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Concurrent Process Tolerancing Based on Manufacturing Cost And Quality Loss

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1. Introduction

In manufacturing practice, actual dimensions are impossible as well as unnecessary to determine exact values. Under stable fabrication conditions, the processed dimensions often vary within certain controlled ranges. Tolerances are specified to control the actual dimensions of processed features within allowable variation zones for product functional requirements and manufacturing costs (Zhang, 1996; Ngoi and Teck, 1997; Lee and Tang, 2000; Fang and Wu, 2000; Huang et al., 2001; Huang and Gao, 2003; Chen et al., 2003).

The contemporary practice of tolerance design has two sequential phases: Product tolerance design and process tolerance design (Ngoi and Teck, 1997).

In product tolerance design, designers use their knowledge and expertise to determine the assembly critical tolerances by computation or design handbooks. These tolerances will then be allocated to component design tolerances (blueprint tolerances) in terms of component structures, assembly restrictions, and given design criteria. If a mathematical model is used, the objective function is usually to minimize manufacturing costs or to maximize weighted component tolerances. The constraints are often tolerance stack-up and economical tolerance ranges of each component part (Swift et al., 1999; Ngoi and Min, 1999; Ngoi and Ong, 1999; Huang and Gao, 2002). Swift *et al* (1999) presented a tolerance optimization model in assembly stacks based on capacity design. In their research, systematic analysis for estimating process capability levels at the design stage is used in conjunction with statistical methods for optimization of tolerances in assembly stacks. Ngoi and Min (1999) presented a new approach for optimum tolerance allocation in assembly. Their method allows all blueprint (BP) tolerances to be determined while ensuring that all as-

sembly requirements are satisfied. Ngoi and Ong (1999) presented a complete tolerance charting in the assembly phase. Their method integrates product tolerance design and process tolerance design. The objective is to maximize the summation of weighted process tolerances. Huang and Gao (2002) presented a discrete hierarchy optimal approach for allocating the optimum component tolerance based on estimated process capability. They minimize the total manufacturing cost by using a cost-tolerance function.

In process tolerance design, manufacturing engineers develop component process planning to determine manufacturing methods, machine tools, fixtures, cutting tools, cutting conditions, manufacturing routines, and process tolerances. At this stage, BP tolerances are the most important factors. If they are too tight and cannot guarantee the economic fabrication for components by using selected process planning, more precise machine tools, special fixtures, and expensive measurements should be introduced (Wu et al., 1998). This inevitably increases the manufacturing cost of the product. The manufacturing engineers may ask for revision of BP tolerances or of the process plan. In process tolerance design, the most popular methods are also the optimal design for minimum manufacturing cost or maximum process tolerances. Huang *et al.* (2002) presented an optimal planar tolerance design approach to allocate dimensional and orientation geometric tolerances. A special relevance graph (SRG) was used to represent the relationships between manufactured elements and their size and tolerance information. In addition, the SRG is also applied for the geometric dimensions and tolerances. A linear programming model was established to solve the problem. Huang and Gao (2003) presented a nonlinear programming model for optimal process tolerance balancing. A linear programming model to determine process dimensions and process tolerances was used in Ji (1993) and Ngoi and Teck (1993). Similar methods to determine optimum process tolerances were proposed by Wei and Lee (1995) and Chang *et al.*, (2000).

Though the above methods have been used successfully to distribute both component design tolerances and process tolerances in two different phases, they over-emphasize manufacturing factors and seldom consider quality aspects. Systematically, product satisfaction conflicts with manufacturing cost. In other words, a better product satisfaction requires smaller tolerances and a higher manufacturing cost. Taguchi quality loss is a useful monetary specification to evaluate the quality factors (Taguchi et al., 1989; Taguchi, 1993; Jeang, 1998). Therefore the best policy is to consolidate manufacturing cost and quality loss in the same optimization objective to best balance quality satisfaction

and tolerances (Taguchi, 1993; Huang and Gao, 2002). Using this method, the research work has been carried out in product design and component process planning stages, respectively. Lee and Tang (Lee and Tang, 2000) presented an optimization model for controlling dimensional tolerances of components with multiple functional characteristics by minimizing the sum of manufacturing cost and quality loss. Jeang (1998) introduced a mathematical optimization model to integrate manufacturing cost and quality loss for tolerance charting balancing during machining process planning. Jeang (1997) also discussed a set of models to determine the optimal product tolerance and to minimize combined manufacturing and related costs.

Although tolerance assignment in the product design and process planning stages is often interdependent and interactive and affects overall production costs and product satisfaction, research into these areas is often conducted separately (Ngoi and Teck, 1997). There are some inherent shortcomings in this method. Firstly, in product tolerance design, designers are unable to allocate the real optimal BP tolerances to components because there is no manufacturing information available at this stage. Secondly, in process tolerance design, manufacturing engineers develop process planning in terms of the component information obtained from mechanical drawings, technical notes, and others such as title bars. They are less concerned with functional roles of components than with their manufacturing capabilities. This sequential tolerance design method would result in some problems in cooperation, continuity, and consistency between two separate design stages. Therefore, rework or redesign cannot be avoided.

