Electricity and magnetism

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C O N N E X I O N S

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Chapter 1

Special theory of relativity[1]

The Newtonian mechanics is considered to be valid in all inertial frames of reference, which are moving at a constant relative velocity with respect to each other. Einstein broadened the scope of this theorem and extended the validity of all physical laws including electromagnetic theory to all inertial frames of reference. Now, constancy of speed of light in vacuum is a core consideration in the electromagnetic theory. Therefore, Einstein postulated that speed of light is a constant in all inertial frames of reference. The speed of light does not depend upon the motion of either the source emitting it or the receiver of the light. This simple assertion about the constancy of the speed of light in vacuum is an epoch making assertion as it contradicts one of the equally fundamental assertion that speed (velocity) is a relative concept and that it essentially depends on the state of motion of observer.

We can comprehend the import of special theory of relativity by a simple example. Let a light pulse is moving in x-direction with its speed "c" and let a space craft is also moving ahead in the same direction with a speed "v". These motions are observed from a position on the ground. Let us also assume that there is no atmosphere and we are observing motions in vacuum. Now, the speed with which light reaches spacecraft should be the relative speed "c-v". This is what we deduce classically. Special theory of relativity, however, asserts that the relative speed of light with respect to spacecraft is "c" only – notwithstanding the speed of spacecraft (v).

[1] This content is available online at <http://cnx.org/content/m32527/1.36/>.

Motion of a light pulse and a spacecraft

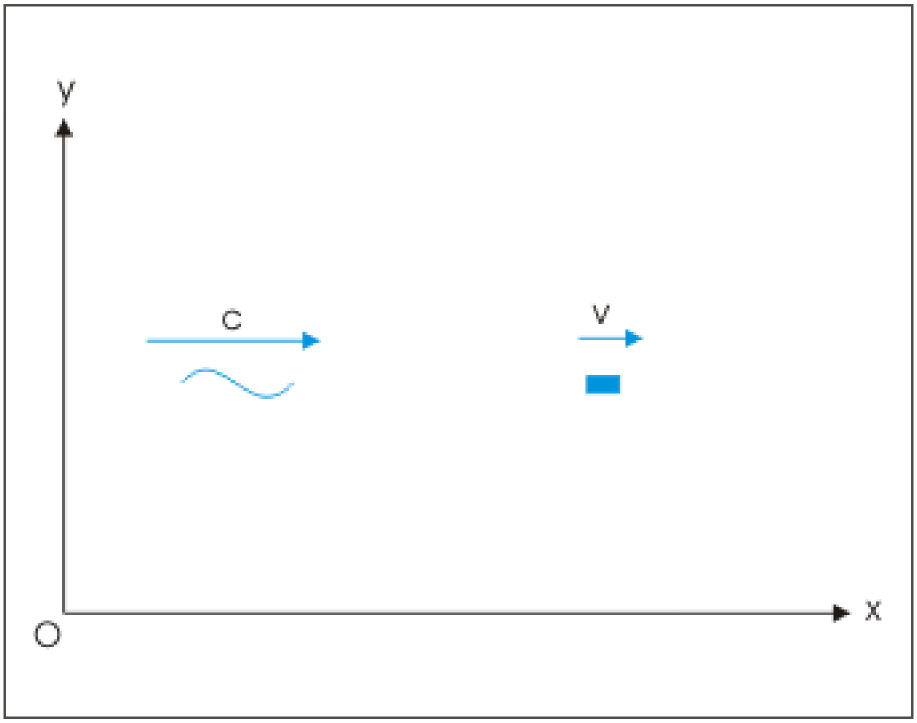

Figure 1.1: Motion of a light pulse and a spacecraft

The physical interpretation of the assertion of special theory of relativity is quite unthinkable classically. The constant relative speed of approach by light in the above example is possible only if the constituents of speed (distance and time) are different for observers having different motions. In the instant example, both "distance" and "time" as measured by spacecraft are different than the corresponding measurements by a ground observer which is observing motions of both light and spacecraft. The measurements of "distance" and "time" in two different frames of reference need to be different such that speed ratio for light in vacuum i.e. "x/t" or "x'/t'" in two inertial references (parameters in one reference is denoted by unprimed varibales whereas parameters in other reference is denoted be primed variables) remains a constant.

