2

An Analytical Analysis of a Wind Power Generation System Including Synchronous Generator with Permanent Magnets, Active Rectifier and Voltage Source Inverter

Sergey A. Kharitonov, Member. IEEE
Novosibirsk State Technical University
Rusia

1. Introduction

For the high-power Wind Power Installation (WPI) with a variable speed wind turbine the system of transformation of mechanical energy into electric energy of the alternating current, constructed under the scheme "the synchronous generator with constant magnets - the active rectifier - the voltage inverter" (fig. 1) is perspective. Then the system is called the Wind Power Generation System – WPGS.

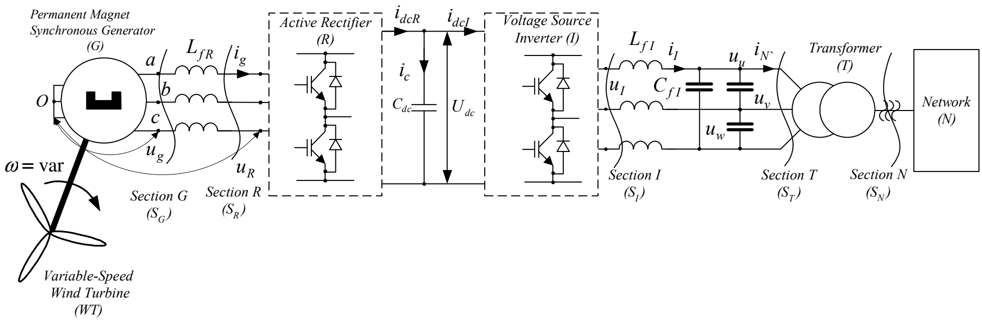

Fig. 1.

WPGS of this type implements the full set of options required from the generation for high-power WPI, namely this: a generating mode at work on the nonlinear, asymmetric and non-stationary loads, electric starter startup mode of the wind turbine, in-phase and parallel operation of an electric network and other WPI.

In this paper we attempt to identify the main energy characteristics of system in sections S_G, S_R, S_I, S_T, S_N (fig. 1) when working on high-power electric network. Processes in the active rectifier and the voltage inverter are well studied at a constant frequency and voltage of the generator. The peculiarity of this study is the consideration of factors that arise when working within the system to generate WPI with a variable rotation speed of the wind turbine. These factors include: changing the frequency and the voltage of the

Source: Wind Power, Book edited by: S. M. Muyeen,
ISBN 978-953-7619-81-7, pp. 558, June 2010, INTECH, Croatia, downloaded from SCIYO.COM

synchronous generator (G), as well as dependence of the generated power on shaft speed of the wind turbine.
Presence in the system of the active rectifier (R) modifies the functional and energy potential of WPGS. The active rectifier with a PWM, which frequency is much higher than the voltage frequency of a synchronous generator (G), allows for a number of modes, significantly affecting the power consumption of G in the WPGS.
As a result of the conducted research where as an example it is accepted that the active rectifier and the inverter are based on the classical scheme of the two-level voltage inverter, the analytical description of WPGS system is obtained at a variable speed of rotation of the wind turbine, the basic expressions for currents, voltage and capacities of the synchronous generator, the voltage inverter are defined, algorithms of the management are offered by the active rectifier, and also the modular principle of construction of the voltage inverter and WPGS system as a whole are considered.

2. Basic assumptions. Mathematical model of the system.

For a generality of results of the analysis in the scheme elements which not always are obligatory are entered:
L_{fR} - inductance of the cable connecting the generator and the active rectifier;
T - the matching transformer;
C_{fI} - capacity of the output filter, for the smoothing of pulsations on a transformer input.
The active rectifier and the voltage inverter are controlled by a high-frequency PWM, and their frequencies ω_{cR} and ω_{cI}, correspondingly, are significantly higher than frequencies of the fundamental harmonic voltage of synchronous generator (ω) and the electric voltage network (Ω). The multiplicities of frequencies are constant, i.e. $\omega_{cR}/\omega = a_R = const$ and $\omega_{cI}/\Omega = a_I = const$.
The electrical network has a capacity much bigger than the power of WPI.
Let's assume also that the synchronous generator does not contain soothing contours and its magnetic system is linear.
The generating mode is a subject to consideration. In this case, the active rectifier (R) entrusted with the tasks of forming a given voltage in the DC link U_{dc} and reactive power control on the frequency ω in sections S_G and S_R, while the voltage inverter (I) is tasked to ensure the specified quality and quantity of the generated current in the electric network .
Let's consider that the capacity C_{dc} in a DC link is big, the voltage regulator U_{dc} in a control system works with the maximum speed and is non-static then in the established mode it is possible to accept that $U_{dc} = const$. In this case in sections S_G, S_R it is possible to consider the electromagnetic processes irrespective of processes in sections S_I, S_N and S_T.
It is convenient to study WPGS in the rotating coordinate systems. In this case in section S_G the coordinate system rotates synchronously with the frequency of the generator voltage (ω), and in sections S_I, S_N respectively, with the frequency of the mains voltage (Ω).
Taking into account the accepted assumptions *the mathematical model of SG* in rotating system of co-ordinates, under condition of axis orientation d on a longitudinal axis of the synchronous generator will look like:

$$\boldsymbol{u}_R = -\boldsymbol{r}_\Sigma \boldsymbol{i}_G - \frac{d}{dt}\boldsymbol{\psi}_\Sigma - \boldsymbol{\omega}\boldsymbol{\psi}_\Sigma, \qquad \boldsymbol{\psi}_\Sigma = \boldsymbol{L}_\Sigma \boldsymbol{i}_G - \boldsymbol{\psi}_0 \tag{1}$$

where: $\boldsymbol{\psi}_\Sigma = \begin{bmatrix} \Psi_{\Psi d} & \Psi_{\Sigma q} \end{bmatrix}^t$, $\Psi_{\Sigma d} = \Psi_d + L_{fR} i_{Gd}$, $\Psi_{\Sigma q} = \Psi_q + L_{fR} i_{Gq}$, Ψ_d, Ψ_q - the magnetic flux of generator in the longitudinal and transverse axes, $\boldsymbol{u}_R = \begin{bmatrix} u_{Rd} & u_{Rq} \end{bmatrix}^t$, $\boldsymbol{i}_g = \begin{bmatrix} i_{Gd} & i_{Gq} \end{bmatrix}^t$ - vectors of the active rectifier voltages and currents of the generator; $\boldsymbol{\psi_o} = \begin{bmatrix} \Psi_o & 0 \end{bmatrix}^t$, $\Psi_o = const$ - the magnetic flux created by permanent magnets; $\boldsymbol{r}_\Sigma = diag\{ r_\Sigma, r_\Sigma \}$, $r_\Sigma = r_s + r_{LfR}$, $\boldsymbol{r}_s$, r_{LfR} - the active resistances of stator phase windings of the generator and cable connecting the generator and active rectifier; $\boldsymbol{L}_\Sigma = diag\{ L_{\Sigma d}, L_{\Sigma q} \}$, $L_{\Sigma d} = L_d + L_{fR}$, $L_{\Sigma q} = L_q + L_{fR}$, L_d, L_q - inductance of the generator in the longitudinal and transverse axes; $\boldsymbol{\omega} = \begin{bmatrix} 0 & -\omega \\ \omega & 0 \end{bmatrix}$, ω - circular frequency of the electromotive force (EMF) of SG ($\omega = \text{var}$).

