p}(\ddot{q}_{pd} + k_p \dot{e}_p) + \hat{M}_{\theta\theta}(k_\theta \dot{e}_\theta) + \hat{B}_{\theta p}(\dot{q}_{pd} + k_p e_p) + \hat{B}_{\theta\theta} k_\theta e_\theta \tag{28}$$

and $\hat{\phi}_1, \hat{\phi}_2$ are the estimates of ϕ_1, ϕ_2 respectively, then the error dynamics can be obtained as

$$\begin{bmatrix} M_{PP} & M_{P\theta} \\ M_{P\theta}^T & M_{\theta\theta} \end{bmatrix} \begin{bmatrix} \dot{s}_p \\ \dot{s}_\vartheta \end{bmatrix} + \begin{bmatrix} B_{PP} & B_{P\theta} \\ B_{\theta P} & B_{\theta\theta} \end{bmatrix} \begin{bmatrix} s_p \\ s_\theta \end{bmatrix} + \begin{bmatrix} K_{vp} & 0 \\ 0 & K_{v\theta} \end{bmatrix} \begin{bmatrix} s_p \\ s_\theta \end{bmatrix} = \begin{bmatrix} \omega_1 \tilde{\phi}_1 - \tau_v \\ \omega_2 \phi_2 + K_{v\theta} s_\theta \end{bmatrix} \tag{29}$$

or more compactly as

$$M(q)\dot{s} + h(q, \dot{q})s + Ks = \begin{bmatrix} \omega_1 \tilde{\phi}_1 - \tau_v \\ \omega_2 \phi_2 + K_{v\theta} s_\theta \end{bmatrix} \tag{30}$$

where

$$\begin{bmatrix} \tilde{\phi}_1 \\ \tilde{\phi}_2 \end{bmatrix} = \begin{bmatrix} \hat{\phi}_1 - \phi_1 \\ \hat{\phi}_2 - \phi_2 \end{bmatrix} \tag{31}$$

Moreover, let the adaptation laws be chosen as

$$\begin{aligned} \dot{\hat{\phi}}_1 &= -k_a \omega_1 s_p \\ \dot{\hat{\phi}}_2 &= -k_b \omega_2 s_\theta \end{aligned} \tag{32}$$

where k_a, k_b are some positive definite gain matrices. In what follows we will show that the error dynamics (30) along with the adaptive laws (32) constitutes an asymptotically stable closed-loop dynamic system. This is exactly stated in the following theorem.

Theorem : *Consider the 3-D overhead crane system as mathematically described in (10) or (12) with all the system parameters unknown. Then, by applying control laws (25)-(28) and adaptive laws (32), the objective for the tracking control problem can be achieved, i.e., all signals inside the closed-loop system (29) are bounded and* $e_x, e_\alpha, e_y, e_\beta \to 0$ *asymptotically in the sense of Lyapunov.*

Proof: Define the Lyapunov function candidate as

$$V(t) = \frac{1}{2}s^T M(q)s + \frac{1}{2}\tilde{\phi}_1^T k_a^{-1}\tilde{\phi}_1 + \frac{1}{2}\tilde{\phi}_2^T k_b^{-1}\tilde{\phi}_2 + \frac{1}{2}Z_x$$
$$= \frac{1}{2}s^T M(q)s + \frac{1}{2}\tilde{\phi}_1^T k_a^{-1}\tilde{\phi}_1 + \frac{1}{2}\tilde{\phi}_2^T k_b^{-1}\tilde{\phi}_2 + \frac{1}{2}k^2$$

It is obvious that, due to the quadratic form of system states as well as the definition of $Z_x(t)$, *V(t)* is always positive-definite and indeed a Lyapunov function candidate. By taking the time derivative of V we get

$$\dot{V}(t) = s^T M(q)\dot{s} + \frac{1}{2}s^T \dot{M}(q)s + \dot{\tilde{\phi}}_1^T k_a^{-1}\tilde{\phi}_1 + \dot{\tilde{\phi}}_2^T k_b^{-1}\tilde{\phi}_2 + k\dot{k}$$

$$= s^T(-B(q,\dot{q})s - K_{vp}s + \begin{bmatrix} -\omega_1\tilde{\phi}_1 - \tau_v \\ \omega_2\phi_2 + K_{v\theta}s_\theta \end{bmatrix}) + \frac{1}{2}s^T\dot{M}(q)s + s_p{}^T\omega_1\tilde{\phi}_1 + s_\theta{}^T\omega_2\tilde{\phi}_2$$

