5

Home energy management problem: towards an optimal and robust solution

Duy Long Ha, Stéphane Ploix, Mireille Jacomino and Minh Hoang Le
G-SCOP lab (Grenoble Institute of Technology)
France

1. Introduction

A home automation system basically consists of household appliances linked via a communication network allowing interactions for control purposes (Palensky & Posta, 1997). Thanks to this network, a load management mechanism can be carried out: it is called *distributed control* in (Wacks, 1993). Load management allows inhabitants to adjust power consumption according to expected comfort, energy price variation and CO_2 equivalent rejection. For instance, during the consumption peak periods when power plants rejecting higher quantities of CO_2 are used and when energy price is high, it could be possible to decide to delay some services, to reduce some heater set points or to run requested services even so according to weather forecasts and inhabitant requests. Load management is all the more interesting that local storage and production means exist. Indeed, battery, photovoltaic panels or wind mills provide additional flexibilities. Combining all these elements lead to systems with many degrees of freedom that are very complex to manage by users.

The objective of this study is to setup a general mathematical formulation that makes it possible to design optimized building electric energy management systems able to determine the best energy assignment plan, according to given criteria. A building energy management system consists in two aspects: the load management and the local energy production management. (House & Smith, 1995) and (Zhou & Krarti, 2005) have proposed optimal control strategies for HVAC (Home Ventilation and Air Conditioning) system taking into account the natural thermal storage capacity of buildings that shift the HVAC consumption from peak-period to off-peak period. Zhou & Krarti (2005) has shown that this control strategy can save up to 10% of the electricity cost of a building. However, these approaches do not take into account the energy resource constraints, which generally depend on the autonomy needs of off-grid systems (Muselli et al., 2000) or on the total power production limits of the suppliers in grid connected systems.

The household load management problem can be formulated as a assignment problem where energy is considered as a resource shared by appliances, and tasks are energy consumptions of appliances. Ha et al. (2006a) presents a three-layers household energy control system that is both able to satisfy the maximum available electrical power constraint and to maximize user satisfaction criteria. This approach carries out more reactivity to adapt consumption to the energy provider requirements. Ha et al. (2006b) proposes a global solution for the household load management problem. In order to adapt the consumption to the available energy, the home automation system controls the appliances in housing by determining the

starting time of services and also by computing the temperature set points of HVAC systems. This problem has been formulated as a multi-objective constraint satisfaction problem and has been solved by a dynamic Tabu Search. This approach can carry out the coordination of appliance consumptions of HVAC system and of services in making it possible to set up a compromise between the cost and the user comfort criteria.
With an energy production management production point of view, Henze & Dodier (2003) has proposed an adaptive optimal control for an off-grid PV-hybrid system using a quadratic cost function and a Q-learning approach. It is more efficient than conventional control but it requires to be trained beforehand with actual data covering a long time period. Generally speaking, studies in literature focus only on one aspect of the home energy management problem: the load management or the local energy production but not on the joined load and production management problem.
This chapter formulates the global approach for the building energy management problem as a scheduling problem that takes into account the load consumption and local energy production points of view. The optimization problem of the building energy management is modeled using both continuous and discrete variables: it is modeled as a mixed integer linear problem.

2. Problem description

In this chapter, energy is restricted to electricity consumption and production. Each service is depicted by an amount of consumed/produced electrical power; it is supported by one or several appliances.

2.1 The concept of service

Housing with appliances aims at providing comfort to inhabitants thanks to services which can be decomposed into three kinds: the end-user services that produce directly comfort to inhabitants, the intermediate services that manage energy storage and the support services that produce electrical power to intermediate and end-user services. Support services deal with electric power supplying thanks to conversion from a primary energy to electricity. *Fuel cells based generators*, *photovoltaic power suppliers*, *grid power suppliers* such as EDF in France, belong to this class. Intermediate services are generally achieved by electrochemical batteries. Among the end-user services, well-known services such as *clothe washing*, *water heating*, *specific room heating*, *cooking in oven* and *lighting* can be found.
A service with index i is denoted $SRV(i)$. Appliances are just involved in services: they are not central from an inhabitant point of view. Consequently, they are not explicitly modelled.

2.2 Caracterisation of services

Let us assume a given time range for anticipating the energy needs (typically 24 hours). A service is qualified as *permanent* if its energetic consumption/production/storage covers the whole time range of energy assignment plan, otherwise, the service is named *temporary service*. The following table gives some examples of services according to this classification.

	temporary services	permanent services
support services	photovoltaic panels	power provider
intermediate services	-	storage
end-user services	washing	room heating

The services can also be classified according to the way their behavior can be modified.

Whatever the service is, an end-user, an intermediate or a support service can be modifiable or not. A service is qualified as *modifiable by a home automation system* if the home automation system is capable to modify its behavior (the starting time for example).
There are different ways of modifying services. Sometimes, modifiable services can be considered as continuously modifiable such as the temperature set points in *room heating services* or the shift of a washing. Some other services may be modified discretely such as the interruption of a *washing service*. The different ways of modifying services can be combined: for instance, a washing service can be considered both as interruptible and as continuously shiftable. A service modeled as discretely modifiable contains discrete decision variables in its model whereas a continuously modifiable service contains continuous decision variables. Of course, a service may contain both discrete and continuous decision variables.
A service can also be characterized by the way it is known by a home automation system. The consumed or produced power may be observable or not. Moreover, for end-user services, the impact of a service on the inhabitant comfort may be known or not.
Obviously, a service can be taken into account by a home automation system if it is at least observable. Some services are indirectly observable. Indeed, all the not observable services can be gathered into a virtual non modifiable service whose consumption/production is deduced from a global power meter measurement and from the observable service consumptions and productions. In addition, a service can be taken into account for long term schedulings if it is predictable. In the same way as for observable services, all the unpredictable services can be gathered into a global no-modifiable predictable service. A service can be managed by a home automation system if it is observable and modifiable. Moreover, it can be long-term managed if it is predictable and modifiable.