Until recently, the concurrent tolerancing method has attracted the attention of some engineers (Zhang, 1996; Ngoi and Teck, 1997; Fang et al., 1998; Fang and Wu, 2000; Huang et al., 2001, Huang and Gao, 2003; Chen et al., 2003). Zhang (1996) first systematically presented mathematical methods for concurrent tolerancing and developed a general model of optimal tolerancing that supports concurrent engineering. Ngoi and Teck (1997) proposed a concurrent tolerancing method for product design in which the assembly tolerance can be allocated to the component design tolerance in an early stage of product design. Fang et al. (1998) proposed a concurrent tolerancing method to determine the optimum process tolerances with manufacturing cost and quality loss being considered simultaneously. But only a single assembly critical tolerance is related. Fang and Wu (2000) proposed a mathematical model to minimize the cost of sum machining. The constraints include assembly functional requirements, machining methods, stock remove tolerances, and economically attain-

able accuracies. Huang *et al.* (2001) proposed a special relative hierarchical hypergraph (SRHG) to represent the assembly. Through use of SRHG, assembly and process tolerance chains can be generated automatically. The method can allocate required assembly tolerances to process tolerances concurrently. Huang and Gao (2003) and Chen *et al.* (2003) proposed a concurrent method to allocate the optimal process tolerances in early product design stages. Here, a nonlinear optimization model is established to minimize the total manufacturing cost.

So far no design method has been presented to directly allocate multiple correlated critical tolerances to their process tolerances in a concurrent design environment. Therefore, the purpose of this paper is to introduce a concurrent optimal tolerancing method to realize this goal. To implement optimal robust tolerance design from product design stage to manufacturing stage, we first derive the quality loss function of multiple correlated critical tolerances in terms of manufacturing tolerances. A nonlinear optimization model is then given to minimize the summation of total component manufacturing cost and product quality loss. Finally the optimal processes are obtained by solving the model.

This chapter is divided into the following sections. Section 2 discusses the models for converting the geometrical tolerances with fixed tolerance zones into equivalent bilateral sized dimensions and tolerances. In section 3, we discuss the methods to present concurrent dimensional and geometrical tolerance chains. Section 4 further describes integrated concurrent dimensioning and dimensioning. In Section 5 we derive the quality loss of multiple correlated critical dimensions in terms of the process tolerances. In Section 6 we develop the optimal tolerance design model, whereas Section 7 examines the implementation for a specific example. The concluding remarks are given in Section 8.

2. Models for interpretation of geometrical tolerances

Geometric tolerances are usually expressed as graphical symbols, which can contain nominal sizes, tolerance values, and data (references). In order to deal with geometric tolerances in integrated tolerance charts, their geometrical characteristics must be addressed first. Generally geometric tolerances can be classified into five types: individual form, profile, orientation, location, and runout. There are fourteen geometric tolerances items altogether but only

those items with fixed tolerance zones will directly affect tolerance chains. Consequently only four geometrical tolerances in the total fourteen can be included in the integrated tolerance chains. These items — profile, position, symmetry, and concentricity — can be converted into the equivalent bilateral dimensional tolerances. The remaining items are treated as additional tolerance constraints (He & Gibson, 1992; Ngoi & Tan, 1995; Ngoi & Soew, 1996; Tseng & Kung, 1999).

2.1. Profile of a line (surface)

Profile of a line (surface) defines a permitted variation zone of a line (surface) relative to the corresponding theoretical geometry. It can be used to specify the geometrical requirements of an individual and a relevant feature in terms of different graphical notations in mechanical drawing. When profile of a line (surface) tolerance is used to denote an individual feature, then this item doesn't contribute to tolerance stack-up. Thus it can be treated as additional tolerance constraints. However, when profile of a line (surface) tolerance is used to specify a relevant feature, this item possesses a fixed tolerance zone. Thus it can be treated as equivalent bilateral dimensional tolerance. Figure 1 is the interpretation of the relevant profile of a surface. The relationship between profile of a line (surface) and their pertinent processed working dimensions and tolerances can be expressed as:

$$GL \pm TGL = \sum_{i=1}^{n} \xi_i WD_i \pm TWD_i \qquad (1)$$

Where GL and TGL is the nominal dimension the tolerance between the controlled line (surface) and the data (reference), respectively. WD_i and TWD_i is the ith working dimension and tolerance, respectively. ξ_i is the unit vector of WD_i, n is the total number of working dimensions and tolerances.
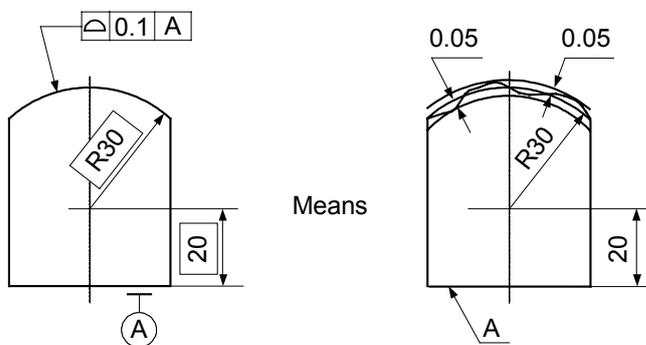
Figure 1. Interpretation of profile of a relevant surface