$$+ (\frac{k\|s_p\|^2 + \varepsilon}{\|s_p\|^2 + \varepsilon})(-s_\theta{}^T\omega_2\hat{\phi}_2 - s_\theta{}^T K_{v\theta}s_\theta)$$

$$= -sKs - s_p{}^T\omega_1\tilde{\phi}_1 - (\frac{(k-1)\|s_p\|^2}{\|s_p\|^2 + \varepsilon})(-s_\theta{}^T\omega_2\hat{\phi}_2 - s_\theta{}^T K_{v\theta}s_\theta) + s_\theta{}^T\omega_2\phi_2 + s_p{}^T\omega_1\tilde{\phi}_1$$

$$+ (\frac{k\|s_p\|^2 + \varepsilon}{\|s_p\|^2 + \varepsilon})(-s_\theta{}^T\omega_2\hat{\phi}_2 - s_\theta{}^T K_{v\theta}s_\theta) + s_\theta{}^T\omega_2\tilde{\phi}_2 + s_\theta{}^T K_{v\theta}s_\theta$$

$$= -s^T Ks \tag{33}$$

It is clear that $\dot{V}(t) < 0$ as long as $K > 0$, which then implies $s, k, \widetilde{\phi}_1, \widetilde{\phi}_2 \in L_\infty$ Now, assume that $k(t) = 0$ instantaneously at t_i. Because the solution $Z_x(t)$ of the equation (21) is well defined and is continuous for all $t \geq 0$, k(t) is continuous at t_i, i.e., $k(t_i-) = k(t_i+)$. Since V is a continuous function of k, it is clear that $V(t)$ remains to be continuous at t_i, i.e. ,$V(t_i-) = V(t_i+)$. Form then hypothesis, $\dot{V}(t_i^-) < 0$ and $\dot{V}(t_i^+) < 0,$ we hence can conclude that V is nonincreasing in t including t_i, which then readily implies that $s, k \in L_\infty$. Therefore, e, τ_v and $\tau_\theta \in L_\infty$ directly from equation (13) and definitions of τ_v and τ_θ. It then follows from (30) that $\dot{s} \in L_\infty$. On the other hand, if the set of time instants $\{t_i\}_{i=1}^{\infty}$ is measure zero, then $-\int_0^\infty \dot{V} dt = V(0) - V(\infty) < \infty$ or equivalently that $-\int_0^\infty \|s\|^2 dt < \infty$ so that $s \in L_2$. Form the error dynamics, we can further conclusion that $\dot{s} \in L_\infty$. Then by Barbalat's lemma we readily obtain that $s \to 0$ as $t \to \infty$ asymptotically as $t \to \infty$ and therefore, $e, \dot{e} \to 0$ as $t \to \infty$ Note that in the above proof we have used the property $(\dot{M}(q) - 2B(q, \dot{q}))$ is skew- symmetric. Finally, to complete the proof in theory, we need to show that the above hypothesis that the set of time instants $\{t_i\}_{i=1}^{\infty}$ is indeed measure zero. However, it is quite straightforward to conclude the result from (21) by simply using the fact that all signals are bounded. This completes our proof.

Remark 3: From the robustness point of view, it would be better if additional feedback term $-k_q s_\theta$ is included in the control law (24). With such an inclusion, the sway stabilization result subject to external disturbance can also be maintained as the cart arrived at its destination. This can be easily checked from the stability proof given in the theorem.

Proof: Let the Lyapunov function candidate be chosen as

$$V(t) = \frac{1}{2} s^T M(q) s + \frac{1}{2} \widetilde{\phi}_1^T k_a^{-1} \widetilde{\phi}_1 + \frac{1}{2} \widetilde{\phi}_2^T k_b^{-1} \widetilde{\phi}_2 + \frac{1}{2} Z_x$$

$$= \frac{1}{2} s^T M(q) s + \frac{1}{2} \widetilde{\phi}_1^T k_a^{-1} \widetilde{\phi}_1 + \frac{1}{2} \widetilde{\phi}_2^T k_b^{-1} \widetilde{\phi}_2 + \frac{1}{2} k^2$$

and take the time derivative of V to get

$$\dot{V}(t) = s^T M(q) \dot{s} + \frac{1}{2} s^T \dot{M}(q) s + \dot{\widetilde{\phi}}_1^T k_a^{-1} \widetilde{\phi}_1 + \dot{\widetilde{\phi}}_2^T k_b^{-1} \widetilde{\phi}_2 + k\dot{k}$$