Motion of a light pulse and a spacecraft

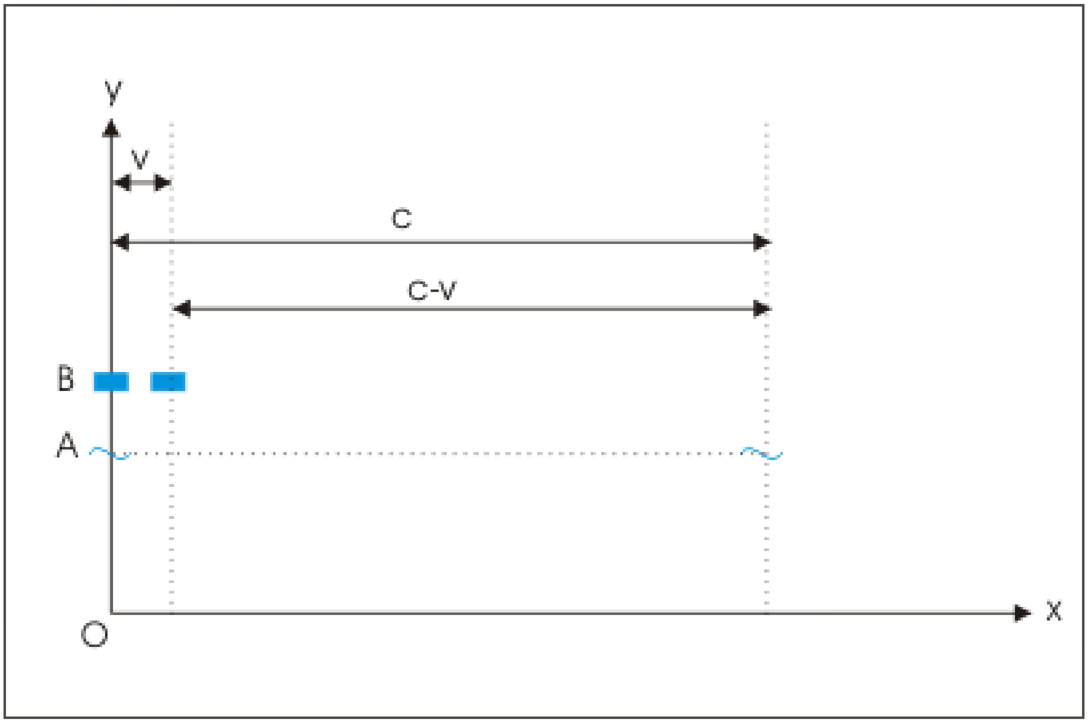

Figure 1.2: Motion of a light pulse and a spacecraft

In the figure above, we consider motion of a light pulse and spacecraft which are moving with speed "c" and "v" respectively in x-direction. They are initially at x=0 when t =0. The positions of light pulse and spacecraft are also shown after 1 second. As seen from the reference of ground (coordinate system), pulse and spacecraft travel "c" and "v" meters respectively. The linear distance between spacecraft and pulse after 1 second is "c-v" in ground reference. But according to special relativity, the linear distance between light pulse and spacecraft after 1 second should be "c" in the reference of spacecraft. As "c-v" can not be "c", it is deduced that measurements of distance and time in two references are different. A part of discripancy is due to difference in the measurement of distance and the remaining due to difference in the measurement of time. These diffferences need to be such that ratio of ditance and time is a constant for the pulse of light in all inertial references.