Selecting the generator currents as variables, after simple transformations, we obtain from equation (1):

$$\boldsymbol{u}_R = -\boldsymbol{r}_\Sigma \boldsymbol{i}_G - \boldsymbol{L}_\Sigma \frac{d}{dt}\boldsymbol{i}_G - \boldsymbol{\omega}\boldsymbol{L}_\Sigma \boldsymbol{i}_G + \boldsymbol{e} \quad \boldsymbol{e} = \begin{bmatrix} 0 & E_o \end{bmatrix}^t, \tag{2}$$

here $E_o = \omega\Psi_o$ - EMF-load of the generator ($E_o = \text{var}$).

Neglecting the active resistance it is possible to write down parity (2) in the scalar form

$$u_{Rd} = -i_{Gd} \cdot r_s - L_{d\Sigma}\frac{di_{Gd}}{dt} + \omega L_{q\Sigma} i_{Gq} \approx -L_{d\Sigma}\frac{di_{Gd}}{dt} + \omega L_{q\Sigma} i_{Gq},$$

$$u_{Rq} = -i_{Gq} \cdot r_s - L_{q\Sigma}\frac{di_{Gq}}{dt} - \omega L_{d\Sigma} i_{Gd} + E_o \approx -L_{q\Sigma}\frac{di_{Gq}}{dt} - \omega L_{d\Sigma} i_{Gd} + E_o \tag{3}$$

In any section S active (P_S), reactive (Q_S) and apparent (S_S) powers will be defined by means of the following parities:

$$P_S = \frac{3}{2}(\boldsymbol{u}, \boldsymbol{i}) = \frac{3}{2}\left(u_d i_d + u_q i_q\right), Q_S = \frac{3}{2}[\boldsymbol{u}, \boldsymbol{i}] = \frac{3}{2}\left(u_d i_q - u_q i_d\right), S_S = \left[P_S^2 + Q_S^2\right]^{\frac{1}{2}}. \tag{4}$$

The mathematical description of the active rectifier and the inverter will be obtained by means of switching functions. We will consider that transistors and diodes are ideal keys. R and I are realized on base of the voltage inverter schematically presented in fig. 2.

The phase voltage on alternating current clips is defined by means of parity:

$$u_m = U_{dc}\left(2F_m - \sum_{\substack{k=1 \\ k \neq m}}^{3} F_k\right) \Bigg/ 3; \quad m = 1, 2, 3, \tag{5}$$

where F_m - the switching functions of transistors VT_m of the inverter which are defined by means of a following parity $F_m = \begin{cases} 1, & VT_m - is\ switched\ on; \\ 0, & VT_m - is\ switched\ off. \end{cases}$

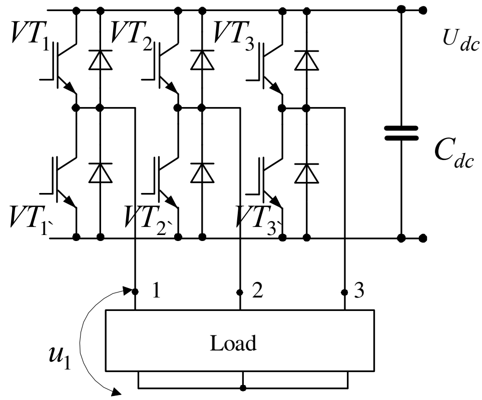

Fig. 2.

They also can be defined by means of a series:

$$F_m = M\sin\theta_m/2 + \sum_{j=1}^{\infty} F_{msj}\sin(j\cdot a_k\vartheta) + F_{mcj}\cos(j\cdot a_k\vartheta);$$

$$F_{msj} = \frac{(-1)^j}{j\pi}[(-1)^j - \cos(j\pi\cdot M\sin\theta_m)]\ ;\ \ F_{mcj} = \frac{(-1)^j}{j\pi}\sin(j\pi\cdot M\sin\theta_m);$$

$$M = \begin{cases} u_c/u_{sc} & \text{SPWM}; \\ 2u_c/\sqrt{3}u_{sc} & \text{SVPWM}; \end{cases}\quad M_{\max} = \begin{cases} 1 & \text{SPWM}; \\ 2/\sqrt{3} & \text{SVPWM}; \end{cases}\quad \theta_m = \vartheta - (m-1)\frac{2\pi}{3} + \phi_c;$$

where: $\vartheta = \omega t$, $a_k = \omega_c/\omega$, ω_c, ω - the cyclic frequencies of PWM and an of operating signal, accordingly, M, $M_{\max}$ – the depth (index) of modulation and its maximum value, u_c - amplitude of the control input wave, u_{sc} - the amplitude of saw tooth carrier wave, φ_c - the phase of the control input wave, is defined by the chosen algorithm of control.

After a number of transformations we will obtain for SPWM

$$u_m = U_{dc}M\sin(\theta_m)/2 + \frac{U_{dc}}{\pi}\sum_{\substack{p=-\infty \\ p\neq 0}}^{\infty}\sum_{k=-\infty}^{\infty}(-1)^{p+3k}/p\cdot J_{3k+1}(p\pi M)\cdot\sin\left[a_k p\vartheta + (3k+1)\theta_m\right];\ ^1 \quad (6)$$

where $J_{3k+1}(....)$ - Bessel functions of the first kind of an order $3k+1$.

Obviously, expression for the fundamental component will look like: $u_{m(1)} = U_{dc}M\sin(\theta_m)/2$.

[1] In this parity and further the high-frequency harmonics are defined for SPWM.

If we introduce the operator of rotation $a = \exp(i \cdot 2\pi/3)$, $i = \sqrt{-1}$, u_m, $m = 1,2,3$ the three voltages can be written in the orthogonal coordinate system:

$$u_{\alpha\beta} = u_\alpha + i \cdot u_\beta = \frac{2}{3}(u_1 + a \cdot u_2 + a^2 \cdot u_3).$$

Using parity (6), we will obtain:

$$u_\alpha = \frac{U_{dc}}{2} M \sin(\theta_1) + \frac{U_{dc}}{\pi} \sum_{\substack{p=-\infty \\ p \neq 0}}^{\infty} \sum_{k=-\infty}^{\infty} \frac{(-1)^{p+3k}}{p} \cdot J_{3k+1}(p\pi \cdot M) \sin[ap\vartheta + (3k+1)\theta_1]; \quad (7)$$

$$u_\beta = -\frac{U_{dc}}{2} M \cos(\theta_1) - \frac{U_{dc}}{\pi} \sum_{\substack{p=-\infty \\ p \neq 0}}^{\infty} \sum_{k=-\infty}^{\infty} \frac{(-1)^{p+3k}}{p} \cdot J_{3k+1}(p\pi \cdot M) \cos[ap\vartheta + (3k+1)\theta_1]. \quad (8)$$

We will determine the current in a direct current link (i_{dc}) by means of parity: $i_{dc} = \sum_{m=1}^{3} i_{gm} F_m$, where i_{gm} - the instant value of phase currents of the generator. The average value of current i_{dc} from the condition of equality of the active power in AC and DC circuits of inverter is: $I_{dco} = 3MI_{(1)} \cos\varphi / 4$, here φ - an angle shift between the fundamental harmonic of phase voltage and an inverter current, $I_{(1)}$ - the amplitude of inverter current, I_{dco} - the mean value of a current in a DC link.