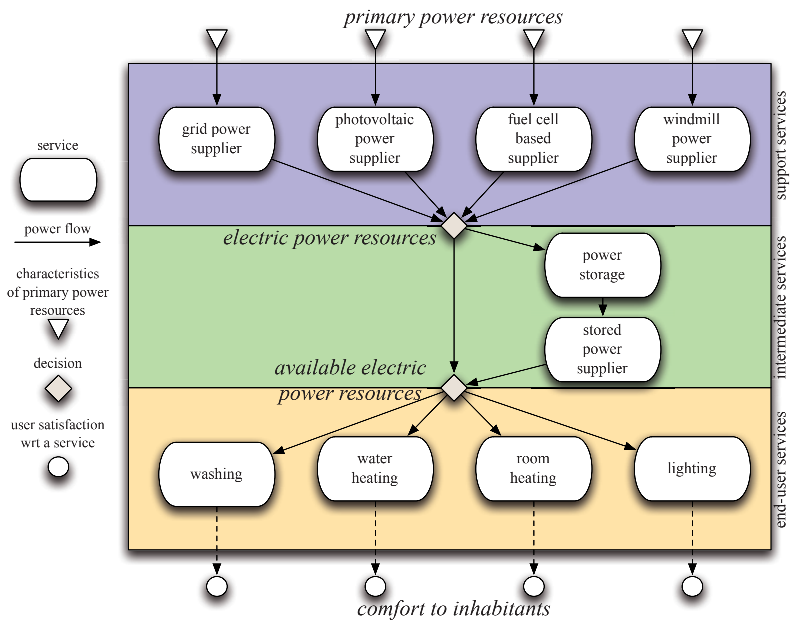

Fig. 1. Structure of services in housing

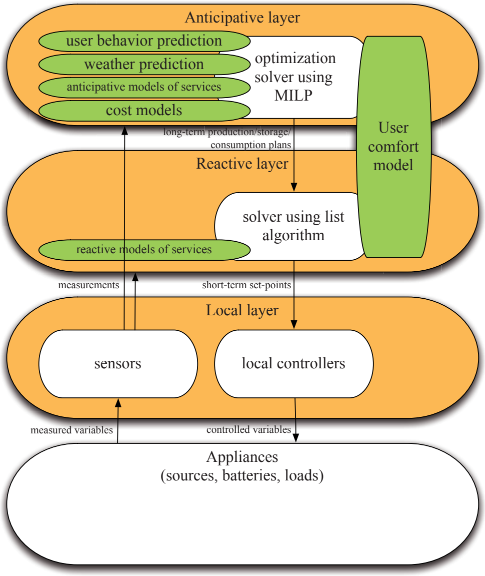

Fig. 2. Schema of the 3 layers control mechanism

2.3 Principle of control mechanism

An important issue in home automation problems is the uncertainties in the model data. For instance, solar radiation, outdoor temperature or services requested by inhabitants may not be predicted with accuracy. In order to solve this issue, a three-layer architecture is presented in this chapter: a local layer, a reactive layer and an anticipative layer (see figure 2).

The *anticipative layer* is responsible for scheduling end-user, intermediate and support services taking into account predicted events and costs in order to avoid as much as possible the use of the *reactive layer*. The prediction procedure forecasts various informations about future user requests but also about available power resources and costs. Therefore, it uses information from predictable services and manage continuously modifiable and shiftable services. This layer has slow dynamics and includes predictive models with learning mechanisms, including models dealing with inhabitant behaviors. This layer also contains a predictive control mechanism that schedules energy consumption and production of end-user services several hours in advance. This layer computes plans according to available predictions. The sampling period of the anticipative layer is denoted Δ. This layer relies on the most abstract models.

The *reactive layer* has been detailed in (Abras et al., 2006). Its objective is to manage adjustments of energy assignment in order to follow up a plan computed by the upper *anticipative layer* in spite of unpredicted events and perturbations. Therefore, this layer manages modifiable services and uses information from observable services (comfort for end-user services and power for others). This layer is responsible for decision-making in case of violation of predefined constraints dealing with energy and inhabitant comfort expectations: it performs arbitrations between services. The set-points determined by the plan computed by the upper *anticipative layer* are dynamically adjusted in order to avoid user dissatisfaction. The control actions may be dichotomic in enabling/disabling services or more gradual in adjusting

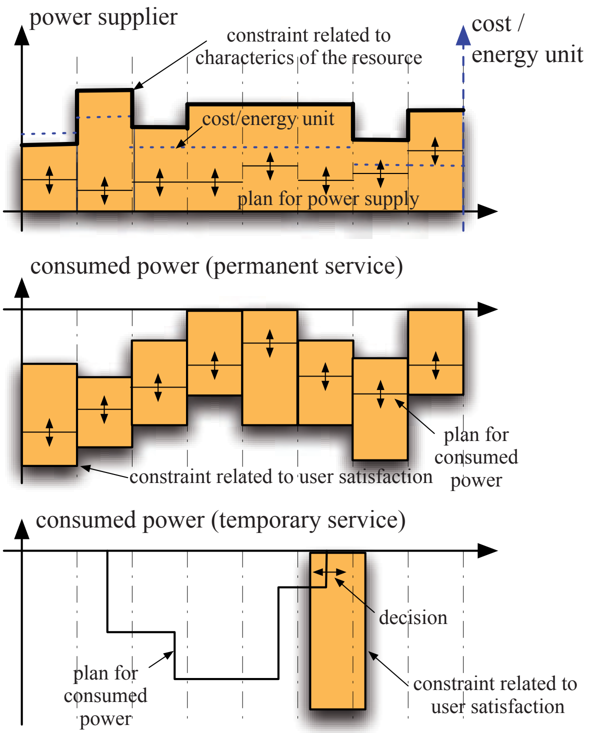

Fig. 3. Plans computed by the anticipative mechanism

set-points such as reducing temperature set point in room heating services or delaying a temporary service. Actions of the reactive layer have to remain transparent for the plan computed by the anticipative layer: it can be considered as a fast dynamic unbalancing system taking into account actual housing state, including unpredicted disturbances, to satisfy energy, comfort and cost constraints. If the current state is too far from the computed plan, the anticipative layer has to re-compute it.
The *local layer* is composed of devices together with their existing local control systems generally embedded into appliances by manufacturers. It is responsible for adjusting device controls in order to reach given set points in spite of perturbations. This layer abstracts devices and services for upper layers: fast dynamics are hidden by the controllers of this level. This layer is considered as embedded into devices: it is not detailed into this chapter.
This chapter mainly deals with the scheduling mechanism of the anticipative layer, which computes anticipative plans as shown in figure 3.

3. Modeling services

Modeling services can be decomposed into two aspects: the modeling of the behaviors, which depends on the types of involved models, and the modeling of the quality of the execution of services, which depends on the types of service. Whatever the type of model it is, it has to be defined all over a time horizon $K\Delta$ for anticipative problem solving composed of K sampling periods lasting Δ each.

3.1 Modeling behavior of services

In order to model the behavior of the different kinds of services in housing, three different types of models have been used: discrete events are modeled by finite state machines, continuous behaviors are modeled by differential equations and mixed discrete and continuous evolutions are modeled by hybrid models that combine the two previous ones.