In essence, special theory of relativity removes "relativity" from "speed of light" and attaches "relativity" to "space (distance)" and "time". This is the difficult part. Classically, we have considered both these elements as universally invariants with respect to all frames of reference which are moving with constant relative velocity (inertial frames of reference). We shall try to come to terms with these new ideas in subsequent modules. But the essence of special theory of relativity is captured as follows : The speed of light in vacuum is invariant whereas distance between two points and time intervals are variant in the system of inertial frames of reference.

The ideas of classical relativity, where in space and time are invarinat and speed of light is variant, is captured by Galilean transformation which enables us to measure motion in two inertial frames of reference. The speed of an object is modified by the relative speed of the frames of reference.

$u' = u \pm v$

where "u" and "u"' are the speeds of an object as measured in two frames of reference which themselves move with a speed "v" with respect to each other.

Einstein employed a different transform called "Lorentz transformation" to capture the idea of invariant speed of light and variant distance and time measurements. The Lorentz tansformation provides the exact relation between coordinates (space and time) of inertial references. We shall discuss these transformations separately in the module.

Further, since we are considering constancy of speed of light in relation to inertial references only, the special theory of relativity is "restricted" to inertial frames of reference and therefore is "special" not "general".

1.1 Postulates of Special Theory of relativity

There are many versions of postulates. The essence of special theory of relativity is finally agreed to be captured by following two principles/ postulates :

1. The principle of relativity : The laws of physics are the same for all observers in uniform motion relative to one another (inertial frames of reference).

2. The principle of constancy of speed of light in vacuum : Light in vacuum propagates with the constant speed through all systems of inertial coordinates, regardless of the state of motion of the light source.

Few scholars consider either of above postulates sufficient to describe special theory of relativity. They are supplementary to each other. As a matter of fact, one can be deduced from other and vice –versa with certain extrapolation.

Proceeding from the principle of relativity, we can arrive at the principle of constancy of speed of light in vacuum. The principle of relativity considers validity of all physical laws across all inertial frames of reference. This means that law of propagation of light (electromagnetic theory) is same across coordinates systems in uniform translatory motion. But, the law of propagation of light says that light moves at a constant speed in vacuum and is independent of the motion of source. Thus, speed of light is constant in terms of any system of inertial coordinates, regardless of the state of motion of the light source. This is exactly the the principle of constancy of speed of light in vacuum.

Similarly, we can proceed from the principle of constancy of speed of light in vacuum to the principle of relativity. If we accept constancy of speed of light in vacuum across all inertial references, then we consider that law of propagation of light in vacuum (electromagnetic theory) is valid in them. Now, the laws of motion are already considered to be independent of inertial frames of reference. Addition of electromagnetic theory to this class of invariants suggests that other physical laws in their simplest form are also valid in all inertial references. This is exactly the principle of relativity.

Clearly, two principles are deducible from each other. Yet, we require to state special theory of relativity in terms of two principles. We see that though we are able to deduce second principle from first, but in the process we have narrowed the scope of principle of relativity. The principle of relativity is a very general principle extending to all physical laws - not only to laws of motion and propagation of light. Similarly, the deduction of first principle from second is not direct deduction - rather an extension. For these reasons, it is generally prudent to state both the principles of special theory of relativity.

The important consideration of special theory of relativity is the inclusion of Maxwell's electromagnetic theory being valid in inertial references. Earlier we limited the scope of validity only to Newton's laws of motion. We should understand that Newton's laws of motion are special case of a more general special theory of relativity. Let us have a look at the validity of the Newton's laws motion in inertial references involving relativistic consideration at higher speed :

1: Newton's first law of motion of motion is valid in inertial references.

2: Newton's second law of motion which defines force in terms of time rate of change of linear momentum is valid in inertial references.

3: Newton's second law of motion which defines force in terms of the product of mass and acceleration is not valid in inertial references, because mass is not a constant in relativistic mechanics. The more general relativistic or modified form of Newton's second law valid in all inertial references, however, reduces to classical Newton's second law of motion at lower speed.