In the analysis of electromagnetic processes in sections S_I, S_T, S_N we will assume that $\omega_{kI} >> \Omega$. In a first approximation it allows to neglect the effect of capacitors C_{fI}. When considering the transformer (*T*), we assume that its magnetic system is unsaturated, active losses and the magnetization current are zero and its influence on the processes we take into account with the total leakage inductance ($L_{\sigma\Sigma T}$) and transformation factor (k_T): $k_T = w_1/w_2$; $L_{\sigma\Sigma T} = L_{\sigma 1T} + L`_{\sigma 2T} = L_{\sigma 1T} + L_{\sigma 2T} \cdot k_T^2$; where w_1, w_2, - the number of turns of primary and secondary windings of the transformer, $L_{\sigma 1T}$, $L_{\sigma 2T}$ - the leakage inductance of primary and secondary windings, respectively.

We will express the voltage of an electric network through the voltage on a primary winding of the transformer using the relation: $u_{N`} = u_N \cdot k_T$. The equivalent inductance in the output circuit of inverter: $L_I = L_f + L_{\sigma\Sigma T}$.

Taking into account the accepted assumptions WPGS can be presented in the form of two equivalent circuits, for example, in phase coordinates fig. 3 and fig. 4.

In these figures dependent sources of voltage which reflect the voltages R and I in alternating current clips are presented in the form of rhombs. These voltages are defined by relation (5).

3. Basic energy indicators in the chain of "synchronous generator - active rectifier"

The power quality parameters of electromagnetic processes in WPGS determine the technical efficiency of converting mechanical energy of a shaft rotating with a variable speed

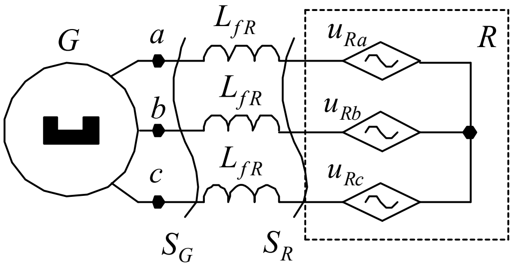

Fig. 3.

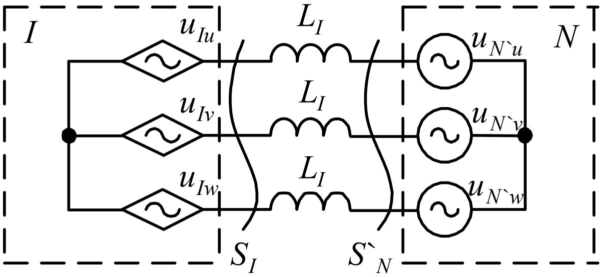

Fig. 4.

wind turbine into electrical energy by means of a synchronous generator and the voltage inverter, and the degree of influence of WPGS on the electric network through quality indicators of the electricity generated.

The basic indicators include: efficiency, power factor, and factors of harmonics and distortions of currents and voltages. For calculation of these indicators the definition of the operating values of currents and voltages in system elements is necessary, therefore they also are indirect power indicators.

For generality of the results, we introduce the relative units, as a basic value we choose the following:

$$U_{dc} = \sqrt{3}U_{N`}; \quad E_б = \frac{U_{dc}}{\sqrt{3}} = U_{N`}; \quad \omega_б = \frac{E_б}{\Psi_o}; \quad X_б = \omega_б\left(L_f + L_d\right) = \omega_б L_d\left(1+q\right);$$

$$I_б = I_{кз} = \frac{E_б}{X_б}; S_б = \frac{3}{2}\cdot E_б I_б; \omega^* = \frac{\omega}{\omega_б}, E_o^* = \frac{E_o}{E_б} = \omega^*, u^* = \frac{u}{E_б}, i^* = \frac{i}{I_б}, a_R = \frac{\omega_{cR}}{\omega}, \tag{9}$$

where $U_{N`}$ - the amplitude value of voltage of electrical network, referred to the primary winding, ω_{cR} the cyclic frequency PWM of an active rectifier.

Denote:

$$q = \frac{L_f}{L_d}; \; k_x = \frac{L_{aq}}{L_{ad}}; \sigma = \frac{L_\sigma}{L_{ad}}; k_L = \frac{L_q}{L_d} = \frac{\sigma + k_x}{\sigma + 1}. \tag{10}$$

Taking into account (9) and (10) we obtain:

$$X_{fR}^* = \frac{X_{fR}}{X_б} = \frac{\omega^* q}{1+q},\ X_d^* = \frac{X_d}{X_б} = \frac{\omega^*}{1+q},\ X_q^* = \frac{X_q}{X_б} = \frac{\omega^*}{1+q} k_L,$$
$$X_{d\Sigma}^* = X_d^* + X_{fR}^* = \omega^*,\ X_{q\Sigma}^* = X_q^* + X_{fR}^* = \omega^* \frac{k_L + q}{1+q}.$$

Considering relative units and the entered designations of the equation (3) will become:

$$\begin{aligned} u_{Rd}^* &= -X_{d\Sigma}^* \frac{di_{Gd}^*}{d\vartheta} + X_{q\Sigma}^* i_{Gq}^* = -\omega^* \frac{di_{Gd}^*}{d\vartheta} + \omega^* \frac{k_L + q}{1+q} \cdot i_{Gq}^*; \\ u_{Rq}^* &= -X_{q\Sigma}^* \frac{di_{Gq}^*}{d\vartheta} - X_{d\Sigma}^* i_{Gd}^* + \omega^* = -\omega^* \frac{k_L + q}{1+q} \cdot \frac{di_{Gq}^*}{d\vartheta} - \omega^* i_{Gd}^* + \omega^*, \end{aligned} \tag{11}$$

where $\vartheta = \omega t$.

Accordingly, we will define the power for the basic harmonics in sections S_G also S_R by means of expressions:

$$\begin{cases} P_{SGo}^* = P_{SRo}^* = P_{SNo}^* = \omega^* i_{gqo}^*, \\ Q_{SGo}^* = u_{Gdo}^* i_{Gqo}^* - u_{Gqo}^* i_{Gdo}^*; \quad Q_{SRo}^* = u_{Rdo}^* i_{Gqo}^* - u_{Rqo}^* i_{Gdo}^*; \\ S_{SGo}^* = [(P_{SNo}^*)^2 + (Q_{SGo}^*)^2]^{\frac{1}{2}}; \quad S_{SRo}^* = [(P_{SNo}^*)^2 + (Q_{SRo}^*)^2]^{\frac{1}{2}}; \end{cases} \tag{12}$$

here it is considered that at the accepted assumptions the active power is identical and equal in all sections to the power generated in an electric network (P_{SNo}^*).

Taking into account the higher harmonics value of active power will not change, and to calculate inactive and a total power it is necessary to apply the following relations:

$$P_{SG}^* = P_{SR}^* = P_{SGo}^*;\ S_{SG}^* = U_{G,rms}^* I_{G,rms}^*; \quad S_{SR}^* = U_{R,rms}^* I_{G,rms}^*;$$
$$Q_{SG}^* = \sqrt{(S_{SG}^*)^2 - (P_{SG}^*)^2}; \quad Q_{SR}^* = \sqrt{(S_{SR}^*)^2 - (P_{SR}^*)^2}$$

We will define the power factor in sections S_R also S_R by means of parities:

$$\chi_G = P_{SG}^* / S_{SG}^* = \nu_{iG} \nu_{uG} \cos\varphi_{SG}; \quad \chi_R = P_{SR}^* / S_{SR}^* = \nu_{iG} \nu_{uR} \cos\varphi_{SR}, \tag{13}$$

where: $\nu_{iSG} = I_{G(1),rms}^* / I_{G,rms}^*$, $\nu_{uSG} = U_{G(1),rms}^* / U_{G,rms}^*$, $\nu_{uSR} = U_{R(1),rms}^* / U_{R,rms}^*$, ν_{iS} , ν_{uS}, φ_S - a fundamental factors of current and voltage, and also a shift angle between the basic harmonics of current and voltage accordingly in sections S_G and S_R, $I_{G(1),rms}^*$, $I_{G,rms}^*$, $U_{G(1),rms}^*$, $U_{G,rms}^*$, $U_{R(1),rms}^*$, $U_{R,rms}^*$ - root-mean-square – RMS of the basic harmonics and full values of a current and a voltage in corresponding sections.