Using finite state machines (FSM)

A finite state machine dedicated to a service SRV is composed of a finite number of states $\{\mathcal{L}_m; m \in \{1, ..., M\}\}$ and a set of transitions between those states $\{\mathcal{T}_{p,q} \in \{0,1\}; (p,q) \in S \subset \{1, ..., M\}^2\}$. Each state $\mathcal{L}_m$ of a service SRV is linked to a phase characterized by a maximal power production $P_m > 0$ or consumption $P_m < 0$.
A transition triggers a state change. It is described by a condition that has to be satisfied to be enabled. The condition can be a change of a state variable measured by a sensor, a decision of the antipative mechanism or an elapsed time for phase transition. If it exists a transition between the state $\mathcal{L}_m$ and $\mathcal{L}_{m'}$ then $\mathcal{T}_{m,m'} = 1$, otherwise $\mathcal{T}_{m,m'} = 0$. An action can be associated to each state: it may be a modification of a set-point or an on/off switching. As an example, let's consider a washing service.
The service provided by a washing machine may be modeled by a FSM with 4 states: the first state is the *stand-by* state $\mathcal{L}_1$ with a maximal power of $P_1 = -5W$ (it is negative because it deals with consumed power). The transition towards the next state is triggered by the anticipative mechanism. The second state is the *water heating* state $\mathcal{L}_2$ with $P_2 = -2400W$. The transition to the next state is triggered after τ_2 time units. The next state corresponds to the *washing* characterized by $\mathcal{P}_3 = -500W$. And finally, after a given duration τ_3 depending on the type of washing (i.e. the type of requested service), the spin-drying state is reached with $\mathcal{P}_3 = -1000W$. After a given duration τ_4, the *stand-by* state is finally recovered. Considering that the initial state is $\mathcal{L}_1$, this behavior can be formalized by:

$$\begin{cases} (state = \mathcal{L}_1) \wedge (t = t_{start}) & \rightarrow state = \mathcal{L}_2 \\ (state = \mathcal{L}_2) \wedge (t = t_{start+\tau_2}) & \rightarrow state = \mathcal{L}_3 \\ (state = \mathcal{L}_3) \wedge (t = t_{start+\tau_2+\tau_3}) & \rightarrow state = \mathcal{L}_4 \\ (state = \mathcal{L}_4) \wedge (t = t_{start+\tau_2+\tau_3+\tau_4}) & \rightarrow state = \mathcal{L}_1 \end{cases} \quad (1)$$

Using differential equations

In buildings, thermal phenomena are continuous phenomena. In particular, the thermal behavior of a HVAC system can be modeled by state space models:

$$\begin{cases} \frac{dx_c(t)}{dt} = A_c x_c(t) + B_c u_c(t) + F_c p_c(t) \\ y_c(t) = C x_c(t) \end{cases} \quad (2)$$

$x_c(t)$ contains state variables, usually temperature. $u_c(t)$ contains controlled input variables such as energy flows. $p_c(t)$ contains known but uncontrolled input variables such as outside temperature or solar radiance. A first order state space thermal model relevant for control purpose has been proposed in Nathan (2001) but the second order model based on an electric analogy proposed in Madsen (1995) has been preferred for our control purpose because it models the dynamic of indoor temperature. For a room heating service $SRV(i)$, it yields:

$$\begin{cases} \frac{d}{dt}\begin{bmatrix} T_{in}(i,t) \\ T_{env}(i,t) \end{bmatrix} = A_c \begin{bmatrix} T_{in}(i,t) \\ T_{env}(i,t) \end{bmatrix} + B_c \begin{bmatrix} P(i,t) \end{bmatrix} + F_c \begin{bmatrix} T_{out}(i,t) \\ \phi_s(i,t) \end{bmatrix} \\ T_{in}(i,t) = C_c \begin{bmatrix} T_{in}(i,t) \\ T_{env}(i,t) \end{bmatrix} \end{cases} \tag{3}$$

with $A_c = \begin{bmatrix} \frac{-1}{r_{in}c_{env}} & \frac{1}{r_{in}c_{env}} \\ \frac{1}{r_{in}c_{in}} & -\frac{r_{env}+r_{in}}{r_{env}r_{in}c_{in}} \end{bmatrix}$, $B_c = \begin{bmatrix} 0 \\ \frac{1}{-c_{in}} \end{bmatrix}$, $F_c = \begin{bmatrix} 0 & 0 \\ \frac{1}{r_{env}c_{in}} & \frac{w}{c_{in}} \end{bmatrix}$ and $C_c = \begin{bmatrix} 1 & 0 \end{bmatrix}$

This model allows a rather precise description of the dynamic variations of indoor temperature with:

- T_{in}, T_{out}, T_{env} the respective indoor, outdoor and housing envelope temperatures
- c_{in}, c_{env} the thermal capacities of first indoor environment and second the envelope of the housing
- r_{in}, r_{env} thermal resistances
- w the equivalent surface of the windows
- P the power consumed by the thermal generator, $P \leq 0$. In this chapter, this flow is assumed to correspond to an electric energy flow.
- ϕ_s the energy flow generated by the solar radiance

In order to solve the anticipative problem, continuous time models have to be discretized according to the anticipation period Δ. Equation (2) modelling service $SRV(i)$ becomes:

$$\begin{aligned} &\forall k \in \{1, \ldots, K\}, \\ &\begin{bmatrix} T_{in}(i,k+1) \\ T_{env}(i,k+1) \end{bmatrix} = A_i \begin{bmatrix} T_{in}(i,k) \\ T_{env}(i,k) \end{bmatrix} + B_i \begin{bmatrix} E(i,k) \end{bmatrix} + F_i \begin{bmatrix} T_{out}(i,k) \\ \phi_s(i,k) \end{bmatrix} \end{aligned} \tag{4}$$

with $A_i = e^{A_c\Delta}$, $B_i = (e^{A_c\Delta} - I_n)A_c^{-1}\Delta^{-1}B_c$, $F_i = (e^{A_c\Delta} - I_n)A_c^{-1}F_c$, $E(i,k) = P(i,k)\Delta$ and $E(i,k) \leq 0$.

Using hybrid models

Some services cannot be modeled by a finite state machine nor by differential equations. Both approaches have to be combined: the resulting model is then based on a finite state machine where each state $\mathcal{L}_m$ actually becomes a set of states which evolution is depicted by a differential equation.

An electro-chemical storage service supported by a battery may be modeled by a hybrid model (partially depicted in figure 4). $x(t)$ stands for the quantity of energy inside the battery and $u(t)$ the controlled electrical power exchanged with the grid network.