4: Newton's third law of motion as stated in the form of equal and opposite action and reaction is not valid in inertial references.

5: The conservation of linear momentum, which is the consequence of Newton's third law, is valid in inertial references.

We shall return to these aspects in detail subsequently.

1.2 Studying special theory of relativity

The idea of constancy of speed of light in all inertial references shakes up well rooted concepts about distance (space) and time. It raises many questions and makes the study of special theory of relativity a bit difficult. Generally the explanations appear to be inadequate or not very convincing. As a matter of fact, there is a temptation to view the theory with a sense of disbelief. But the fact of life is that relativity (we shall use this term to mean "special theory of relativity" for brevity), there is not even a single "exception of" or "departure from" the predictions of special relativity as on date (spanning a period of more than a century).

After many readings of relativity theories, it emerges that it would be futile to follow the conventional approach of studying relativity by explaining the "unthinkable" first and then derive conclusions. No description, however good, satisfies a reader that incidence of time for an event or length of a rod is different in two inertial references. Keeping this aspect of study in mind, we shall attempt a slightly different approach here. Upfront, we shall accept relativistic assertions about distance (space) and time. This is a better approach as it allows us to proceed with the theory and come back to the lingering thoughts when we are equipped with the basic or working knowledge of the theory. After all, electromagnetic theory of light (and hence constancy of speed of light in vacuum) is such an elegant and complete theory that we can only be more than willing to accept assertions which are based on it.

Yet another aspect of the study of relativity is that it relates phenomena which occur over a very large spatial extent. The consideration of motion of light even for 0.1 second involves a linear extent of 30000000 meters. Clearly, there is limitation to pick examples or illustration from our real world. Most of the experiments or illustrations cited in the study of relativity are reasonable as imagined. Conception of special theory of relativity is more an outcome of "experiments in head" than the actual ones, but such experiments are rigorous and subject to direct or indirect scientific verification. This process of performing mental experiments is known by a German term "Gedankenexperimenten". Einstein used this process often to reach conclusions. Clearly, we shall also be required to do a bit of "Gedankenexperimenten" to understand his theories. In a nutshell, we should be ready to imagine spacecrafts or space objects moving at very high speeds without any inhibition. We may even imagine ourselves sitting in those high speed spacecraft. Similarly, we may imagine a train which is moving at a speed of say 100000 km/hr. Apart form the scientific validity of reasoning, there is no constraint in imagining experiments or examples which otherwise can not be realized in our small world.

1.3 What is time ?

We do not know exactly what is time. But we know some of its properties. The closest that we come to define time is about the manner in which we measure it. This measurement is essentially an outcome of the characteristic of time known as "simultaneity". Einstein wrote "That train arrives here at 7 o'clock", I mean something like this : "The pointing of the small hand of my watch to 7 and the arrival of the train are simultaneous events." Thus, we measure time of an event by way of the simultaneity of two events – one belonging to measuring device and other belonging to an arbitrary event like arrival of a train. This argument clinches the issue of time from the relativistic perspective. If we are able to prove that two events which are simultaneous in one inertial frame of reference but "not" simultaneous in another, then we can be sure that measurements of time in two inertial references could indeed be different.

In special theory of relativity, time and time rate in two inertial references are treated as different. If t and t' be the time recorded for a given event in two inertial references, then t may not equal to t'. We shall return to this topic again.

1.4 Absolute and stationary reference

There is no preferred inertial reference frame. This idea predates special relativity. It means that there is no absolute reference frame. Had there been an absolute reference, then we would have a fixed universal space in which all other objects (references) would be considered to be either in rest or moving. But the concept of space is a variant. In other words, the perception of space changes from one reference (say ground) to other reference (say moving train). If we drop a stone from a train, then the trajectory of the stone is a straight vertical line for an observer on the train. The same stone, however, is seen to follow a parabolic trajectory for an observer on the ground. Referring to these trajectories for a single motion of a stone, Einstein questioned the very concept of fixed space.