Assuming that the EMF-load of generator (e_{Gm}^*) and the control voltage of an active rectifier (u_{Rcm}^*) varies according to the law:

$$e_{Gm}^{*} = \omega^{*} \cos\left[\vartheta - (m-1)\frac{2\pi}{3}\right];\ u_{Rcm}^{*} = M\sin(\theta_m);$$
$$\theta_m = \vartheta - (m-1)\frac{2\pi}{3} + \frac{\pi}{2} - \phi_{Rc};\ \vartheta = \omega t;\ m = 1,2,3\ (a,b,c);$$

and taking into account the relation (7, 8), we obtain expressions for the quantities u_{Rd}^{*} and u_{Rq}^{*}: $u_{Rdq}^{*} = u_{Rd} + i\cdot u_{Rq} = u_{R\alpha\beta}^{*}\exp[-i\cdot\gamma(\vartheta)]$, where $\gamma(\vartheta) = \vartheta - \pi/2$.

At the analysis in «*d q*» co-ordinates it is convenient to present the three control input waves of the active rectifier in the form of two orthogonal projections on *d* and *q* axes, then $M_d = M\sin\varphi_{Rc};\quad M_q = M\cos\varphi_{Rc}$, it is obvious that $M = \sqrt{M_d^2 + M_q^2}$.

After the transformations we will obtain an expression for voltage of the active rectifier in «*d q*» co-ordinates:

$$u_{Rd}^{*} = u_{Rdo}^{*} + \Delta u_{Rd}^{*};\quad u_{Rq}^{*} = u_{Rqo}^{*} + \Delta u_{Rq}^{*};\quad u_{Rdo}^{*} = \sqrt{3}\,M_d/2;\quad u_{Rqo}^{*} = \sqrt{3}\,M_q/2;$$

$$\Delta u_{Rd}^{*} = \frac{\sqrt{3}}{\pi}\sum_{\substack{p=-\infty\\ p\neq 0}}^{\infty}\sum_{k=-\infty}^{\infty}\frac{(-1)^{p+k}}{p}\cdot J_{3k+1}(p\pi\cdot M)\cos\left[a_R p\vartheta + 3k\vartheta + (3k+1)(\pi/2 - \varphi_{Rc})\right]; \tag{14}$$

$$\Delta u_{Rq}^{*} = \frac{\sqrt{3}}{\pi}\sum_{\substack{p=-\infty\\ p\neq 0}}^{\infty}\sum_{k=-\infty}^{\infty}\frac{(-1)^{p+k}}{p}\cdot J_{3k+1}(p\pi\cdot M)\sin\left[a_R p\vartheta + 3k\vartheta + (3k+1)(\pi/2 - \varphi_{Rc})\right],$$

here u_{Rdo}^{*}, u_{Rqo}^{*} - the orthogonal components in *d* and *q* coordinates of the basic harmonic of voltage of the active rectifier; Δu_{Rd}^{*}, Δu_{Rq}^{*} - the orthogonal components in *d* and *q* co-ordinates of the high-frequency harmonics of voltage of the active rectifier.

In the steady operating mode for a particular value of generator voltage frequency (ω^{*}) with the help of relations (11) and (14) we can determine an analytical expression for the generator currents. To do this in (14) we will allocate sinus and cosine components ($U_{Rds\,pk}^{*}, U_{Rdc\,pk}^{*}$, $U_{Rqs\,pk}^{*}, U_{Rqc\,pk}^{*}$) of the harmonics with frequencies $\nu_{pk} = a_R p\omega + 3k\omega$:

$$\begin{aligned} U_{Rds\,pk}^{*} &= -g_{kp}\sin\left[(3k+1)(\pi/2 - \phi_{Rc})\right];\ U_{Rqs\,pk}^{*} = g_{kp}\cos\left[(3k+1)(\pi/2 - \phi_{Rc})\right];\\ U_{Rdc\,pk}^{*} &= g_{kp}\cos\left[(3k+1)(\pi/2 - \phi_{Rc})\right];\quad U_{Rqc\,pk}^{*} = g_{kp}\sin\left[(3k+1)(\pi/2 - \phi_{Rc})\right], \end{aligned} \tag{15}$$

here $g_{kp} = \frac{\sqrt{3}}{\pi}\frac{(-1)^{p+k}}{p}\cdot J_{3k+1}(p\pi\cdot M)$.

The equation for the generator current can be represented as a sum of components from the fundamental (i_{Gdo}^{*}, i_{Gqo}^{*}) and the high-frequency ($\Delta i_{Gd}^{*}, \Delta i_{Gq}^{*}$) harmonics

$$i_{Gd}^{*} = i_{Gdo}^{*} + \Delta i_{Gd}^{*};\quad i_{Gq}^{*} = i_{Gqo}^{*} + \Delta i_{Gq}^{*}.$$

Then, using equations (11) and (15), we obtain

$$i^*_{Gqo} = \frac{u^*_{Rdo}}{X^*_{q\Sigma}} = \frac{1+q}{\omega^*(k_L+q)} \cdot u^*_{Rdo} = \frac{\sqrt{3}}{2}\frac{1+q}{\omega^*(k_L+q)}M_d, \quad i^*_{Gdo} = 1 - \frac{u^*_{Rqo}}{\omega^*} = 1 - \frac{\sqrt{3}}{2}\frac{1}{\omega^*}M_q.$$

$$\Delta i^*_{Gd}(\vartheta) = \sum_{\substack{p=-\infty \\ p\neq 0}}^{\infty} \sum_{k=-\infty}^{\infty} \left[I^*_{Gds\,pk}\sin(a_R p + 3k)\vartheta + I^*_{Gdc\,pk}\cos(a_R p + 3k)\vartheta \right];$$
$$\Delta i^*_{Gq}(\vartheta) = \sum_{\substack{p=-\infty \\ p\neq 0}}^{\infty} \sum_{k=-\infty}^{\infty} \left[I^*_{Gqs\,pk}\sin(a_R p + 3k)\vartheta + I^*_{Gqc\,pk}\cos(a_R p + 3k)\vartheta \right]; \tag{16}$$

where:

$$I^*_{Gds\,pk} = gd_{kp}[U_{Rqspk} - U_{Rdcpk}(a_R p + 3k)]\,; I^*_{Gdc\,pk} = gd_{kp}[U_{Rqcpk} - U_{Rdspk}(a_R p + 3k)]\,;$$

$$I^*_{Gqs\,pk} = gq_{kp}[U_{Rdspk} + U_{Rqcpk}(a_R p + 3k)];\, I^*_{Gqc\,pk} = gq_{kp}[U_{Rdcpk} + U_{Rqspk}(a_R p + 3k)]\,,$$

here $gd_{kp} = 1/[(a_R p + 3k)^2 - 1]X_{d\Sigma}$, $gq_{kp} = -1/[(a_R p + 3k)^2 - 1]X_{q\Sigma}$.
Voltage of the synchronous generator is defined as follows:

$$u^*_{Gd} = \Delta u^*_{Gd} + u^*_{Gdo}; \quad u^*_{Gq} = \Delta u^*_{Gq} + u^*_{Gqo}$$