$$= s^T(-B(q,\dot{q})s - K_{vp}s + \begin{bmatrix} -\omega_1 \widetilde{\phi}_1 - \tau_v - k_q s_\theta \\ \omega_2 \phi_2 + K_{v\theta} s_\theta \end{bmatrix}) + \frac{1}{2} s^T \dot{M}(q) s + s_p^T \omega_1 \widetilde{\phi}_1 + s_\theta^T \omega_2 \widetilde{\phi}_2$$

$$+(\frac{k\|s_p\|^2+\varepsilon}{\|s_p\|^2+\varepsilon})(-s_\theta{}^T\omega_2\hat{\phi}_2-s_\theta{}^T K_{v\theta}s_\theta)$$

$$=-s^T Ks-k_q s_\theta s_p$$

$$\dot{V}(t)=-s^T K_v s-k_q s_\theta s_p$$

$$\dot{V}(t)\le-\lambda_{\min}(K_v)(s_p{}^2+s_\theta{}^2)+\frac{1}{2}k_q(s_p{}^2+s_\theta{}^2)$$

$$=-(\lambda_{\min}(K_v)-\frac{1}{2}k_q)(s_p{}^2+s_\theta{}^2)$$

Thus, the same conclusion can be made as preciously if

$$\lambda_{\min}(K_v)>\frac{1}{2}k_q$$

3. Computer Simulation

In this subsection, several simulations are performed and the results also confirm the validity of our proposed controller. The desired positions for *X* and *Y* axes are 1 m. Figure 3 shows the time response of X-direction. Figure 5 show the time responses of Y-direction. It can be seen that the cart can simultaneously achieve the desired positions in both X and Y axes in approximately 6 seconds with the sway angles almost converging to zero at the same time. Figure 4 and Figure 6 show the response of the sway angle with the control scheme. Figure 7 and Figure 8 show the velocity response of both X-direction and Y-direction. Figure 9 and Figure 10 show the control input magnitude. In Figure 11~14, the parameter estimates are seen to converge to some constants when error tends to zero asymptotically and the time response of the tuning function *k(t)* is plotted in Figure 15.
The control gains are chosen to be

$$k_p=\begin{bmatrix}1.5 & 0\\ 0 & 1\end{bmatrix}, k_\theta=\begin{bmatrix}2.35 & 0\\ 0 & 1\end{bmatrix},$$

$$k_{vp} = \begin{bmatrix} 1.5 & 0 \\ 0 & 1.8 \end{bmatrix}, k_{v\theta} = \begin{bmatrix} 1.35 & 0 \\ 0 & 1.2 \end{bmatrix}$$