Trajectory of motion

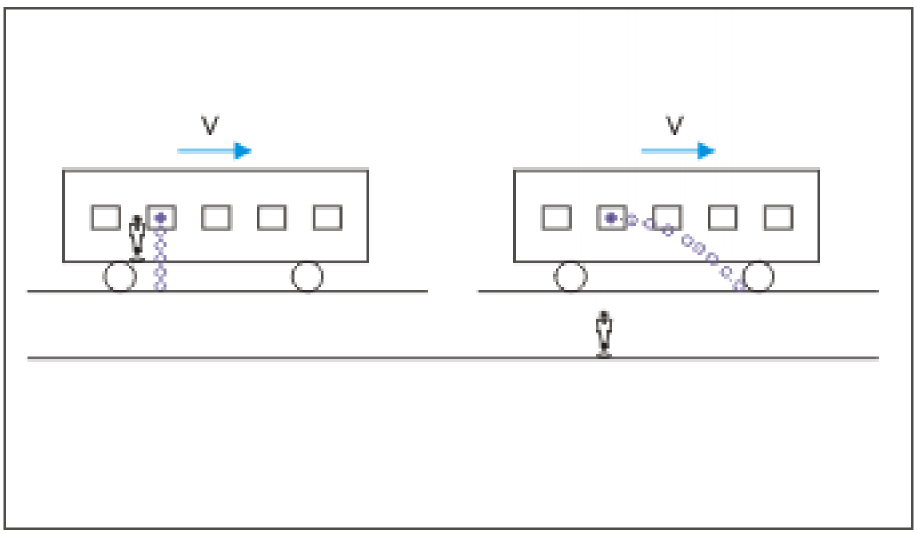

Figure 1.3: Trajectory of motion

Though there is no absolute reference, but the notion of a stationary reference is a powerful idea which stems from our life long perception of stationary objects in Earth's reference. Despite the fact that Earth is moving at about 107,278 km/hr (29.8 km/s) around Sun, we are generally not aware about it unless we make detailed observations about celestial objects. But as the concept of stationary system is ingrained in our perception, we employ this concept with great effect in the study of relative motion. We refer either of the moving systems as stationary in which an observer is making the measurements. Consider for example two spacecrafts moving with uniform velocities. We can refer either of spacecrafts as stationary and other as moving with a velocity which is measured from the referred stationary reference (spacecraft). There is no preference. In the case of a motion of a train, for example, we consider Earth as stationary and train as moving reference. There is no bar though that we consider moving train as stationary reference with the observer and Earth as moving reference in the opposite direction to the moving train.

The "rest" is local concept. The object like a house is at rest in Earth's reference. But the same object as seen from a spaceship is not stationary.

1.5 Constancy of speed of light

Constancy of speed of light has two different considerations in the study of relativity. In the first place, it is the central idea of special relativity. But besides this consideration, the constancy of light has other important consideration in that it is one of the measuring standards which can not be challenged in any inertial reference. This aspect is important as meaning of space and time in different inertial frames is not very explicit. We shall see subsequently that they are in fact entangled. Further, the distance (space) and time are relative and are therefore very subjective in conception and measurement.

On the other hand, speed of light in vacuum is invariant in inertial references (though it is not invariant in accelerated references). As such, it can be used as a parameter to measure "time" and "distance". A linear distance, for example, can be expressed in terms of "time" taken by light to cover a given distance. Alternatively, a particular time interval can be expressed in terms of "linear distance" covered by the light in a given time.

The official measure of speed of light in vacuum is as given here :

c = 299,792,458 meters/second

1.6 Galilean transformation

The transforms are mathematical constructs which allow us to convert one set of spatial (x,y,z) and time (t) measurements in one frame of reference to another frame of reference based on certain physical principle or law. Our current context is limited to inertial frames of reference. Therefore, we shall study transforms which refer to inertial frames of reference. Here, we shall study Galilean and Lorentz transforms. The Galilean transform encapsulates the ideas of non-relativistic mechanics whereas Lorentz transform encapsulates the ideas of relativistic mechanics.