Using static models

Power sources are usually modelled by static constraints. Local intermittent power resources, such as photovoltaic power system or local electric windmill, and power suppliers are considered here. Using weather forecasts, it is possible to predict the power production $w(i,k)$

during each sampling period $[k\Delta, (k+1)\Delta]$ of a support service $SRV(i)$. The available energy for each sampling period k is then given by:

$$E(i,k) = w(i,k)\Delta \ \forall k \in \{1, ..., K\} \tag{5}$$

with $w(i,k) \geq 0$
According to the subscription between inhabitants and a power supplier, the maximum available power is given. It may depends on time. For a service of power supply $SRV(i)$, it can be modelled by the following constraint:

$$E(i,k) \leq p_{max}(i,k)\Delta \ \forall k \in \{1, ..., K\} \tag{6}$$

where $p_{max}(i,k)$ stands for the maximum available power.

3.2 Modeling quality of the execution of services

Depending on the type of service, the quality of the service achievement may be assessed in different ways. End-user services provide comfort to inhabitants, intermediate services provide autonomy and support services provide power that can be assessed by its cost and its impact on the environment. In order to evaluate these qualities different types of criteria have been introduced.

End-user services

Generally speaking, modifiable permanent services use to control a physical variable: the user satisfaction depends on the difference between an expected value and an actual one. Let's consider for example the HVAC controlling a temperature. A flat can usually be split into several HVAC services related to rooms (or thermal zones) assumed to be independent.
According to the comfort standard 7730 (AFNOR, 2006), three qualitative categories of thermal comfort can be distinguished: A, B and C. In each category, (AFNOR, 2006) proposes typical value ranges for temperature, air speed and humidity of a thermal zone that depends on the type of environment: office, room,... These categories are based on an aggregated criterion named Predictive Mean Vote (PMV) modelling the deviation from a neutral ambience. The absolute value of this PMV is an interesting index to evaluate the quality of a HVAC service. In order to simplify the evaluation of the PMV, typical values for humidity and air speed are used. Therefore, only the ambient temperature corresponding to the neutral value of PMV (PMV=0) is dynamically concerned. Under this assumption, an ideal temperature T_{opt} is obtained. Depending on the environment, an acceptable temperature range coming from

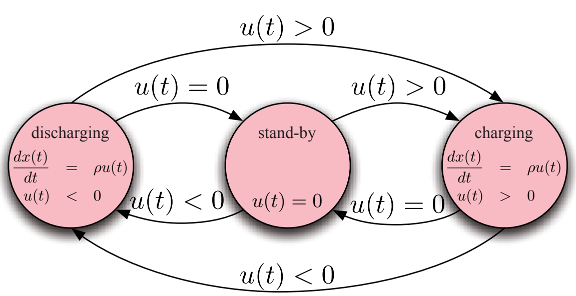

Fig. 4. Hybrid model of a battery

the standard leads to an interval $[T_{min}, T_{max}]$. For instance, in an individual office in category A, with typical air speed and humidity conditions, the neutral temperature is $T_{opt} = 22°C$ and the acceptable range is $[21°C, 23°C]$.
Therefore, considering the HVAC service $SRV(i)$, the discomfort criterion $D(i,k)$, which is more usable than comfort criterion here, is modelled by the following formula where assumptions are depicted by two parameters a_1 and a_2:

$$D(i,k) = |PMV(T_{in}(i,k))| = \begin{cases} a_1 \times \dfrac{(T_{opt} - T_{in}(i,k))}{T_{opt} - T_{Min}} & \text{if } T_{in}(i,k) \leq T_{opt} \\ a_2 \times \dfrac{(T_{in}(i,k) - T_{opt})}{T_{Max} - T_{opt}} & \text{if } T_{in}(i,k) > T_{opt} \end{cases} \quad (7)$$

The global comfort criterion is defined as following:

$$D(i) = \sum_{k=1}^{K} D(i,k) \quad (8)$$

Generally speaking, modifiable temporary end-user services do not aim at controlling a physical variable. Temporary services such as washing are expected by inhabitants to finished at a given time. Therefore, the quality of achievement of a temporary service depends on the amount of time it is shifted. Therefore, in the same way as for permanent services, a user dissatisfaction criterion for a service $SRV(i)$ is defined by:

$$D(i) = \begin{cases} \dfrac{f(i) - f_{opt}(i)}{f_{max}(i) - f_{opt}(i)} & \text{if} \quad f(i) > f_{opt}(i) \\ \dfrac{f_{opt}(i) - f(i)}{f_{opt}(i) - f_{min}(i)} & \text{if} \quad f(i) \leq f_{opt}(i) \end{cases} \quad (9)$$

where f_{opt} stands for the requested end time and f_{min} and f_{max} stand respectively for the minimum and maximum acceptable end time.

Intermediate services

Intermediate services are composed of two kinds of services: the *power storage services*, which store energy to be able to face difficult situations such as off-grid periods, and then lead to the availability of the *stored power supplier services* (see figure 1). A power storage service $SRV(i)$ and a stored power service $SRV(j); j \neq i$ are associated to each storage system.
The quality of a *power storage service* has to be evaluated: it is related to the amount of stored energy. This quality is called *autonomy*.
Let us consider a electric storage system modelled by a power storage service $SRV(i)$ and by a stored power supplier service $SRV(j)$. The stock $E^{stock}(k)$ of the storage system is modelled by:

$$E^{stock}(k) = E^{stock}_{initial} - \sum_{\zeta=1}^{k} (E(i,\zeta) + E(j,\zeta)) \quad (10)$$

with $E(i,\zeta) \leq 0$ and $E(j,\zeta) \geq 0$.
Let P_{ref} be the reference power taken into account for the computation of the autonomy duration $\tau_{autonomy}$. The autonomy objective $A(k)$ can be defined by:

$$A(k) = \sum_{k \in \{1,\ldots,K\}} E^{stock}(k) \quad (11)$$

Depending on the inhabitant expectations, autonomy can also be formulated by constraints to be satisfied at any sample time: $P_{ref}\tau_{autonomy} - E^{stock}(k) = 0,\ \forall k \in \{1, \dots, K\}$.
Let's now focus on *stored power supplier* service. What is the quality for this service i.e. the service that provides stored energy to the housing. It is not a matter of economy nor of ecology because costs is already taken into account when power production services provide power to the storage system. It is not also a matter of stored energy: there is no quality of service defined for *stored power supplier* service.