The corresponding adaptive gains are set to be $k_a = k_b = 1$

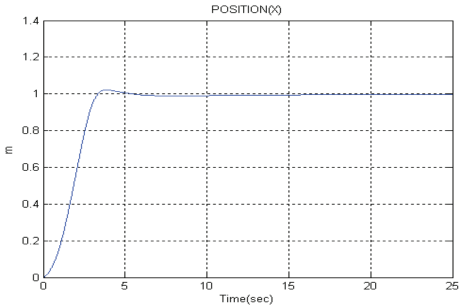

Fig. 3. Gantry Tracking Response $x(t)$ with Adaptive Algorithm

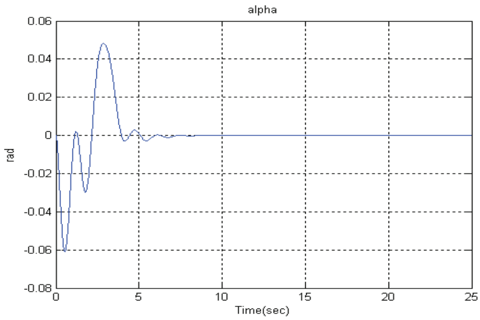

Fig. 4. Sway Angle Response $\alpha(t)$ with Adaptive Algorithm

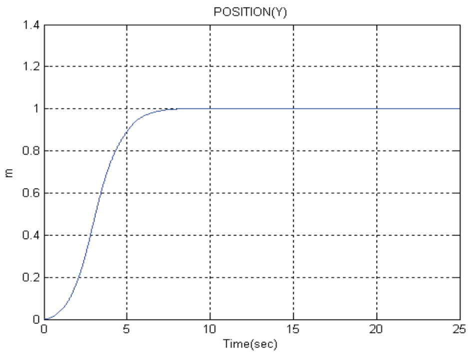

Fig. 5. Gantry Tracking Response $y(t)$ with Adaptive Algorithm

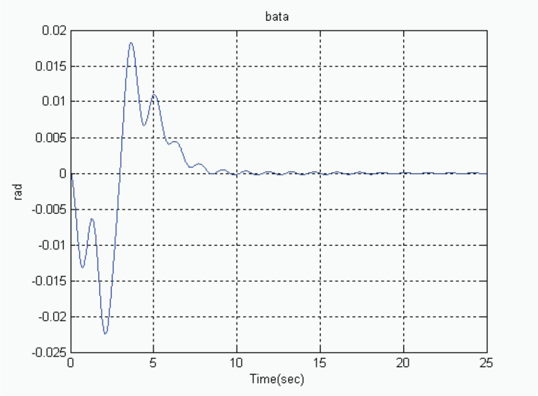

Fig. 6. Sway Angle Response $\beta(t)$ with Adaptive Algorithm

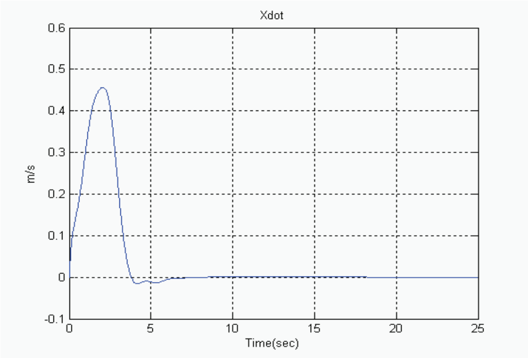

Fig. 7. Gantry Velocity Response $\dot{x}(t)$ with Adaptive Algorithm

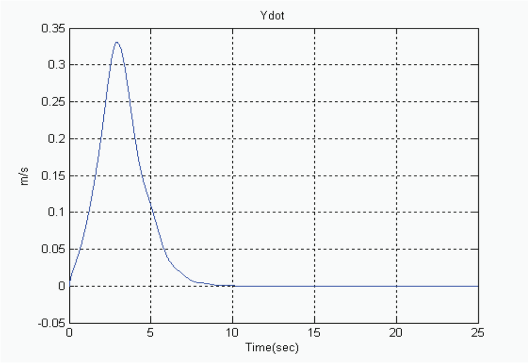

Fig. 8. Gantry Velocity Response $\dot{y}(t)$ with Adaptive Algorithm

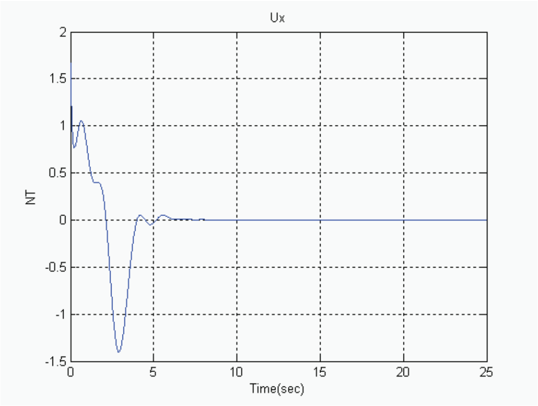

Fig. 9. Force Input u_x

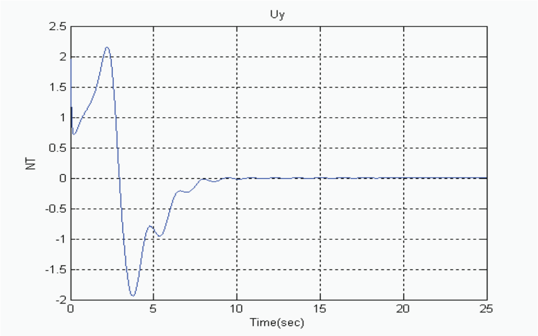

Fig. 10. Force Input u_y

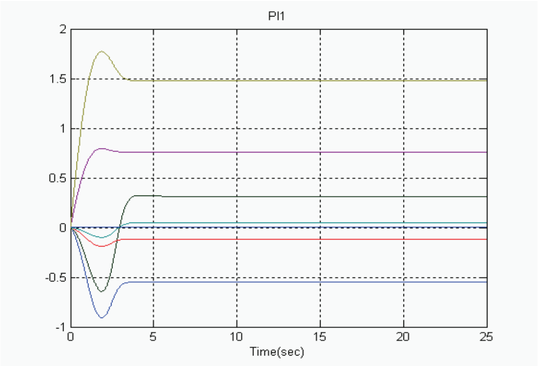

Fig. 11. Estimated Parameters $\phi_{1x}(t)$