2.2 Position

Position tolerance defines the true position of a feature with respect to the references or the data. Because position tolerance holds a fixed tolerance zone with respect to the data, it can be transformed into equivalent bilateral dimensional tolerance. All the pertinent dimensions and tolerances in determining position of the controlled feature with respect to the data will be the link members of the position tolerance. Figure 2 is the interpretation of position tolerance. The transform model between position tolerance and their pertinent processed working dimensions and tolerances is:

$$GP \pm TGP = \sum\nolimits_{i=1}^{n} \xi_i WD_i \pm TWD_i \qquad (2)$$

Where GP and TGP is the nominal dimension and position tolerance from the controlled feature to the data, respectively. WD_i and TWD_i is the ith working dimension and tolerance, respectively. ξ_i is the unit vector of WD_i, n is the total number of working dimensions and tolerances. In Figure 2 the position tolerance value is specified when the controlled hole is under the maximum material condition.
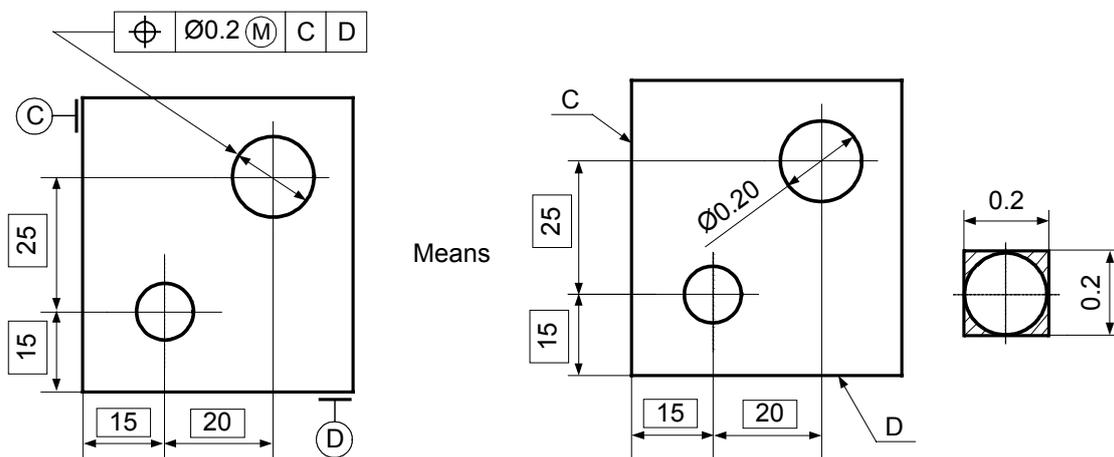
Figure 2. Interpretation of position

2.3 Concentricity

Concentricity tolerance expresses the requirement that the controlled axis should locate within the given allowable cylinder zone whose axis is the datum axis. Thus all the pertinent dimensions contribute to the dimension be-

tween the controlled axis and the datum axis will be the link members of this specification. Figure 3 shows a simple example for interpretation of concentricity into its equivalent bilateral dimensional tolerance. The model for interpretation of concentricity is:

$$GA \pm TGA = \sum_{i=1}^{n} \xi_i WD_i \pm TWD_i \qquad (3)$$

Where GA and TGA is the nominal dimension and concentricity tolerance between the controlled axis and the datum axis, respectively. Generally this dimension is zero. WD_i and TWD_i is the working dimension and tolerance for the ith link member of GA, respectively. ξ_i is the unit vector of WD_i. n is the number of link members.

2.4 Symmetry

Symmetry tolerance presents the requirement that the controlled centre relevant feature such as the centre line of a hole, or the centre plane of a slot should locate within the given zone with respect to the datum. So all the related dimensions contribute to the dimension for determining the location of the controlled feature with respect to the datum will be the link member of this specification. Figure 4 gives a simple example for interpretation of symmetry into its equivalent dimensional tolerance specification. The model for interpretation of symmetry is:

$$GB \pm TGB = \sum_{i=1}^{n} \xi_i WD_i \pm TWD_i \qquad (4)$$

Where GB and TGB is the nominal dimension and symmetry tolerance between the controlled center features with respect to the datum, respectively. Generally, this dimension is zero. WD_i and TWD_i is the working dimension for the ith link member of GB, respectively. ξ_i is the unit vector of WD_i. n is the number of link members.
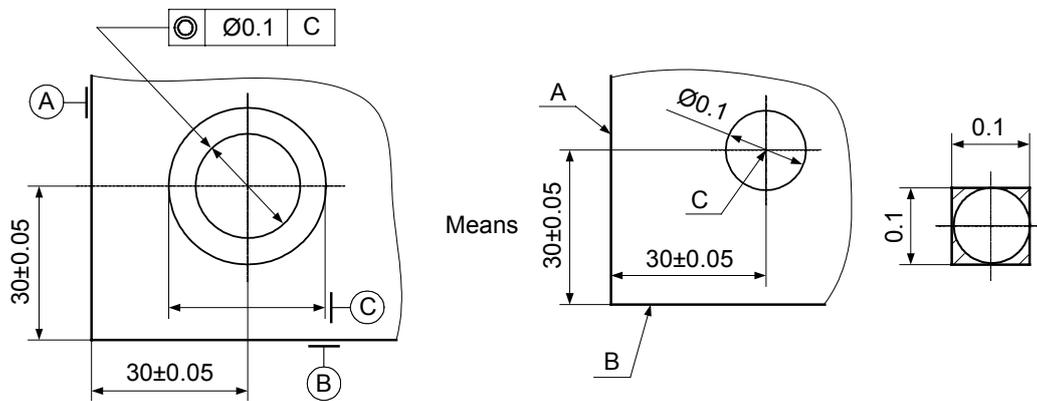
Figure 3. Interpretation of concentricity
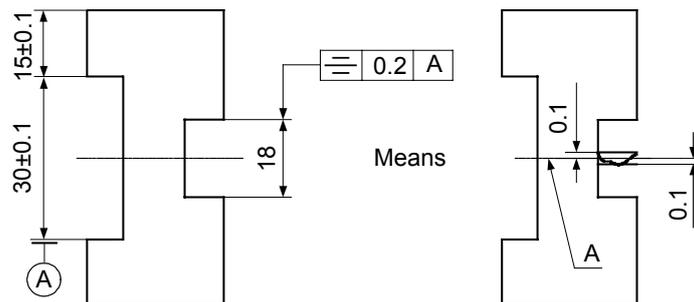
Figure 4. Interpretation of symmetry

3. Concurrent dimensional and geometric tolerance chains

In a concurrent tolerancing environment one of the most important issues is presentation of the concurrent integrated dimensional and geometric tolerance (DGT) chains. In a conventional system, tolerance design is being executed in two separate sequential stages: BP tolerance design and process tolerance design. Unlike the methods presented by several researchers (Ngoi & Tan, 1995; Zhang, 1996; Huang et al., 2001; Huang and Gao, 2002; Gao and Huang, 2003; Chen et al., 2003), this paper presents a general methodology for concurrent allocation of the required assembly functional DGTs to the component process ones.

In the stage of product design, let all the required assembly functional dimen-

sions and tolerances be the set $S_{AD} = \{L_{ADi} \pm T_{ADi} / 2, i = 1, \ldots, n\}$, where n is the number of functional dimensions and tolerances, L_{ADi} is the ith assembly functional dimension, T_{ADi} is the tolerance of L_{ADi}. Also all the assembly functional geometric tolerances which can be modeled as equivalent dimensions and tolerances be the set $S_{AG} = \{L_{AGi} \pm T_{AGi} / 2, i = 1, \ldots, m\}$, where m is the number of functional geometric tolerances which can be treated as equivalent bilateral dimensional tolerances, L_{AGi} is the ith equivalent assembly functional dimension, T_{AGi} is the geometric tolerance of L_{AGi}. And all the assembly functional geometric tolerances which can be modeled as additional tolerance constraints be the set $S'_{AG} = \{L_{AGi}(T_{AGi}), \; i = m+1, \ldots, m+\beta\}$, where β is the number of functional geometric tolerances which can be treated as additional tolerance constraints, T_{AGi} is the geometric tolerance treated as additional tolerance constraint, $L_{AGi}(T_{AGi})$ is the ith equivalent assembly functional dimension.

For simplicity the set notation is introduced as $S_{AF} = \{S_{AD}, S_{AG}, S'_{AG}\} = \{L_{AFi} \pm T_{AFi}/2, i = 1, \ldots, n+m, L_{AGi}(T_{AGi}), i = n+m+1, \ldots, n+m+\beta\}$. Where $\{L_{AFi} \pm T_{AFi}/2, i = 1, \ldots, n\}$ corresponds to $S_{AG} = \{L_{ADi} \pm T_{ADi}/2, i = 1, \ldots, n\}$, $\{L_{AFi} \pm T_{AFi}/2, i = n+1, \ldots, n+m\}$ corresponds to $S_{AG} = \{L_{AGi} \pm T_{AGi}/2, i = 1, \ldots, m\}$, and $\{L_{AFi}(T_{AFi}), i = n+m+1, \ldots, n+m+\beta\}$ corresponds to $S'_{AG} = \{L_{AGi}(T_{AGi}), i = m+1, \ldots, m+\beta\}$.

In a given assembly assume that all the component functional dimensions and tolerances be the set $S_{CD} = \{L_{CDj} \pm T_{CDj}/2, j = 1, \ldots, r\}$, where r is the number of functional dimensions and tolerances of all the components, L_{CDj} is the jth component functional dimension, T_{CDj} is the tolerance of L_{CDj}. And all the component functional geometric tolerances which can be converted into the equivalent bilateral dimensional tolerances be the set $S_{CG} = \{L_{CGj} \pm T_{CGj} / 2, j = 1, \ldots, p\}$, where p is the number of functional geometric tolerances, which can be treated as the equivalent bilateral dimensional tolerances of the components, T_{CGj} is the jth component functional geometric tolerance, L_{CGj} is the nominal dimension of T_{CGj}. Also the functional component geometric tolerances which can be treated as the additional tolerance constraints be the set $S'_{CG} = \{L_{CGj}(T_{CGj}), j = p+1, \ldots, p+\delta\}$, where δ is the number of the functional geometric tolerances which can be treated as the additional tolerance constraints of the components, T_{CGj} is the jth component functional geometric tolerances which is treated as the additional tolerance constraint, $L_{CGj}(T_{CGj})$ is the nominal dimension of T_{CGj}.

