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Kinematic Design and Description of Industrial Robotic Chains

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1. Introduction

Today, industrial robots can replace humans in carrying out various types of operations. They can as well serve machine tools as to carry out various tasks like welding, handling, painting, assembling, dismantling, foundering, forging, packaging, palletizing ….in different areas of the mechanical, car, aerospace, automotive, electronics … and other industries. However, the complexity of the industrial process poses difficult problems for insertion and generation of the movements of a robot: the working environment of the robot is often complex and varied (presence of obstacles during the execution of a task for example).

One of the objectives concerning the problems of computer-aided design (CAD) of robots is the validation of their topological structures. The robot-design engineer puts forth assumptions as regards the provision of the links and the joints of the mechanical articulated system. A first validation of this choice is of a geometrical nature (Merlet, 1996). At first sight the design of a mechanical architecture for a robot appears rather simple and yet it presents a very complex basic difficulty, as it must take into account not only the mechanical possibilities of realization but also the possibilities of control's development, which passes by generating of a mathematical model. The latter strongly affects the mechanical design if good performances are sought. Many methods for mechanism description appeared with the creation of CAD systems (Warnecke, 1977) and (Coiffet, 1992). The existing methods may be separated into two categories:

- description methods for classification (Roth, 1976),
- methods for mathematical modelling (Borel, 1979), (Khalil, 1976), (Renaud, 1975) and (Touron, 1984).

Mechanisms and Machines Theory (MMT) contributed greatly to planar and spatial mechanism synthesis with different degrees of freedom (Hwang & Hwang, 1992; Hervè, 1994; Gonzales, 1996; Karouia & Hervè, 2005). Some of

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the current industrial robots with planar chains have a structure created by the kinematic graphs of MMT (Manulescu et al. 1987; Ma & Angeles, 1991).
The morphological (topological) synthesis of kinematic chains has, for a long time, been the subject of many papers. There are different methods for number synthesis of planar kinematic chains with simple revolute joints, with different degrees of mobility and different numbers of links and joints. These chains are usually called "planar pin-joined" chains in MMT. While number synthesis originates from kinematics of mechanisms, all methods entail operations on graphs, which in one way or another, represent kinematic chains.
There exist different methods for kinematic synthesis of planar chains with simple joints (Tischler et al., 1995; Belfiore, 2000; Rao & Deshmukh, 2001): intuition and inspection (Crossley, 1964), graphs theory (Dobrjanskyi and Freudenstein, 1967; Woo, 1967). Others consist in transformation of binary chains (Mruthyunjaya, 1979; Mruthyunjaya, 1984-a; Mruthyunjaya, 1984-b; Mruthyunjaya, 1984-c), in the concept of Assur groups (Manolescu et al., 1987; Manolescu, 1964; Manolescu, 1979; Manolescu, 1987), or Franke's notation (Davies & Crossley, 1966; Crossley, 1966). Recently, new methods based on genetic algorithms or neuronal networks are also used (Tejomurtula & Kak, 1999; Abo-Hamour et al., 2002; Cabrera et al., 2002; Laribi et al., 2004).
The analysis of existing methods shows that there are several methods applied to the development of a mathematical model concerning its application for the control design of the robot. However, concerning the topological description of the chains and robots, only Roth-Piper's method (Roth, 1976; Pieper & Roth, 1969) tends towards mechanism description with a view to classify robots.
Generally speaking, the problem of synthesis of mechanism falls into three sub problems:

- specification of the problem: topological and functional specifications and constraints imposed by the environment,
- topological synthesis of the mechanism: enumeration and evaluation of possible topologies,
- dimensional synthesis: choice of dimensions of the mechanism for the selected types of morphologies.

This chapter relates in particular to the second sub problem. Its *principal goal* is to present an overview concerning the chronology of the design of an industrial robot kinematic chain. The chapter starts with a brief reminder of the theory of Modular Structural Groups (MSG), and of the *connectivity* and *mobility* laws of MMT presented in § 2. Afterwards, a new method for structural synthesis of planar link chains in robotics is presented in § 3. It is based on the notion of *logical equations*. Various levels of abstraction are studied concerning the complexity of the structure. This permits the synthesis of planar chains with various degrees of complexity expressed by the number of links, joints and the degree of mobility. The logical equations allow the association of MSGs of type A and closed chains of type G. The rules for associations of groups are also presented. The aim is to execute all the possible combinations to *join* or *transform* links in order to obtain as many structures as possible by avoiding those which are isomorphic. The association of two groups allows the elaboration of different closed chains of upper level. However there are some defective structures, which do not respect the connectivity and mobility laws. Therefore great care has been taken to avoid them. The problem of degenerated structures is central in their synthesis. It especially concerns chains with two and more degrees of mobility. The problem of *defect*, *degeneration* and *isomorphism* is then approached. Later, a method for description of chains by *contours* and *molecules* is proposed in § 4. It enables to reduce the number of the topological structures of the robots concerning its frame and end-effector position by comparing their respective molecules. This method is then applied in § 5 to describe the structures thus obtained (by logical equations approach) and the topology of the *principal structure* of the industrial robots in the initial phase of their design. Finally a classification of industrial robot structures by different levels of complexity is presented in § 6.