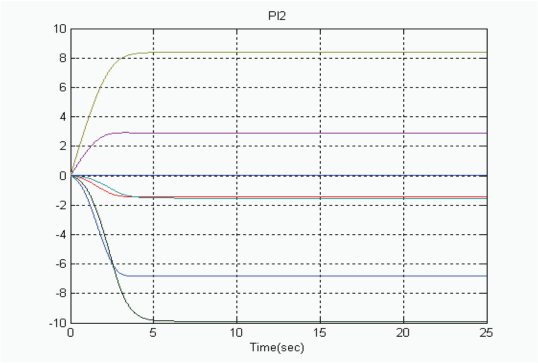

Fig. 12. Estimated Parameters

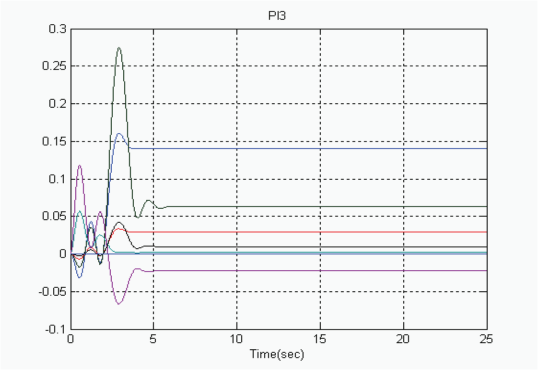

Fig. 13. Estimated Parameters $\phi_{1y}(t)$

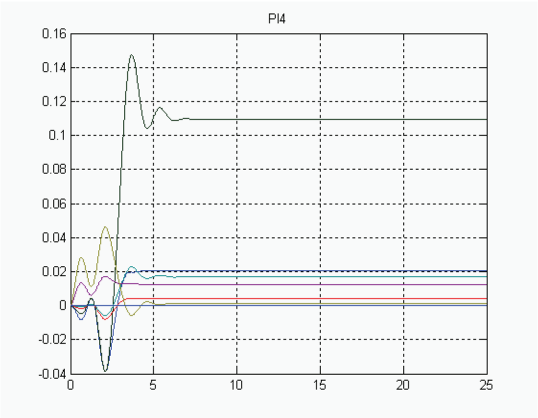

Fig. 14. Estimated Parameters $\phi_{2\alpha}(t)$

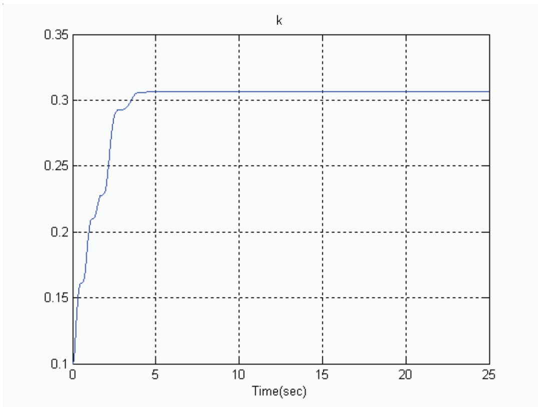

Fig. 15. Response Trajectory of $k(t)$

4. Experimental Verification

In this section, to validate the practical application of the proposed algorithms, a three degree-of-freedom overhead crane apparatus, is built up as shown in Figure 16. Several experiments are also performed and indicated in the subsequent section for demonstration of the effectiveness of the proposed controller.

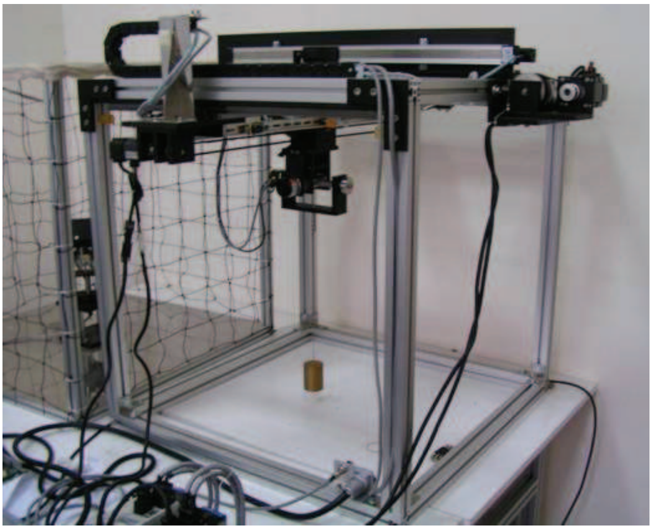

Fig. 16. Experimental setup for the overhead crane system

The control algorithm is implemented on a xPC Target for use with real time Workshop® manufactured by The Math Works, Inc., and the xPC target is inserted in a Pentium4