The set notation is introduced as $S_{CF} = \{S_{CD}, S_{CG}, S'_{CG}\} = \{L_{CFj} \pm T_{CFj}/2, j = 1, \ldots, r+p, L_{CFj}(T_{CFj}), j = r+p+1, \ldots, r+p+\delta\}$. Where $\{L_{CFj} \pm T_{CFj}/2, j = 1, \ldots, r\}$ corresponds to $S_{CD} = \{L_{CDj} \pm T_{CDj}/2, j = 1, \ldots, r\}$, $\{L_{CFj} \pm T_{CFj}/2, j = r+1, \ldots, r+p\}$ corresponds to $S_{CG} = \{L_{CGj} \pm T_{CGj}/2, j = 1, \ldots, p\}$, and $\{L_{CFj}(T_{CFj}), j = r+p+1, \ldots, r+p+\delta\}$ corresponds to $S'_{CG} =$

$\{L_{CGj}(T_{CGj}),\ j = p+1,\ \dots,\ p+\delta\}$.

Using of the assembly drawing, the required functional nominal dimensions of the assembly can be expressed as the related component BP nominal dimensions:

$$L_{AFi} = \sum_{j=1}^{r+p} \alpha_{ij}\xi_{ij}K_{ij}L_{CFij} \quad i = 1,\cdots,n+m \qquad (5)$$

where α_{ij} is the BP dimension selection coefficient. When the functional component BP dimension L_{CFij} is the link member of dimension L_{AFi}, $\alpha_{ij} = 1$, otherwise, $\alpha_{ij} = 0$. ξ_{ij} is the unit vector for L_{CFij}. $K_{ij} = \partial L_{AFi} / \partial L_{CFij}$ is the dimension coefficient of L_{CFij}, $0 \leq K_{ij} \leq 1$, $L_{CFij} \in S_{CF}$. L_{AFi} is an assembly functional dimension, $L_{AFi} \in S_{AF}$.

With above dimensional equations, a set of assembly functional DGT inequalities can be derived to represent the relationship between the assembly functional tolerances and the component functional BP tolerances. The general formulation with the worst-case model is:

$$T_{AFi} \geq \sum_{j=1}^{r+p} \alpha_{ij}K_{ij}T_{CFij} \quad i = 1,\cdots,n+m \qquad (6)$$

where T_{CFij} is the tolerance of component functional dimension L_{CFij}, $T_{CFij} \in S_{CF}$, T_{AFi} is the tolerance of the required assembly functional dimension L_{AFi}, $T_{AFi} \in S_{AF}$.

In the stage of process planning, the task of tolerancing, however, is to allocate the obtained component functional BP DGTs to the pertinent process tolerances. In most cases, because the design data, the measurement data, and the process data do not always coincide with each other, the tolerance stack-up is inevitable. Assume that there are ϕ manufactured components in an assembly and the subscription variable u denotes the sequence number of the component, thus $u \in [1,\ \dots,\ \phi]$. The subscription variable v denotes the sequence number of the operations related to each component, thus $v \in [1,\ \dots,\ \theta_u]$. Where θ_u is the total operations of the uth component. Let processing working dimensions and tolerances of the uth component be the set $S_{MD\,u} = \{L_{MD\,u\,v} \pm T_{MD\,u\,v}/2,\ u = 1,\ \dots,\ \phi,\ v = 1,\ \dots,\ f_u\}$, where f_u is the number of process dimensions and tolerances of the uth component. Let processing geometric tolerances of the uth component that can be treated as equivalent bilateral dimensional tolerances be the set S_{MG}

$_u$ = {$L_{MG\,u\,v} \pm T_{MG\,u\,v} / 2$, $u = 1, \ldots, \phi$, $v = 1, \ldots, g_u$}, where g_u is the number of geometric tolerances that can be interpreted as equivalent bilateral process dimensional tolerances related to the uth component. Let processing geometric tolerances of the uth component that can be treated as additional processing tolerance constraints be the set $S'_{MG\,u}$ = {$L_{MG\,u\,v}(T_{MG\,u\,v})$, $u = 1, \ldots, \phi$, $v = g_u+1, \ldots, g_u+\varepsilon_u$}, where ε_u is the number of geometric tolerances that can be interpreted as additional processing tolerance constraints related to the uth component, $T_{MG\,u\,v}$ is the component BP geometric tolerances, $L_{MG\,u\,v}(T_{MG\,u\,v})$ is the process dimension of tolerance $T_{MG\,u\,v}$.