Support services

Support services dealing with power resources do not interact directly with inhabitants. However, inhabitants do care about their cost and their environmental impact. These two aspects have to be assessed.
In most cases, the economical criterion corresponds to the cost of the provided, stored or sold energy. This cost may contain depreciation of the device used to produce power.
Let $SRV(0)$ be a photovoltaic support service and $SRV(1)$ be a power supplier service. Let's examine the case of power provider such as EDF in France. Energy is sold at a given price $C(1,k)$ to the customer for each consumed kWh at time k. In order to promote photovoltaic production, power coming from photovoltaic plants is bought by the supplier at higher price $C(0,k) > C(1,k)$.
Different power metering principles can be subscribed with a French power supplier. Only the most widespread principle is addressed. The energy cost is thus given by the following equation:

$$C(k) = C(1,k)E(1,k) - C(0,k)E(0,k),\ \forall k \in \{1, \dots, K\} \tag{12}$$

The equivalent mass of carbon dioxide rejected in the atmosphere has been used as ecological criterion for a support service. This criterion is easy to establish for most power devices: photovoltaic cells, generator and even for energy coming from power suppliers. Powernext energy exchange institution publishes the equivalent mass of carbon dioxide rejected in the atmosphere per power unit in function of time (see http://www.powernext.fr). For instance, in France, electricity coming from the grid network produces $66g/kWh$ of CO_2 during off-peak periods and $383g/kWh$ during peak period (Angioletti & Despretz, 2003). Energy coming from photovoltaic panels is considered as free of CO_2 rejection (grey energy is not taken into account). For each support service $SRV(i)$, a CO_2 rejection rate $\tau_{CO2}(i,k)$ can be defined as the equivalent volume of CO_2 rejected per kWh. Therefore, the total rejection for a support service $SRV(i)$ during the sampling period k is given by $\tau_{CO2}(i,k)E(i,k)$ where $E(i,k)$ corresponds to the energy provided by the support service $SRV(i)$ during the sampling period k.

4. Formulation of the anticipative problem as a linear problem

The formulation of the energy management problem contains both behavioral models with discrete and continuous variables, differential equation and finite state models, and quality models with nonlinearities such as in the PMV model. In order to get mixed linear problems which can be solved by well known efficient algorithms, transformations have to be done. The ones that have been used are summarized in the next section.

4.1 Transformation tools

Basically, a proposition denoted $\mathcal{X}$ is either *true* or *false*. It can result from the combination of propositions thanks to connecting operators such as "$\wedge$"(and), "$\vee$"(or), "$\oplus$" (exclusive or), "$\neg$"

(not), "$\rightarrow$" (implies), "$\leftrightarrow$" (if and only if),... Whatever the proposition $\mathcal{X}$ is, it can be associated to a binary variable $\delta \in \{0,1\}$ such as: $\mathcal{X} = (\delta = 1)$.
Therefore, (Williams, 1993) has shown that, in integer programming, connecting operators may be modelled by:

$$\begin{array}{lcl} \neg\mathcal{X} & \leftrightarrow & \delta = 0 \\ \mathcal{X}_1 \wedge \mathcal{X}_2 & \leftrightarrow & \delta_1 + \delta_2 = 2 \\ \mathcal{X}_1 \vee \mathcal{X}_2 & \leftrightarrow & \delta_1 + \delta_2 \geq 1 \\ \mathcal{X}_1 \oplus \mathcal{X}_2 & \leftrightarrow & \delta_1 + \delta_2 = 1 \\ \mathcal{X}_1 \rightarrow \mathcal{X}_2 & \leftrightarrow & \delta_1 - \delta_2 \leq 0 \\ \mathcal{X}_1 \leftrightarrow \mathcal{X}_2 & \leftrightarrow & \delta_1 - \delta_2 = 0 \end{array} \tag{13}$$

According to (Bemporad & Morari, 1998), the transformation into a standard linear problem can be achieved using lower and upper bounds of $dom(f(x); x \in dom(x)) = dom(ax - b; x \in dom(x)) \subset [m, M]$. Then, Binary variables can be connected to linear conditions as follows:

$$\delta = (ax - b \leq 0) \leftrightarrow \begin{cases} ax - b \leq M(1-\delta) \\ ax - b > m\delta \end{cases} \tag{14}$$

Consider for instance the statement $a_1 x \leq b_1 \leftrightarrow a_2 x' \leq b_2$. Using the previous transformation, it can be formulated as:

$$\begin{cases} a_1 x - b_1 \leq M(1-\delta) \\ a_1 x - b_1 \leq m\delta \\ a_2 x' - b_2 \leq M(1-\delta) \\ a_2 x' - b_2 \leq m\delta \end{cases}$$

with $dom(a_1 x - b_1; x \in dom(x)) \cup dom(a_2 x' - b_2; x' \in dom(x')) \subset [m, M]$.
In many cases, such as in presence of absolute values like in PMV evaluation, products of discrete and continuous variables appear. They have to be reformulated in order to get mixed linear problems. Auxiliary variables may be used for this purpose. First consider the product of 2 binary variables δ_1 and δ_2: $\delta_3 = \delta_1 \times \delta_2$. It can be transformed into a discrete linear problem:

$$\delta_3 = \delta_1 \times \delta_2 \leftrightarrow \begin{cases} -\delta_1 + \delta_3 & \leq & 0 \\ -\delta_2 + \delta_3 & \leq & 0 \\ \delta_1 + \delta_2 - \delta_3 & \leq & 1 \\ \delta_1,\ \delta_2,\ \delta_3 \in \{0,1\} & & \end{cases} \tag{15}$$

Consider now the product of a binary variable with a continuous variable: $z = \delta \times x$ where $\delta \in \{0,1\}$ and $x \in [m, M]$. It means that $\delta = 0 \rightarrow z = 0$ and $\delta = 1 \rightarrow z = x$. Therefore, the semi-continuous variable z can be transformed into a mixed linear problem:

$$z = \delta \times x \leftrightarrow \begin{cases} z & \leq M \times \delta \\ z & \geq m\delta \\ z & \leq x - m(1-\delta) \\ z & \geq x - M(1-\delta) \end{cases} \tag{16}$$

These transformations can now be used to remove nonlinearities from the PMV computations, time shifting of services and power storage.