$$u^*_{Gdo} = u^*_{Rdo} - X^*_{Rf} i^*_{Gqo} = \frac{\sqrt{3}}{2}\frac{k_L}{k_L+q} M\sin(\varphi_{Rc});$$
$$u^*_{Gqo} = u^*_{Rqo} + X^*_{Rf} i^*_{Gdo} = \frac{1}{1+q}\left[\omega^* q + \frac{\sqrt{3}}{2} M\cos(\varphi_{Rc})\right]; \tag{17}$$

$$\Delta u^*_{Gd} = \Delta u^*_{Rd} + X^*_{Rf}\frac{di^*_{Gd}}{d\vartheta} - X^*_{Rf}\Delta i^*_{Gq}; \quad \Delta u^*_{Gq} = \Delta u^*_{Rq} + X^*_{Rf}\frac{d\Delta i^*_{Gq}}{d\vartheta} + X^*_{Rf}\Delta i^*_{Gd}.$$

here u^*_{Gdo}, u^*_{Gqo} - the orthogonal components in the *d* and *q* coordinates of the fundamental harmonic of generator voltage, $\Delta u^*_{Gd}, \Delta u^*_{Gq}$ - the orthogonal components in the *d* and *q* coordinates of the high-frequency harmonics of generator voltage.
From (16a) and (17) can easily be obtained the following useful relations:

$$u^*_{Gdo} = k_L u^*_{Rdo} / (k_L + q), \quad u^*_{Gqo} = (u^*_{Rqo} + q\omega^*)/(1+q)\,. \tag{18}$$

RMS of the active rectifier voltage ($U^*_{R,rms}$) and its fundamental factor can be determined if we use the following properties of switching functions F_m [1]:

$$\frac{1}{2\pi}\cdot\int_0^{2\pi}(F_j)^2 d\vartheta = \frac{1}{2}; \quad \frac{1}{2\pi}\cdot\int_0^{2\pi} F_j\cdot F_i d\vartheta \approx \frac{1}{2}\cdot(1-0.5M); \quad i,j = 1,2,3; (i \approx j).$$

Then:

$$U^*_{R,rms} = \sqrt{M/2}; \quad \nu_{uR} = \frac{U_{R(1),rms}}{U_{R,rms}} = \sqrt{3M}/2.$$

We define RMS of the generator current through the equation:

$$I^*_{G,rms} = \sqrt{(i^*_{Gdo})^2 + (i^*_{Gqo})^2 + (\Delta I^*_G)^2} \Big/ \sqrt{2}\ , \tag{19}$$

where

$$\Delta I^*_G \approx \left(\frac{\sqrt{6}}{\pi \cdot \omega^*}\right) \cdot \left(J_1(\pi \cdot M)^2 \frac{a_R^2 + 1}{(a_R + 1)^2 (a_R - 1)^2}\right)^{\frac{1}{2}}. \tag{19a}$$

Then the fundamental factor of the generator current can be estimated using the relation:

$$\nu_{iG} = \sqrt{\frac{(i^*_{Gdo})^2 + (i^*_{Gqo})^2}{(i^*_{Gdo})^2 + (i^*_{Gqo})^2 + (\Delta I^*_{Gdq})^2}}\ .$$

We will determine RMS of the generator voltage considering that, $k_L \to 1$, then:

$$U^*_{G,rms} = \frac{1}{\sqrt{2}} \sqrt{\left(\omega^* \cos\theta\right)^2 + \frac{1}{(1+q)^2} \cdot \left[M - \left(\frac{\sqrt{3}}{2} M \cos\phi_c\right)^2\right]}\ ,$$

where θ - the angle between the main harmonics of the EMF and the generator voltage. In accordance with the vector diagram (fig. 5) θ is given by: $\theta = arctg\, u^*_{Gdo} \big/ u^*_{Gqo}$.

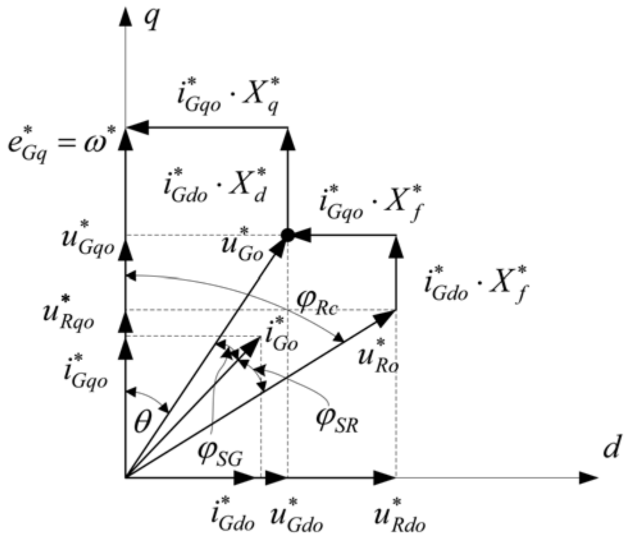

Fig. 5.

In fig.6 as an illustration of the possibilities suggested by the mathematical model of the system the calculated dependences of some power indicators as change of frequency of rotation of the wind turbine for a mode $i^*_{Gdo} = 0$, i.e. when a phase of a current and EMF of the generator coincide are presented. In this case the active filter increases the voltage $u^*_{Ro} > e^*_G$.

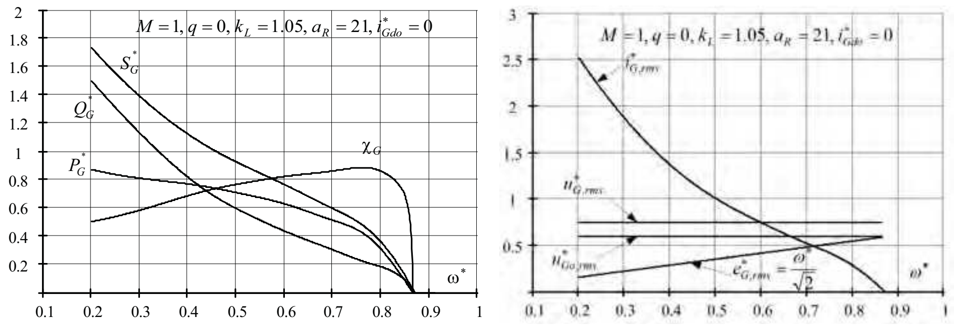

Fig. 6.

Dependence of the fundamental factor of the generator current on the modulation depth and speed of the wind turbine for different multiplicities of frequencies is presented in fig. 7. It follows from these graphs, in engineering calculations, and when $a_R > 15 \div 20$ and $\omega^* = 0.4 \div 0.8$ you can take $\nu_{iR} \approx 1$. This means that the active power generated by the system is determined by the fundamental harmonics of current and voltage.

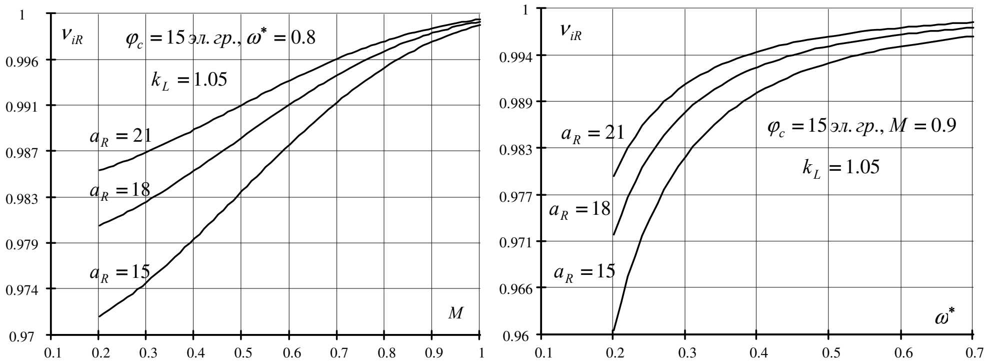

Fig. 7.