The concepts of a transform, physical laws and inertial frames of reference are entangled with each other. The physical laws are required to be valid across all inertial frames of references.

Galilean transform gives the relation between two inertial systems which are moving at a constant velocity with respect to each other. If space (co-ordinates) and time values in one reference are known, then we can find out space and time values using Galilean transform in another reference which is moving at a constant velocity "v' with respect to first in x-direction. Let two inertial reference systems are denoted by unprimed and primed variables and their spatial origins coincide at t = t' = 0. Then, space (x',y',z') and time (t') co-ordinates of a "single arbitrary event" in primed inertial reference is related to space (x,y,z) and time (t) of unprimed inertial reference as :

Galilean transformation

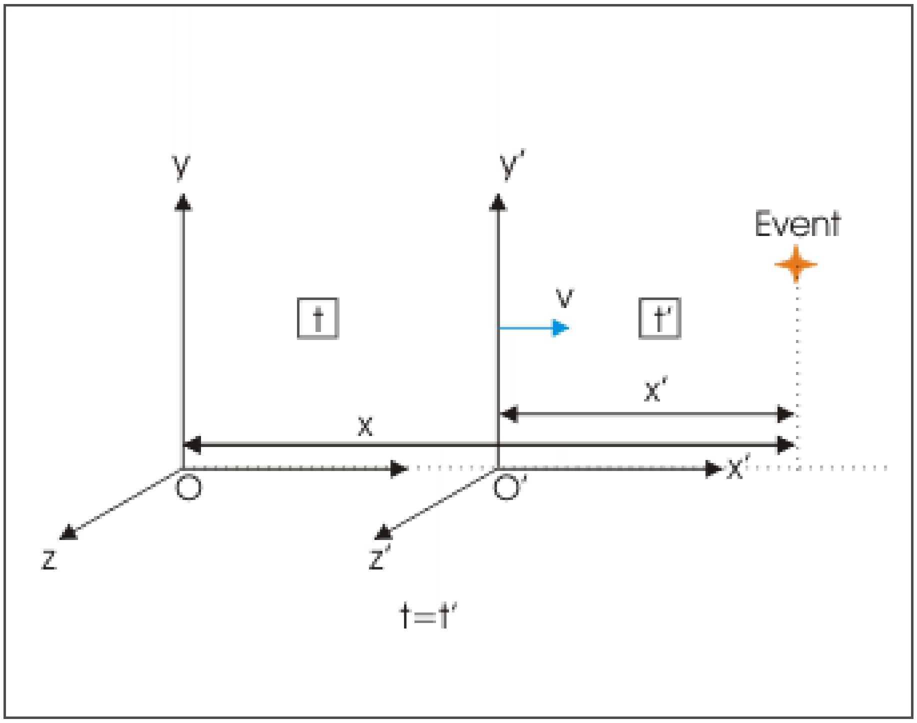

Figure 1.4: Time is same in two inertial references.

$$x\prime = x - vt$$

$$y\prime = y$$

$$z\prime = z$$

$$t\prime = t$$

We can also express unprimed variables in terms of primed variable by solving for unprimed variable as :

$$x = x\prime + vt$$

$$y = y\prime$$

$$z = z\prime$$

$$t = t\prime$$

The most important aspect of Galilean transform is the last equation, t' = t, denoting that time is an invariant for inertial frames of references. The constancy of time across inertial frames of reference is the key consideration here. With the advent of special theory of relativity, however, this transform is considered as a restricted case as it is valid for small relative speed,v, only. At higher values of relative speed “v”, we need to employ Lorentz transform in accordance with special theory of relativity such that speed of light in vacuum is constant in all inertial references.