The set notation related to the uth component is introduced as $S_{CP\,u}$ = {$L_{CP\,u\,v} \pm T_{CP\,u\,v} / 2$, $u = 1, \ldots, \phi$, $v = 1, \ldots, f_u+g_u$, $L_{CP\,u\,v}+T_{CP\,u\,v}$, $v = f_u+g_u+1, \ldots, f_u+g_u+\varepsilon_u$}. Where {$L_{CP\,u\,v} \pm T_{CP\,u\,v} / 2$, $v = 1, \ldots, f_u$} corresponds to $S_{MD\,u}$ = {$L_{MD\,u\,v} \pm T_{MD\,u\,v} / 2$, $v = 1, \ldots, f_u$}, {$L_{CP\,u\,v} \pm T_{CP\,u\,v} / 2$, $v = f_u+1, \ldots, f_u+g_u$} corresponds to $S_{MG\,u}$ = {$L_{MG\,u\,v} \pm T_{MG\,u\,v} / 2$, $v = 1, \ldots, g_u$}, and {$L_{CP\,u\,v} + T_{CP\,u\,v}$, $v = f_u+g_u+1, \ldots, f_u+g_u+\varepsilon_u$} corresponds to $S'_{MG\,u}$ = {$L_{MG\,u\,v} + T_{MG\,u\,v}$, $v = g_u+1, \ldots, g_u+\varepsilon_u$}.

Using the process planning of each related components, the required nominal functional BP dimensions can be expressed as the process dimensions:

$$L_{CFj} = \sum_{v=1}^{\theta_u} \alpha_{uv} \xi_{uv} K_{uv} L_{CPuv} \qquad u = 1, \cdots, \varphi \qquad (7)$$

where α_{uv} is the process dimension selection coefficient. For the given process planning of the uth component, when a process dimension L_{CPuv} is the link member of dimension L_{CFj}, $\alpha_{uv} = 1$, otherwise, $\alpha_{uv} = 0$. ξ_{uv} is the unit vector of L_{CPuv}. $K_{uv} = \partial L_{CFj} / \partial L_{CPuv}$ is the dimension coefficient of L_{CPuv}, $0 \leq K_{uv} \leq 1$. L_{CPuv} is the vth process dimension of the uth component, $L_{CPuv} \in S_{CP\,u}$. L_{CFj} is the component functional dimension, $L_{CFj} \in S_{CF}$.

With above equation, the allocation of the component functional BP DGTs to the process DGTs can be formulated by following inequalities with the worst-case model:

$$T_{CFj} \geq \sum_{v=1}^{\theta_u} \alpha_{uv} K_{uv} T_{CPuv} \qquad u = 1, \cdots, \varphi \qquad (8)$$

where T_{CPuv} is the vth process DGT specification corresponds to process dimension L_{CPuv} of the uth component, T_{CFj} is the component functional BP DGT corresponds to BP dimension L_{CFj}.

In a conventional tolerancing system, the process tolerances are acquired by allocation of the functional component BP DGT specifications to the process ones. The disadvantages of this method are that the obtained process tolerances are just under the constraints of BP tolerances and process accuracies. Moreover, component BP tolerances are first determined in the product tolerance design stage. In this stage, the assembly functional DGT specifications cannot be allocated to the relevant component functional BP DGTs in an optimal way without manufacturing information. Therefore some process DGT specifications obtained in the process stage will be beyond the economical bounds and the manufacturing costs will increase unnecessarily.

In concurrent tolerance design, the assembly functional DGT specifications can be directly expressed as the process DGT specifications through using the process planning information of each related component. When the design criterions such as maximum total manufacturing tolerances or minimum manufacturing costs have been presented, the optimal process tolerances can be obtained through establishing and solving an optimization model. Therefore by substituting Equation (7) into (5), the concurrent integrated dimension chains are obtained as:

$$L_{AFi} = \sum_{u=1}^{\varphi} \sum_{v=1}^{\theta_u} \alpha_{uv}^* \xi_{uv}^* \lambda_{uv}^* L_{CPuv} \quad i = 1, \cdots, n+m \qquad (9)$$

where α_{uv}^* is the concurrent dimension selection coefficient. For the given process planning of the uth component, when a process dimension L_{CPuv} is the link member of dimension L_{AFi}, $\alpha_{uv}^* = 1$, otherwise, $\alpha_{uv}^* = 0$. ξ_{uv}^* is the unit vector of dimension L_{CPuv}. $\lambda_{uv}^* = \partial L_{AFi} / \partial L_{CPuv}$ is the dimension coefficient of L_{CPuv}, $0 \leq \lambda_{uv}^* \leq 1$.

With above equation, the concurrent integrated DGT chains, which will be used for directly allocating of the assembly functional DGTs to the component process DGTs, are formulated as:

$$T_{AFi} \geq \sum_{u=1}^{\varphi} \sum_{v=1}^{\theta_u} \alpha_{Tuv} \lambda_{Tuv} T_{CPuv} \quad i = 1, \cdots, n+m \qquad (10)$$

The concurrent integrated DGT chains are main constraints and the technical bridge to link substantially the assembly functional DGT specifications and the component process DGT specifications. The approaches used in this paper for establishing the concurrent DGT chains are divided into three steps. First, the

assembly functional product DGT chains will be formulated by using the related mechanical structures of the components and the assembly constraints as the input data. The assembly functional DGTs are expressed as the related functional component BP DGTs by using the integrated tolerance charts in product tolerance design stage. Second, in terms of the given process planning of each component, the component functional BP DGT specifications will be formulated by process DGTs. In this stage, the pertinent structures and the processing plans of the components are used as the input data. Finally, when each component functional BP DGT equation is substituted into the required assembly functional product DGT chains, the required concurrent integrated DGT chains are obtained.