Let's consider the character of change of generated power, and also currents and voltages using (11), (12) and (16a,). For the real synchronous generators it is characteristic $L_d \approx L_q$, i.e. $k_L \to 1$; besides this, usually, the inequality $L_f < L_d$ takes place, so $q < 1$.

For WPI with a variable speed wind turbine the generated active power (P^*_{WTo}) in the working range ($\omega^* \in \{\omega^*_{WT\min}, \omega^*_{WT\max}\}$) is determined by the speed of the wind and can be calculated for the known characteristics of the turbine using the relation:

$$P^*_{WTo} = \gamma \cdot (\omega^*)^3 . \tag{20}$$

where γ - a constant coefficient determined by the design of wind turbine; $\{\omega^*_{WT\min}, \omega^*_{WT\max}\}$ - the working range, also characterized by the value $D_{WT} = \omega^*_{WT\max} / \omega^*_{WT\min}$; $\omega^*_{WT\min}, \omega^*_{WT\max}$ - the minimum and maximum operating speed of WT, respectively.

Obviously, the active power generated by the system (P^*_{Ro}) should satisfy the inequality:

$$P^*_{Ro} \geq P^*_{WTo} .$$

The orthogonal components of voltages in the sections S_R and S_G for a given active power are determined, as it follows from (12), according to the expression:

$$u^*_{Rqo} = \omega^* \cdot \left(1 - \frac{P^*_{Ro}}{u^*_{Rdo}}\right)\frac{k_L + q}{k_L - 1}; \quad u^*_{Gqo} = \omega^* \cdot \left(1 - \frac{P^*_{Ro}}{u^*_{Gdo}(1+q)}\right)\frac{k_L}{k_L - 1};$$

On the other hand, the active power generated by system

$$P^*_{Ro} = e^*_{Gq} i^*_{Gqo} = \omega^* i^*_{Gqo} = \omega^* \frac{u^*_{Rdo}}{X^*_{q\Sigma}} = \omega^* \frac{u^*_{Gdo}}{X^*_q} = \frac{1+q}{k_L + q} u^*_{Rdo} = \frac{1+q}{k_L} u^*_{Gdo} . \tag{21}$$

At $k_L \to 1$ $P^*_{Ro} \approx u^*_{Rdo} = (1+q) u^*_{Gdo}$.

Fig. 8 shows the dependence u^*_{Rqo} of u^*_{Rdo} for different values P^*_{Ro} . Constancy of the active power is carried out on the sites of characteristics between the points of «a» and «b» outside of these points the modulation index (M) is limited and the active power decreases.

The total power and power factor in the section S_R are defined by the relations:

$$S^*_{Ro} = [(Q^*_{Go})^2 + (P^*_{Go})^2]^{\frac{1}{2}} , \ \cos\varphi_R = P^*_{Ro} / S^*_{Ro} ,$$

where the reactive power Q^*_{Ro} is determined by the ratio

$$Q^*_{Ro} = \frac{1+q}{\omega^*(k_L + q)} (u^*_{Rdo})^2 + \frac{1}{\omega^*} (u^*_{Rqo})^2 - u^*_{Rqo} . \tag{22}$$

On fig. 9, 10 the dependences of S^*_R and $\cos(\varphi_R)$ on ω^* are presented. As can be seen from figure 9 at a certain frequencies, there is a minimum total power that takes place at zero values of the inactive power. Denote the frequency at which there is a minimum of full power as ω^*_0 , while its value is determined using the relation

$$\omega^*_0 = u^*_{Rqo} + \frac{(u^*_{Rdo})^2}{u^*_{Rqo} \cdot} \frac{(1+q)}{(1+k_L)} = u^*_{Gqo} + \frac{(u^*_{Gdo})^2}{u^*_{Gqo} \cdot} \frac{1}{k_L}$$

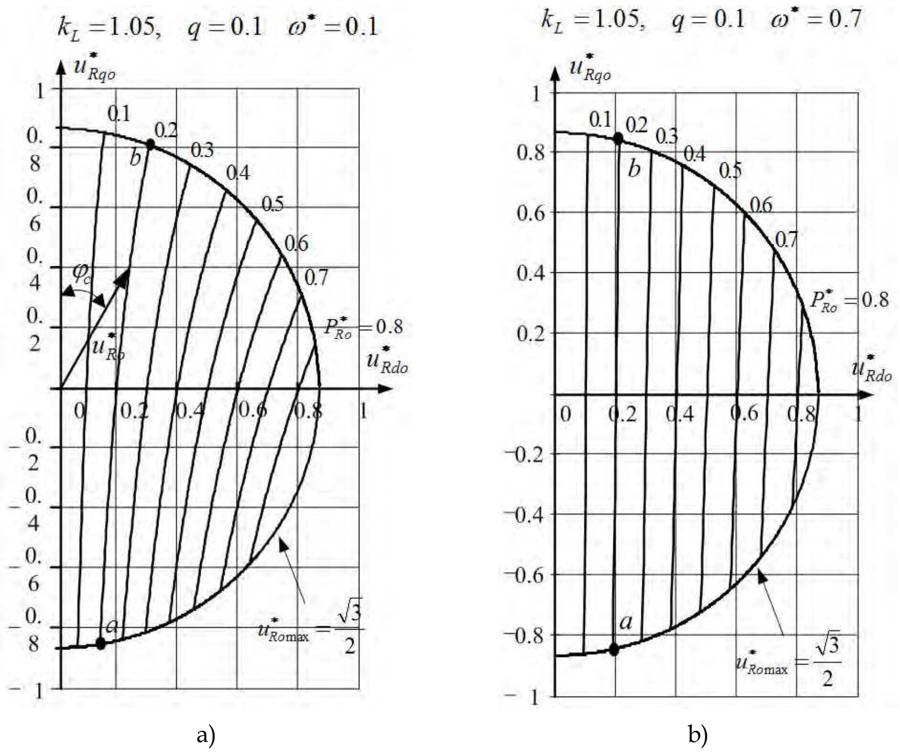
$k_L = 1.05, \quad q = 0.1 \quad \omega^* = 0.1$
u^*_{Rqo}
u^*_{Rdo}
$P^*_{Ro} = 0.8$
$u^*_{Romax} = \frac{\sqrt{3}}{2}$
a)
$k_L = 1.05, \quad q = 0.1 \quad \omega^* = 0.7$
u^*_{Rqo}
u^*_{Rdo}
$P^*_{Ro} = 0.8$
$u^*_{Romax} = \frac{\sqrt{3}}{2}$
b)

Fig. 8.

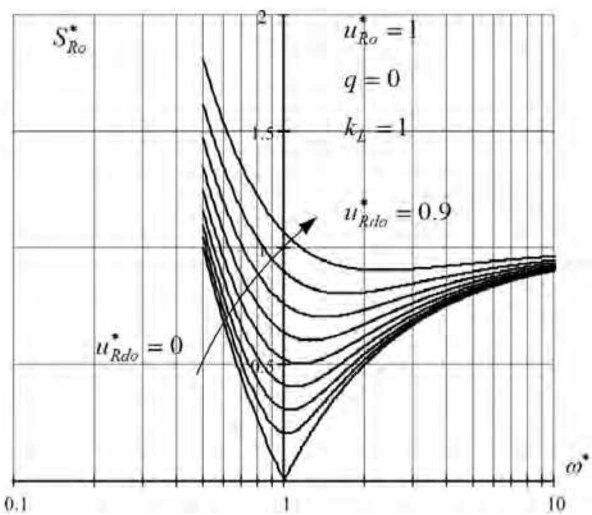
S^*_{Ro}
$u^*_{Ro} = 1$
$q = 0$
$k_L = 1$
$u^*_{Rdo} = 0.9$
$u^*_{Rdo} = 0$
ω^*
0.1
1
10

Fig. 9.