Further, we get the equation for the velocities of a particle or object at position "x" or "x’" in the unprimed and primed references respectively by differentiating first equation of the transform,

u’ = u - v

where “u” and “u”’ are the speeds of a particle or object as measured in two frames of reference which themselves move with a speed “v” with respect to each other.

1.7 Lorentz transformation

Like Galilean transform, Lorentz transform provides relation for space and time between inertial systems for all possible range of relative velocity. Importantly, it satisfies the postulate of special theory of relativity that speed of light in vacuum is a constant. The derivation of Lorentz transform has elaborate historical perspectives and is also the subject of insight into the relativistic space and time concepts. For this reason, we shall keep the derivation of this separate to be dealt later. Here, we shall restrict our consideration to the final form of Lorentz transform only. Let two inertial reference systems are denoted by unprimed and primed variables and their spatial origins coincide at t = t’ = 0. Then, space (x’,y’,z’) and time (t’) co-ordinates of a "single arbitrary event" in primed inertial reference is related to space (x,y,z) and time (t) of primed inertial reference as :

Lorentz transformation

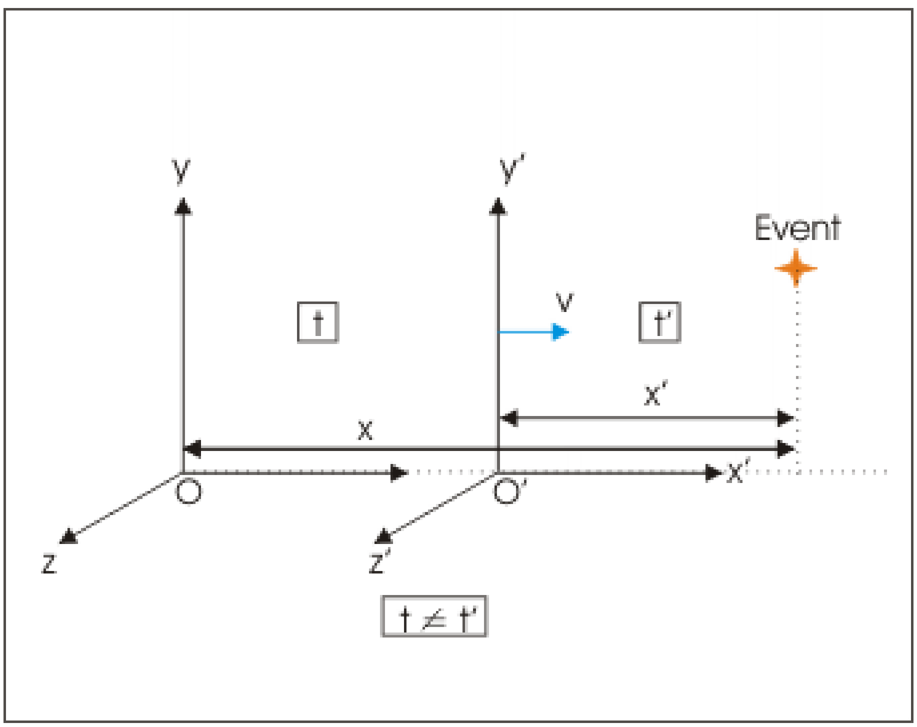

Figure 1.5: Time is not same in two inertial references.

$$x\prime = \gamma \left(x - vt\right)$$

$$y\prime = y$$

$$z\prime = z$$

$$t\prime = \gamma \left(t - \frac{vx}{c^2}\right)$$

where,

$$\gamma = \frac{1}{\sqrt{(1-\beta^2)}} = \frac{1}{\sqrt{\left(1-\frac{v^2}{c^2}\right)}}$$

The dimensionless γ is called Lorentz factor and dimensionless β is called speed factor. For small relative velocity, v, the terms $v^2/c^2 \rightarrow 0$, $v/c^2 \rightarrow 0$ and $\gamma \rightarrow 1$. In this case, the Lorentz transform is reduced to Galilean transform as expected. Further, we can write transformation in the direction from primed to unprimed reference as :

$$x = \gamma \left(x\prime + vt\prime\right)$$

$$t = \gamma \left(t\prime + \frac{vx\prime}{c^2}\right)$$

Note the change of the sign between terms on right hand side.