4. Concurrent integrated dimensioning and tolerancing

In assembling a complex product, normally several critical dimensions evaluate the functional performance requirements. These critical dimensions are controlled simultaneously within certain variation ranges for the best working performances. Let the critical dimension vector $y = [y_1 \ y_2 \ \dots \ y_p]^T$, and the deviation vector $w = [w_1 \ w_2 \ \dots \ w_p]^T$, $w_i = y_i - y_{0i}$, $i = 1, 2, \dots, p$, where y_{0i} is the nominal/target value of y_i. In a concurrent design environment, the assembly restrictions, topological relationships, and nominal dimensions of the main component have been determined by the assembly structure design. Let $x = [x_1 \ x_2 \ \dots \ x_n]^T$ be the vector of component design dimensions. These dimensions include sized dimensions and geometrical dimensions. For the geometrical dimensions with fixed tolerance zones, their dimensions and corresponding tolerances can be converted into equivalent bilateral sized dimensions and tolerances. The remaining geometric tolerances are treated as additional tolerance constraints. For simplicity, we denote both sized dimensions and equivalent bilateral sized dimensions as component design dimensions and process dimensions in their different design and manufacturing stages. Therefore, x_j ($j = 1, 2, \dots, n$) is the combination of a set of pertinent process dimensions of a component. Let the process dimension vector $z_j = [z_{j1} \ z_{j2} \ \dots \ z_{jm_j}]^T$, ($j = 1, 2, \dots, n$), where m_j is the number of the operations related to dimension x_j. Finally the assembly functional equations (Zhang, 1996) are expressed:

$$y_i = f_i(x) \qquad i = 1, 2, \dots, p \tag{11}$$

In process planning, the machining equations (Zhang, 1996) are generally expressed as:

$$x_j = g_j(z_j) \quad j = 1, 2, \ldots, n \tag{12}$$

Since there is no need or way for critical dimensions to be controlled in the exact nominal/target value, a rational variation zone should be assigned for each design dimension. From Equation (11), the actual critical dimension deviations due to their design dimension deviations are expressed as:

$$w_i = y_i - f_i(\overline{x}) = \sum_{j=1}^{n} \left. \frac{\partial f_i(x)}{\partial x_j} \right|_{\overline{x}} \Delta x_j \tag{13}$$

where $f_i(\overline{x})$ is the nominal value obtained by evaluating the assembly functional Equation (1) with its nominal design dimension vector $\overline{x}$. Δx_j is the algebraic difference between x_j and $\overline{x}_j$.

In tolerance design, accumulated design tolerances must be less than or equal to their critical tolerance, so Equation (13) needs some adjusting. For worst-case tolerance stack-up, each differential coefficient is positive, therefore, absolute value of each differential coefficient is required. w_i and Δx_j are replaced by t_i and tx_j. Where t_i and tx_j are respectively the tolerance of critical dimension y_i and design dimension x_j. With these substitutions, Equation (13) changes into inequality:

$$t_i \geq \sum_{j=1}^{n} \left| \left. \frac{\partial f_i(x)}{\partial x_j} \right|_{\overline{x}} \right| tx_j \tag{14}$$

Similarly, from Equation (12) the actual design dimension deviations due to their process dimension deviations can be expressed as:

$$x_j - g_j(\overline{z}_j) = \sum_{k=1}^{m_j} \left. \frac{\partial g_j(z_j)}{\partial z_{jk}} \right|_{\overline{z}_j} \Delta z_{jk} \tag{15}$$

where $g_j(\overline{z}_j)$ is the nominal value obtained by evaluating the machining Equation (12) with its nominal process dimension vector $\overline{z}_j$. Δz_{jk} is the algebraic difference of z_{jk} and $\overline{z}_{jk}$.

When component design tolerances are allocated to process tolerances, Equa-

tion (15) changes into inequality:

$$tx_j \geq \sum_{k=1}^{m_j} \left| \frac{\partial g_j(z_j)}{\partial z_{jk}} \right|_{\bar{z}_j} \left| t_{jk} \right. \tag{16}$$

where t_{jk} is jk-th process tolerance of design dimension z_{jk}.