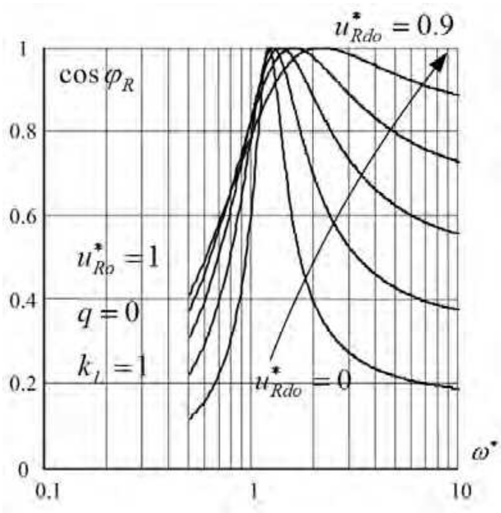

Fig. 10.

As appears from (22), the orthogonal components u^*_{Rdo} also u^*_{Rqo} are connected by the equation

$$\left(\frac{u^*_{Rdo}}{\gamma_Q R_Q}\right)^2 + \left(\frac{u^*_{Rqo} - \dfrac{\omega^*}{2}}{R_Q}\right)^2 = 1\,, \tag{23}$$

where:

$$\gamma_Q = \sqrt{(k_L + q)/(1+q)}, \quad R_Q = \sqrt{\omega^* Q^*_{Ro} + (\omega^*/2)^2}\ . \tag{24}$$

Expression (23) is the equation of an ellipse with the major axis - $2a_d$ and the minor axis - $2b_q$ that we find from the following expressions:

$$a_d = \gamma_Q R_Q; \quad b_q = R_Q\ .$$

Thus the ellipse centre in «*d q*» co-ordinates is located in a point $\left(0, \omega^*/2\right)$.
Considering the known relation [8], equation (23) in polar coordinates takes the form:

$$\rho(\phi)^2 \cdot [1 - (\varepsilon\cos\phi)^2] - 2\rho_{ucr}(\phi)\rho_0\left[\left(\frac{b_q}{a_d}\right)^2 \cos\phi\cos\phi_0 + \sin\phi\sin\phi_0\right] + \rho_0^2 \cdot [1 - (\varepsilon\cdot\cos\phi_0)^2] - {b_q}^2 = 0,$$

where co-ordinates of the centre of the ellipse ρ_0 , ϕ_0 and the parameter ε are defined by means of expressions

$$\rho_0 = \frac{n^*}{2}; \quad \phi_0 = \frac{\pi}{2}; \quad \varepsilon^2 = 1 - \left(\frac{b_q}{a_d}\right)^2 = \frac{k_L - 1}{k_L + q}. \tag{25}$$

Considering (24) ÷ (25) we will obtain the following expression for the locus of voltage $u^*_{Ro} = \sqrt{(u^*_{Rdo})^2 + (u^*_{Rqo})^2}$ in section S^*_R

$$u^*_{Ro} = \left[\omega^* \sin\phi + \sqrt{\left(\omega^* \sin\phi\right)^2 + 4\omega^* Q^*_{Ro}\left(1-\varepsilon^2\cos\phi\right)}\right] \Big/ \left[2\left(1-\varepsilon^2\cos\phi\right)\right]; \quad \phi \in (0, 2\pi).$$

The dependence $u^*_{Ro}(\phi)$ for different values ω^* and the value of reactive power Q^*_{Ro} are shown in Figure 11. Here and below, a circle with a radius $u^*_{Ro\max} = \sqrt{3}/2$ is limiting mode with $M = 1$, i.e. outside this circle the modulation depth is limited, and therefore the ratios obtained above are valid only inside the circle. The negative value of the reactive power Q^*_{Ro} means that in the given section the current of the basic harmonic lags behind of a voltage phase.

Considering $P^*_{Ro} \approx u^*_{Rdo}$, from fig. 11b it follows that the maximum active power ($P^*_{Ro\max}$) which is defined by the maximum projection of locus on a «d» axis, essentially depends on size of the reactive power (Q^*_{Ro}), and at a negative value of Q^*_{Ro} $P^*_{Ro\max}$ decreases. Indeed from (23) and (24) we obtain:

$$P^*_{Ro\max} \approx \gamma_Q R_Q \approx \sqrt{\omega^* Q^*_{Ro} + (\omega^*/2)^2} \ .$$

By changing the coordinates u^*_{Rdo} , u^*_{Rqo} we obtain the possibility to control the active power generated and the reactive power consumed from the generator on the fundamental harmonic.

Using the relation $Q^*_{Ro} = tg\varphi_{SR} P^*_{Ro}$ and assuming $q = 0$ that equation (23) can be rewritten in the variables P^*_{Ro} and u^*_{Rqo}

$$\left(\frac{P^*_{Ro} - \frac{\omega^*}{2} tg\varphi_{SR} \cdot k_L}{\sqrt{k_L} \cdot \frac{\omega^*}{2}\sqrt{1+k_L\left(tg\varphi_{SR}\right)^2}}\right)^2 + \left(\frac{u^*_{Rqo} - \frac{\omega^*}{2}}{\frac{\omega^*}{2}\sqrt{1+k_L\left(tg\varphi_{SR}\right)^2}}\right)^2 = 1 \qquad (26)$$

$$a_d = \frac{\omega^*}{2}\sqrt{k_L}\sqrt{1+k_L\left(tg\varphi_{SR}\right)^2}; \quad b_q = \frac{\omega^*}{2}\sqrt{1+k_L\left(tg\varphi_{SR}\right)^2} \ .$$

Thus the ellipse centre in «*d q*» co-ordinates is located in a point $\left(\omega^* tg\varphi_{SR} \cdot k_L \cdot (1+q)/2, \omega^*/2\right)$

$$\rho_0 = \frac{n^*}{2}\sqrt{1+(tg\varphi_{SR} \cdot k_L)^2}; \quad \phi_0 = arctg\left(\frac{1}{tg\varphi_{SR} \cdot k_L}\right); \varepsilon^2 = 1 - \left(\frac{b}{a}\right)^2 = \left(1 - \frac{1}{k_L(1+q)^2}\right).$$

The maximum active power generated for the each set of parameters is determined by the point on the graph, as shown, for example, in fig.12a. From the relation (26) we obtain

$$P^*_{Ro\max} = a_d + \omega^* k_L tg\varphi_{SR}/2 = \omega^* \big/ 2[\sqrt{k_L} tg\varphi_{SR} + \sqrt{1+k_L(tg\varphi_{SR})^2}\,] \ .$$