4.2 Linearization of PMV

Generally speaking, behavioral models of HVAC systems is given by Eq. (2) and an example is given by (3). Model (4) is already linear but nonlinearities come up with the absolute value of the PMV evaluation. Let's introduce a binary variable $\delta_a(k)$ satisfying $\delta_a(k) = 1 \leftrightarrow T_{in}(k) \leq T_{opt}\ \forall k$. Then, the PMV function (7) can be reformulated into a mixed linear form for every service $SRV(i)$:

$$\begin{aligned} |PMV(T_{i,a}(k))| &= \delta_a(k) \times a_1 \times \frac{(T_a(i,k) - T_{opt})}{T_{opt} - T_{Min}} + (1 - \delta_a(k)) \times a_2 \times \frac{(T_{opt} - T_a(k))}{T_{Max} - T_{opt}} \\ &= F_1\delta_a(k) + F_2 T_a(k) + F_3 z_a(k) + F_4 \end{aligned} \tag{17}$$

Using eq. (14) to transform the absolute value, the equivalent form of the condition that contains $T_a(k) \leq T_{opt}$ is given by:

$$\begin{cases} T_a(k) - T_{opt} \leq (T_{max} - T_{opt})(1 - \delta_a(k)) \\ T_a(k) - T_{opt} \geq \epsilon + (T_{min} - T_{opt} - \epsilon)\delta_a(k) \end{cases} \tag{18}$$

A semi-continuous variable $z_a(k)$ is added to take place of the product $\delta_a(k) \times T_{in}(k)$ in eq. (17). According to eq. (16), the transformation of $z_a(k) \triangleq \delta_a(k) \times T_{in}(k)$ leads to:

$$\begin{cases} z_a(k) & \leq (T_{max} - T_{opt})\delta_a(k) \\ z_a(k) & \geq (T_{min} - T_{opt})\delta_a(k) \\ z_a(k) & \leq T_{in}(k) - (T_{min} - T_{opt})(1 - \delta_a(k)) \\ z_a(k) & \geq T_{in}(k) - (T_{max} - T_{opt})(1 - \delta_a(k)) \end{cases} \tag{19}$$

After the linearization of PMV, let's now consider the linearization of the time shifting of services.

4.3 Formalizing time shifting

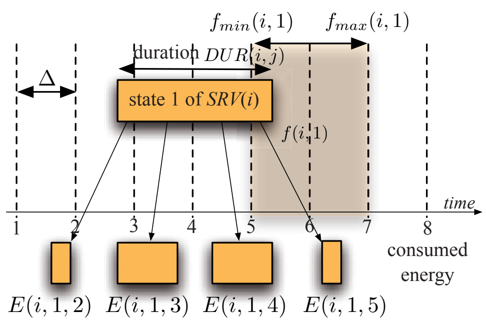

Fig. 5. Shift of temporary services

Temporary services are modelled by finite state machines. The consumption of a state can be shifted such as task in scheduling problems. The starting and ending times of services can be synchronized to an anticipative period such as in (Duy Ha, 2007). It leads to a discrete-time formulation of the problem. However, this approach is both a restriction of the solution space and an approximation because the length of a time service has to be a multiple of Δ. The general case has been considered here.

In the scientific literature, continuous time formulations of scheduling problems exist (Castro & Grossmann, 2006; Pinto & Grossmann, 1995; 1998). However, these results concerns scheduling problems with disjunctive resource constraints. Instead of computing the starting time of tasks, the aim is to determine the execution sequence of tasks on shared resources. In energy management problems, the matter is not restricted to determine such sequence because several services can be achieved at the same time.

An alternative formulation based on transformations (14) and (16), suitable for the energy management in housings, is introduced.

Temporary services can be continuously shifted. Let $DUR(i,j)$, $f(i,j)$ and $p(i,j)$ be respectively the duration of the state j of service $SRV(i)$, the ending time and the power related to the service $SRV(i)$ during the state j. $f(i,j)$ is defined according to inhabitant comfort models: they correspond to extrema in the comfort models presented in section 3.2.

According to (Esquirol & Lopez, 1999), the potential consumption/production duration (effective duration if positive) $d(i,j,k)$ of a service $SRV(i)$ in state j during a sampling period $[k\Delta,(k+1)\Delta]$ is given by (see figure 5):

$$d(i,j,k) = \min(f(i,j),(k+1)\Delta) - \max(f(i,j) - DUR(i,j),k\Delta) \quad (20)$$

Therefore, the consumption/production energy $E(i,j,k)$ of the service $SRV(i)$ in state j during a sampling period $[k\Delta,(k+1)\Delta]$ is given by:

$$E(i,j,k) = \begin{cases} d(i,j,k)p(i,j) & \text{if } d(i,j,k) > 0 \\ 0 & \text{otherwise} \end{cases} \quad (21)$$

$E(i,j,k)$ can be modelled using a binary variable: $\delta_{t0}(i,j,k) = (d(i,j,k) \geq 0)$ and a semi-continuous variable $z_{t_0}(i,j,k) = \delta_{t0}(i,j,k)d(i,j,k)$ such as in (14) and in (16). It leads to the following inequalities:

$$\begin{aligned}
d(i,j,k) &\leq \delta_{t0}(i,j,k)K\Delta && (22)\\
d(i,j,k) &> (\delta_{t0}(i,j,k)-1)\,K\Delta && (23)\\
E(i,j,k) &= z_{t_0}(i,j,k)p(i,j) && (24)\\
z_{t_0}(i,j,k) &\leq \delta_{t0}(i,j,k)K\Delta && (25)\\
z_{t_0}(i,j,k) &\geq -\delta_{t0}(i,j,k)K\Delta && (26)\\
z_{t_0}(i,j,k) &\leq d(i,j,k) + (1-\delta_{t0}(i,j,k))\,K\Delta && (27)\\
z_{t_0}(i,j,k) &\geq d(i,j,k) - (1-\delta_{t0}(i,j,k))\,K\Delta && (28)
\end{aligned}$$

But the model still contains nonlinear functions min and max in the expression of $d(i,j,k)$. Therefore, equation (20) has to be transformed into a mixed-linear form. Let's introduce 2 binary variables $\delta_{t1}(i,j,k)$ and $\delta_{t2}(i,j,k)$ defined by:

$$\begin{aligned}
\delta_{t1}(i,j,k) &= (f(i,j) - k\Delta \geq 0)\\
\delta_{t2}(i,j,k) &= (f(i,j) - DUR(i,j) - k\Delta \geq 0)
\end{aligned}$$

Using (14), it yields:

$$\begin{aligned}
f(i,j) - k\Delta &\leq \delta_{t1}(i,j,k)K\Delta && (29)\\
f(i,j) - k\Delta &\geq (\delta_{t1}(i,j,k)-1)\,K\Delta && (30)\\
f(i,j) - DUR(i,j) - k\Delta &\leq \delta_{t2}(i,j,k)K\Delta && (31)\\
f(i,j) - DUR(i,j) - k\Delta &\leq (\delta_{t2}(i,j,k)-1)\,K\Delta && (32)
\end{aligned}$$

Therefore, min and max of equation (20) become:

$$f_{min}(i,j,k) = \delta_{t1}(i,j,k+1)(k+1)\Delta + (1-\delta_{t1}(i,j,k+1))\,f(i,j) \tag{33}$$

$$s_{max}(i,j,k) = \delta_{t2}(i,j,k)(f(i,j) - DUR(i,j)) + (1-\delta_{t2}(i,j,k))\,k\Delta \tag{34}$$

with $\min(f(i,j),(k+1)\Delta) = f_{min}(i,j,k)$ and $\max(f(i,j) - DUR(i,j), k\Delta) = s_{max}(i,j,k)$.
The duration $d(i,j,k)$ can then be evaluated:

$$d(i,j,k) = f_{min}(i,j,k) - s_{max}(i,j,k) \tag{35}$$

Equations (22) to (35) model the time shifting of a temporary service.
Let's now consider nonlinearities inherent to power storage services modelled by hybrid models.