Assume that all process dimensions are of normal distributions. Because design dimensions are functions of process dimensions and assembly critical dimensions are functions of design dimensions, according to statistical theory, both critical dimensions and design dimensions are of normal distributions. From Equation (11), we get variance equations:

$$\text{var}(w_i) = \sum_{j=1}^{n} \left(\frac{\partial f_i(x)}{\partial x_j} \bigg|_{\bar{x}} \right)^2 \text{var}(\Delta x_j) \quad i = 1, 2, \cdots, p \tag{17}$$

where variance $\text{var}(\Delta x_j)$ is obtained from Equation (13) and expressed as:

$$\text{var}(\Delta x_j) = \sum_{k=1}^{m_j} \left(\frac{\partial g_j(z_j)}{\partial z_{jk}} \bigg|_{\bar{z}_j} \right)^2 \text{var}(z_{jk}) \quad k = 1, 2, \cdots, n \tag{18}$$

where $\text{var}(w_i)$, $\text{var}(\Delta x_j)$, and $\text{var}(z_{jk})$ are variances of w_i, Δx_j, and z_{jk}, respectively. Equations (14) and (16) reveal the worst-case tolerance stack-up effect related to two stages, respectively. In Equation (14), component design stack-up tolerance must be less than or equal to functional critical tolerances. Similarly in Equation (16), component process stack-up tolerance must be less than or equal to design tolerances. As discussed above, interdependent tolerancing is divided into two separate stages. In initial product design, designers care more about product satisfaction than about subsequent production capabilities and costs. On the other hand, process planners are more concerned about component manufacturing capabilities than their functional roles in assembly. This conventional method can obtain only the optimum solutions within two separate stages. The best policy is to integrate the two stages into one.

In concurrent engineering, however, the two separate phases are integrated into only one stage (Zhang, 1996; Ngoi and Teck, 1997). This makes it easy for design and manufacturing to collaborate. Essentially, the product designer can consider more fabrication issues when initially designing the product, while

manufacturing engineers can cope with the manufacturing problems based on the component functional roles. This balances the different targets related to product satisfaction and production costs. Mathematically, by substituting machining equation into functional equations the concurrent design equation can be obtained as:

$$t_i \geq \sum_{j=1}^{n} \sum_{k=1}^{m_j} \left| \frac{\partial f_i(x)}{\partial x_j} \right|_{\bar{x}} \left\| \frac{\partial g_j(z_j)}{\partial z_{jk}} \right\|_{\bar{z}_j} t_{jk} \quad i = 1,2,\ldots,p \qquad (19)$$

5. Quality loss of multiple correlated critical dimensions

High quality and low cost are two fundamental requirements for product design and manufacturing. In an assembly, critical tolerances must be guaranteed for functional requirements. It is well known that the tighter tolerance is, the higher the cost is, and vice versa. For a selected machining operation, if process tolerance becomes smaller and smaller until it reaches a certain value, it will result in the infinite theoretical manufacturing cost. To simplify computation, let best product performance be the point where tolerance is zero. At that point, the theoretical manufacturing cost is infinite. For a single critical dimension case, when critical dimension deviates from its target, the symmetric quadratic Taguchi quality loss function is (Taguchi et al., 1989):

$$L(y) = k(y - \bar{y})^2 \qquad (20)$$

where y and $\bar{y}$ are respectively the actual and target values of critical dimension, and k is a positive constant coefficient

To determine the value of k, provided that when dimension y deviates from its target in value w, will cause the loss of $A\$$. Thus the following equation will be satisfied:

$$k = A / w^2 \qquad (21)$$

where $w = y - \bar{y}$.

For a p-dimensional multivariate vector w, Le and Tang (2000) presented a general formula to evaluate the total quality loss due to w:

$$L(w) = w^T K w \qquad (22)$$

where K is a $p \times p$ symmetric constant matrix. $k_{ij} = k_{ji}$, for $i \neq j$, $i, j = 1, 2, \ldots, p$. If $p(p+1)/2$ set of product quality deviations and corresponding quality losses are available. The elements of K are related by:

$$\sum_{i=1}^{p}\sum_{j=1}^{p} k_{ij} w_i^{(k)} w_j^{(k)} = A_k \qquad k = 1,2,\ldots, p(p+1)/2 \qquad (23)$$

Since manufacturing dimension distribution is dependent upon the related manufacturing process random factors such as machine tools, fixtures, tool wearing, system vibration, temperature fluctuation, operators, and measurement devices, etc, each actual process dimension z_{jk} is obviously a random variable. In terms of Equations (12) and (11), design dimension x_j is the combination of process dimension z_{jk} and critical dimension y_i is the combination of design dimension x_j, so design dimension x_j and critical dimension y_i are also random variables. The distribution of critical dimension y_i is finally dependent upon the density distribution functions of pertinent process dimensions. The product quality loss is determined by all critical dimension distributions. For a batch of products, average quality loss rather than individual loss should be considered. When a product has only a single critical dimension y, let the density function of y be function $\psi(w)$, the average loss of a batch product could be obtained by integration:

$$E(L(w)) = \int_{-\infty}^{+\infty} \psi(w) k w^2 dw \qquad (24)$$

As for the multiple critical dimensions, the expectation loss is obvious the summation of individual contributions derived from Equation (24):

$$E(L(w)) = \sum_{k} \Psi(w^{(k)})(w^{(k)^T} K \, w^{(k)}) \qquad (25)$$

where

$$\sum_{k} \Psi(w^{(k)}) = 1 \qquad (26)$$

For the design vector x, the density function is continuous within an interval. Expected quality loss function is (Lee and Tang, 2000):

$$E(L(w)) = \text{Trace } [KV(w)] \qquad (27)$$

where $V(w)$ is the variance-covariance matrix of the parameter vector w expressed by:

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