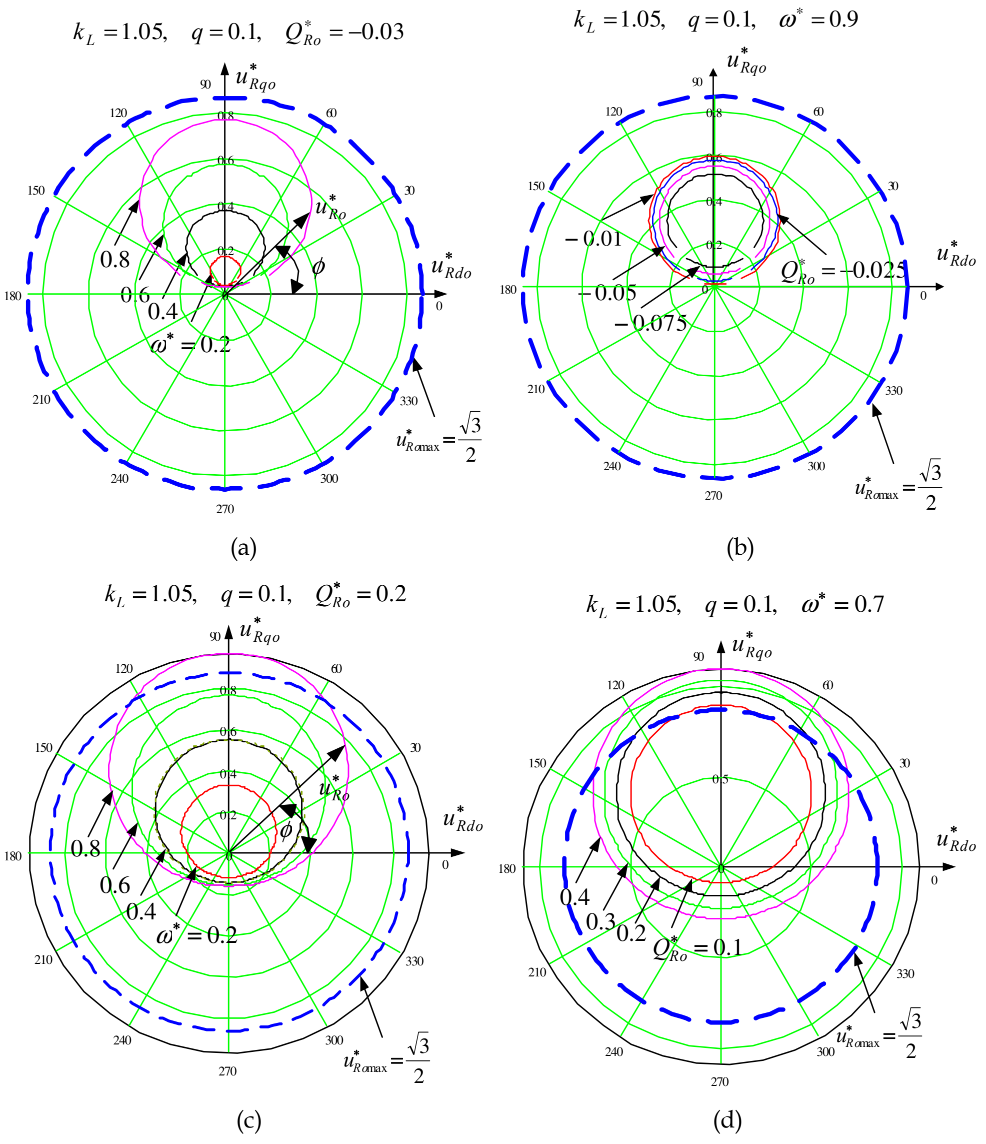

Fig. 11.

The graph of dependence $P^*_{Ro\max}$ on the frequency of rotation ω^* and φ_{SR} for the various k_L is presented in fig. 13.

The above reasoning and results of calculations allow drawing a conclusion that thanks to possibility of independent regulation by means of the active rectifier of orthogonal components of the resultant voltage vector u_R, modes with various $\cos(\varphi_R)$ values at change ω^* are possible in the system.

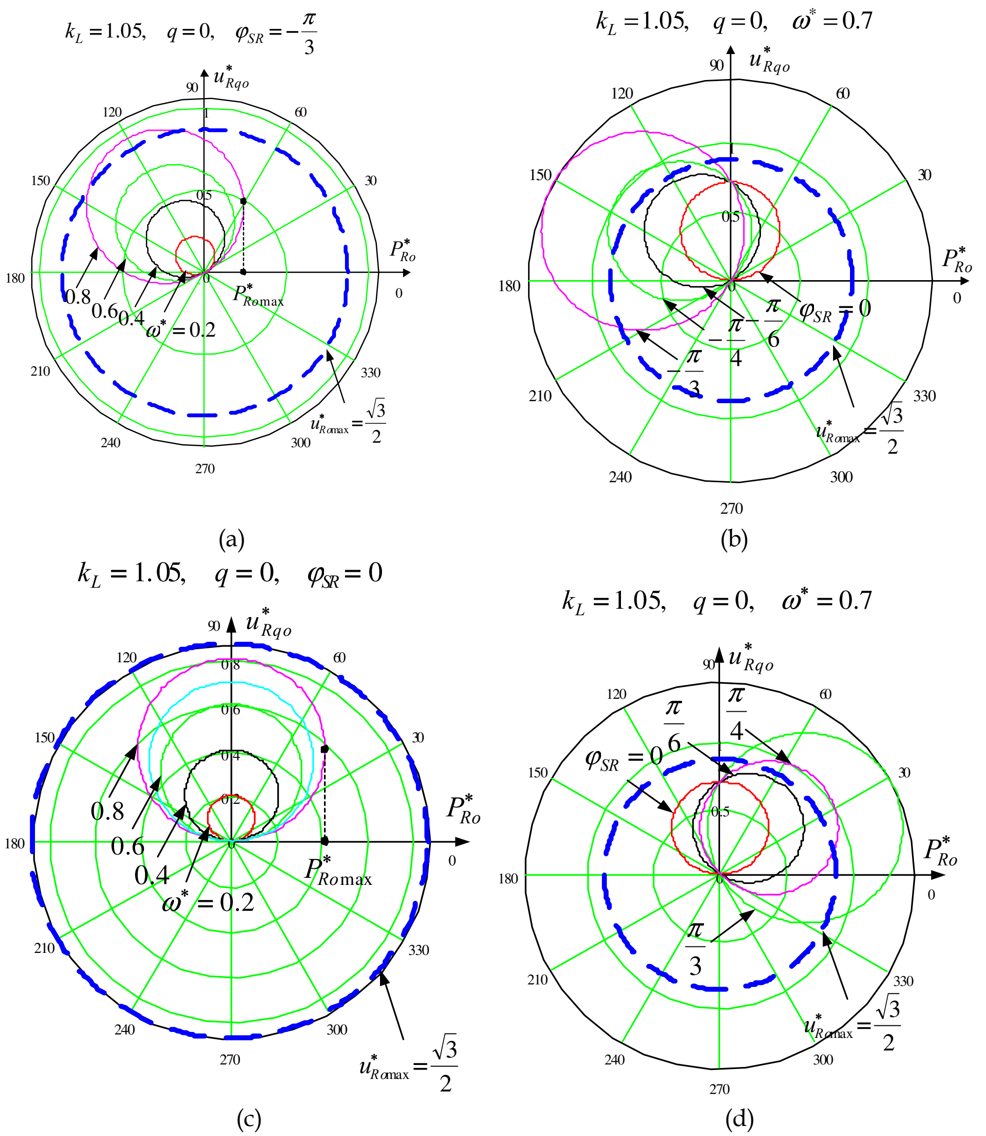

Fig. 12.

Phases of the fundamental harmonics of current and voltage of the generator coincide.

The vector diagram for the fundamental harmonics of current and voltage in the given mode is shown in fig. 14. As appears from the given diagram

$$tg(\theta) = i_{do}^{*} / i_{qo}^{*} = u_{gdo}^{*} / u_{gqo} .$$