4.4 Linearization of power storage

A storage service $SRV(i)$ with a maximum capacity of E_{stock}^{max} can be modelled at time k by:

$$E_{stock}(i,k) = max(min(E_{stock}^{max}, E_{stock}(i,k-1) + E(i,k-1)), 0)$$

Let's define the following binary variables: $\delta_1(i,k) = (E_{stock}(i,k) \leq E_{stock}^{max})$ and $\delta_2(i,k) = (E_{stock}(i,k) \geq 0)$. Using (14), it yields:

$$\begin{aligned}
E_{stock}(i,k) - E_{stock}^{max} &\leq (1-\delta_1(i,k))\,E_{stock}^{max} && (36)\\
E_{stock}(i,k) - E_{stock}^{max} &> -\delta_1(i,k)E_{stock}^{max} && (37)\\
E_{stock}(i,k) &\leq \delta_2(i,k)E_{stock}^{max} && (38)\\
E_{stock}(i,k) &> (\delta_2(i,k)-1)\,E_{stock}^{max} && (39)
\end{aligned}$$

The stored energy can then be written:

$$\begin{aligned}
E_{stock}(i,k) = {} & \delta_1(i,k-1)\delta_2(i,k-1)\,(E_{stock}(i,k-1) + E(i,k-1))\ldots\\
& \cdots + (1-\delta_1(i,k))E_{stock}^{max}
\end{aligned}$$

With variables $\delta_3(i,k) = \delta_1(i,k)\delta_2(i,k)$, $z_1(i,k) = \delta_3(i,k)E_{stock}(i,k)$ and $z_2(i,k) = \delta_3(i,k)E(i,k)$ and using transformations (15) and (16), the energy $E_{stock}(i,k)$ can be rewritten into a linear form:

$$E_{stock}(i,k) = z_1(i,k-1) + z_2(i,k-1) + (1-\delta_1(i,k))E_{stock}^{max} \tag{40}$$

The following constraints must be satisfied:

$$\begin{aligned}
-\delta_1(i,k) + \delta_3(i,k) &\leq 0 && (41)\\
-\delta_2(i,k) + \delta_3(i,k) &\leq 0 && (42)\\
\delta_1(i,k) + \delta_2(i,k) - \delta_3(i,k) &\leq 1 && (43)\\
z_1(i,k) &\leq \delta_3(i,k)E_{stock}^{max} && (44)\\
z_1(i,k) &\geq -\delta_3(i,k)E_{stock}^{max} && (45)\\
z_1(i,k) &\leq E_{stock}(i,k) + (1-\delta_3(i,k))E_{stock}^{max} && (46)\\
z_1(i,k) &\geq E_{stock}(i,k) - (1-\delta_3(i,k))E_{stock}^{max} && (47)\\
z_2(i,k) &\leq \delta_3(i,k)E_{stock}^{max} && (48)\\
z_2(i,k) &\geq -\delta_3(i,k)E_{stock}^{max} && (49)\\
z_2(i,k) &\leq E(i,k) + (1-\delta_3(i,k))E_{stock}^{max} && (50)\\
z_2(i,k) &\geq E(i,k) - (1-\delta_3(i,k))E_{stock}^{max} && (51)
\end{aligned}$$

Equations (40) to (51) are a linear model of a power storage service.
Main services have been modelled by mixed integer linear form. Other services can be modelled in the same way. Let's now focus on how to solve the resulting mixed integer linear problem.

5. Solving approach

Anticipative control in home energy management can be formulated as an multicriteria mixed-linear programming problem represented by a set of constraints and optimization criteria.

5.1 Problem summary

In a actual problem, the number of constraints is so large they cannot be detailed in this chapter. Nevertheless, the fundamental modelling and transformation principles have been presented in sections 3 and 4.
HVAC services are representative examples of permanent services. They have been modelled by equations like (4) and (19). The decision variables are heating powers $\Phi_s(i,k)$.
Temporary services, such as clothe washing, are modelled by equations like (22) to (35). The decision variables are the ending times: $f(i,j)$.
Storage services are modelled by equations like (40) to (51). The decision variables are energy exchange with the storage systems: $E(i,j)$.
Power supplier services are modelled by equations like (5). There is no decision variable for these services.
These results can be adapted to fit most situations. If necessary, more details about modelling can be found in (Duy Ha, 2007). As a summary, the following constraints may be encountered:

- linearized behavioral models of services
- linearized comfort models related to end-user services

In addition, a constraint modelling the production/consumption balance has to be added. Generally speaking, this constraint can be written:

$$\forall k \in \{1,\ldots,K\},\ \sum_{i\in\mathcal{I}} E(i,k) = 0 \tag{52}$$

where $\mathcal{I}$ contains the indexes of available predictable services.
If there is a grid power supplier modelled by a support service $SRV(0)$, the imported energy can be adjusted to effective needs (it is also true for fuel cells based support services). Therefore, $E(0,k)$ has to be set to the maximum available energy for a sampling period: $E(0,k) = P^{max}(0,k)\Delta$ where $P^{max}(0,k)$ stands for the maximum available power during sampling period k. Consequently, (52) becomes:

$$\forall k \in \{1,\ldots,K\},\ \sum_{i\in\mathcal{I}} E(i,k) \geq 0 \tag{53}$$

All the predictable but not modifiable services provide data to the optimization problem. Their indexes are contained in $\mathcal{I}^{modifiable} \subset \mathcal{I}$. Decision variables are all related to predictable and modifiable services: they may be binary or continuous decision variables. The problem to be solved is thus a mixed-linear programming problem. Moreover, the optimization problem is a multi-criteria problem using the following criteria: economy, dissatisfaction, CO2eq and autonomy criteria.