1.7.1 Transformation involving two events

If two events, separated by a distance, occur along x axis at two instants, then we can write Lorentz transformations of space and time differences using following notations :

$$\Delta x = x_2 - x_1; \quad \Delta t = t_2 - t_1; \quad \Delta x\prime = x_2\prime - x_1\prime; \quad \Delta t\prime = t_2\prime - t_1\prime$$

The subscripts 1 and 2 denote two events respectively. The transformations in the direction from unprimed to primed references are :

$$\Delta x\prime = \gamma \left(\Delta x - v\Delta t\right)$$

$$\Delta t\prime = \gamma \left(\Delta t - \frac{v\Delta x}{c^2}\right)$$

The transformations in the direction from primed to unprimed references are :

$$\Delta x = \gamma \left(\Delta x\prime + v\Delta t\prime\right)$$

$$\Delta t\prime = \gamma \left(\Delta t\prime + \frac{v\Delta x\prime}{c^2}\right)$$

1.7.2 Constancy of speed of light in inertial references

We can test Lorentz transform against the basic assumption that speed of light in vacuum is constant. Let a light pulse moves along x-axis. Then, consideration in unprimed reference gives speed of light as :

$$c = \frac{x}{t}$$

If Lorentz transform satisfies special theory of relativity for constancy of speed of light, then the propagation of light as seen from the primed reference should also yield the ratio x'/t' equal to c i.e. speed of light in vacuum. Now,

$$\frac{x\prime}{t\prime} = \frac{\gamma\left(x - vt\right)}{\gamma\left(t - \frac{vx}{c^2}\right)} = \frac{\left(x - vt\right)}{\left(t - \frac{vx}{c^2}\right)}$$

Dividing numerator and denominator by "t" and substituting x/t by c, we have :

$$\Rightarrow \frac{x\prime}{t\prime} = \frac{\left(\frac{x}{t} - v\right)}{\left(1 - \frac{vx}{tc^2}\right)}$$

$$\Rightarrow \frac{x\prime}{t\prime} = \frac{\left(c - v\right)}{\left(1 - \frac{vc}{c^2}\right)} = \frac{\left(c - v\right)}{\left(1 - \frac{v}{c}\right)}$$

$$\Rightarrow \frac{x\prime}{t\prime} = \frac{c\left(c - v\right)}{\left(c - v\right)} = c$$

Clearly, Lorentz transform meets the requirement of special theory of relativity in so far as to guarantee that speed of light in vacuum is indeed a constant.

1.8 Lorentz factor

Lorentz factor ,γ, is the multiplicative factor in the transformation equations for x-coordinate and time. It is a dimensionless number whose value depends on the relative speed "v". Note that the relativistic transformation for x-coordinate is just Lorentz factor times the non-relativistic or Galilean transformation.

$$x\prime = \gamma\left(x - vt\right)$$

Lorentz factor appears in most of the relativistic equations including the calculation of relativistic effects like time dilation, length contraction, mass etc. An understanding of the beahviour of this factor at different relative velocity is intuitive for assessing the extent of relativistic effect. Few values of Lorentz factor are tabulated here.

Lorentz factors

Speed (v)	**0 0.1c 0.2c 0.3c 0.4c 0.5c 0.6c 0.7c 0.8c 0.9c 0.99c 0.999c**
Lorentz factor (γ)	1.000 1.005 1.021 1.048 1.091 1.115 1.250 1.400 1.667 2.294 7.089 22.366

Table 1.1

Lorentz factor begins at 1 and as v->c, y-> infinity. It is either equal to 1 or greater than 1. In other words, it is never less than 1. A plot of Lorentz factor .vs. relative speed is shown here.