Economy criterion is given by (12) when there is only a grid power supplier and a photovoltaic power supplier. Depending of the predictable support services $\mathcal{I}^{support*}$ excluding photovoltaic power supplier and on the existence of photovoltaic power supplier $SRV(0)$,

$$J^{autonomy} = \sum_{k=1}^{K} \left(\sum_{i \in \mathcal{I}^{support*}} C(i,k)E(i,k) - C(0,k)E(0,k) \right) \quad (54)$$

where $C(i,k)$ stands for the kWh cost of the support service i.
Dissatisfaction criterion comes from expressions like (7) and (9). Let $\mathcal{I}^{end-user} \subset \mathcal{I}$ be the indexes of predictable end-user services. The comfort criteria may be given by:

$$J^{discomfort} = \sum_{i \in \mathcal{I}^{end-user}} sum_{k \in \{1,\ldots,K\}} D(i,k) \quad (55)$$

The autonomy criterion comes from (11). It is given by:

$$J^{autonomy} = sum_{k \in \{1,\ldots,K\}} A(k) \quad (56)$$

If there are several storage systems, the respective $A(k)$ have to be summed up in the criterion $J^{autonomy}$.
Finally, the CO2 equivalent rejection can be computed like the autonomy criteria:

$$J^{CO2eq} = \sum_{k=1}^{K} \sum_{i \in \mathcal{I}^{support}} \tau_{CO2}(i,k)E(i,k) \quad (57)$$

where $\tau_{CO2}(i,k)$ stands for the CO2 equivalent volume rejection for 1 kWh consummed by the support service i and $\mathcal{I}^{support}$ gathers the indexes of predictable support services.
All these criteria can be aggregated into a global criterion. α-criterion approaches can also be used.

5.2 Decomposition into subproblems

In section 2.2, services have been split into permanent and temporary services. Let $\mathcal{I}^{temporary}$ be the indexes of modifiable and predictable temporary services. It is quite usual in housing that some modifiable and predictable temporary services cannot occur at the same time, whatever the solution is. Using this property, the search space can be reduced.
Let's defined the horizon of a service.

Definition 1. *The horizon of a service $SRV(i)$, denoted $H(SRV(i))$, is a time interval in which $SRV(i)$ may consume or produce energy.*

The horizon of a service $SRV(i)$ is denoted: $[\underline{H}(SRV(i)), \overline{H}(SRV(i))] \subseteq [0, K\Delta]$. A permanent service has an horizon equal to $[0, K\Delta]$. A temporary service $SRV(i)$ has an horizon given by $\underline{H}(SRV(i)) = s_{min}(i)$ (the earliest starting of the service) and $\overline{H}(SRV(i)) = f_{max}(i)$ (the latest ending of the service).
Only predictable and modifiable services are considered in the following because they contain decision variables. Two predictable and modifiable services may interact if and only if there is a non empty intersection between their horizons.

Definition 2. *Two predictable and modifiable services $SRV(i)$ and $SRV(j)$ are in direct temporal relation if $H(SRV(i)) \bigcap H(SRV(j)) \neq \emptyset$. The direct temporal relation between $SRV(i)$ and $SRV(j)$ is denoted $\overbrace{SRV(i), SRV(j)} = 1$ if it exists, and $\overbrace{SRV(i), SRV(j)} = 0$ otherwise.*

If $H(SRV(i)) \bigcap H(SRV(j)) = \emptyset$, $SRV(i)$ and $SRV(j)$ are said temporally independent. Even if two services $SRV(i)$ and $SRV(j)$ are not in direct temporal relation, it may exists an indirect relation that can be found by transitivity. For instance, consider an additional service $SRV(l)$. If $\overbrace{SRV(i), SRV(l)} = 1$, $\overbrace{SRV(i), SRV(l)} = 1$ and $\overbrace{SRV(i), SRV(j)} = 0$, $SRV(i)$ and $SRV(j)$ are said to be indirect temporal relation.
Direct temporal relations can be represented by a graph where nodes stands for predictable and modifiable services and edges for direct temporal relations. If the direct temporal relation graph of modifiable and predictable services is not connected, the optimization problem can be split into independent sub-problems. The global solution corresponds to the union of sub-problem solutions (Diestel, 2005). This property is interesting because it may lead to important reduction of the problem complexity.

6. Application example of the mixed-linear programming

After the decomposition into independent sub-problems, each sub-problem related to a specific time horizon can be solved using Mixed-Linear programming. The open source solver GLKP (Makhorin, 2006) has been used to solve the problem but commercial solver such as CPLEX (ILOG, 2006) can also be used. Mixed-Linear programming solvers combined a branch and bound (Lawler & Wood, 1966) algorithm for binary variables with linear programming for continuous variables.
Let's consider a simple example of allocation plan computation for a housing for the next 24h with an anticipative period Δ =1h. The plan starts at 0am. Energy coming from a grid power supplier has to be shared between 3 different end-user services:

- $SRV(1)$ is a room HVAC service whose model is given by (3). According to the inhabitant programming, the room is occupied from 6pm to 6am. Out of the occupation periods, the inhabitant dissatisfaction $D(1,k)$ is not taken into account. Room HVAC service is thus considered here as a permanence service. The thermal behavior is given by:

$$\begin{bmatrix} T_{in}(1,k+1) \\ T_{env}(1,k+1) \end{bmatrix} = \begin{bmatrix} 0.299 & 0.686 \\ 0.203 & 0.794 \end{bmatrix} \begin{bmatrix} T_{in}(1,k) \\ T_{env}(1,k) \end{bmatrix} + \begin{bmatrix} 1.264 \\ 0.336 \end{bmatrix} E(1,k) + \begin{bmatrix} 0.015 & 0.44 \\ 0.004 & 0.116 \end{bmatrix} \begin{bmatrix} T_{ext}(k) \\ \phi_s(1,k) \end{bmatrix} \tag{58}$$

The comfort model of service $SRV(1)$ in period k is

$$D(1,k) = \begin{cases} \dfrac{22 - T_{in}(i,k)}{5} & \text{if } T_{in}(i,k) \leq 22 \\ \dfrac{T_{in}(i,k) - 22}{5} & \text{if } T_{in}(i,k) > 22 \end{cases} \tag{59}$$

The global comfort of service $SRV(1)$ is the sum of comfort model of the whole period:

$$D(1) = \sum_{k=1}^{K} D(1,k) \tag{60}$$

- Service $SRV(2)$ corresponds to an electric water heater. It is considered as a temporary preemptive service. Its horizon is given by $H(SRV(2)) = [3,22]$. The maximal power consumption is 2kW and 3.5kWh can be stored within the heater.