2. Topology of a linked mechanical structure, definitions, terminologies and restrictions

The Manipulation System (MS) of the robot is a mechanism composed of links (elements) joined by kinematic joints often with one degree of mobility (rotary or prismatic joints). These elements are considered as rigid bodies which form a kinematic chain, plane or spatial, which can be open or closed. The type of a kinematic link noted "j" is given by the number of joints enabling its association with other links (Erdman & Sandor, 1991). There are links of “binary”, “ternary”, “quaternary”… “polynary” type with j=1,2,3…, following from the fact that the link contains 1,2,3,… kinematic joints :

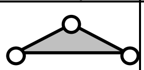
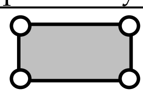
link	binary	ternary	quaternary	...
representation				...
notation	N2	N3	N4	…

Table 1. Planar link types

MMT proposes various ways of representing kinematic structures. The most common, the *kinematic graph*, consists in conserving a shape for the links in order to better appraise the topology of the structure. Nevertheless this presentation is difficult to manipulate. Any kinematic structure may be transformed into Crossley's *inverse graph* (Crossley, 1964) replacing every link (binary, ternary…) by a point. Lines linking the points concerned represent the joints themselves.

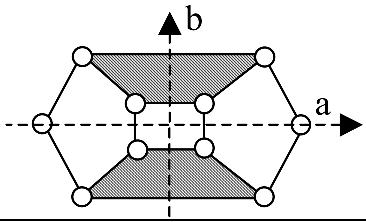
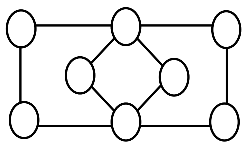
kinematic graph	inverse graph
	

Figure 1. Representation of a structure by kinematic and Crossley's inverse graph

Kinematic *chain* (named Grübler chain in MMT) is a structure which "floats" in space and all its links have identical status. If a frame and an input link (links) is (are) indicated the chain becomes a *mechanism*. The possible motions in a chain are called *degree of freedom* (or absolute degree of mobility) whereas for a mechanism they are called *degree of mobility* (or relative degree of mobility) of N-link Grübler chain.
As it has been said, the arm of a robot or the MS is a mechanism composed of a set of rigid links, which form an open or closed kinematic chain. Existing robots may be topologically classified in two categories according to whether their structure is open or closed:

- robots with simple (open) structure: one can traverse all the kinematic joints by making an open chain. They are noted $A_{i,j}$ where *i* and *j* are respectively the degree of mobility and the number of links of the structure (cf. § 3.1).
- robots with complex (arborescent or closed) structure: one can traverse all the kinematic joints by making a closed loop. They are noted $G_{k,l}$ where *k* and *l* are respectively the degree of mobility and the number of links of the structure (cf. § 3.1).

Another method allows them to be classified mechanically as robots with planar or spatial chains.

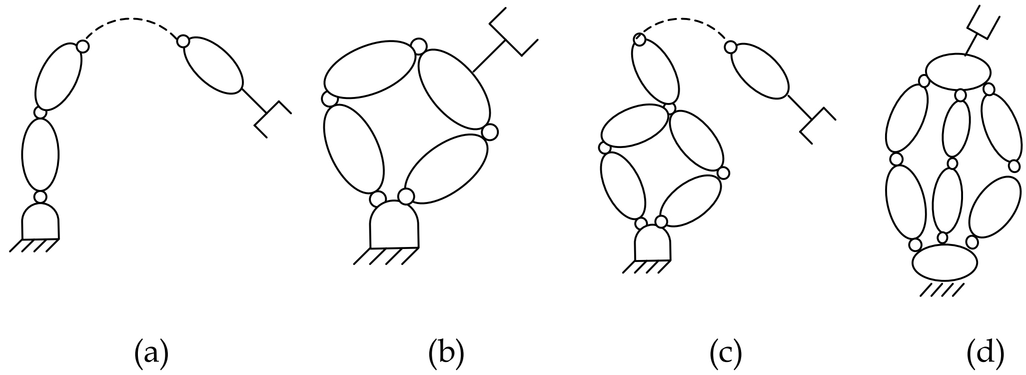


Figure 2. Robots with: simple-open (a), closed (b), arborescent (c) and complex structures (d)

Robots, being complex mechanical systems with an important interaction between their links, their architecture is defined by (Mitrouchev, 1999):

- the *main structure* which generates the main motion of the robot and upon which is situated the rest of the MS,
- the *regional structure* which is composed of the arm and of the forearm (mechanical arm),
- the *local structure* which is the wrist of the robot often with three degrees of freedom in rotation.

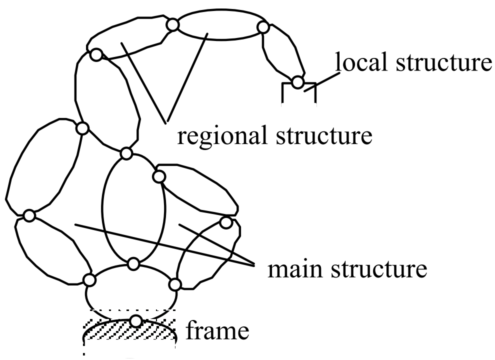


Figure 3. Topological structure of a robot

The number M of degrees of mobility of an isostatic chain relative to a fixed frame is given by Somov-Malisev's formula (Artobolevski, 1977):

$$M = 6N^* - \sum_{k=1}^{5} kC_k \tag{1}$$

with: C_k – number of simple joints of class "k",
N* - number of mobile links (in general N* =N-1).

If the kinematic chain only contains simple rotary joints of class 5 (one degree of freedom joint), instead of equation (1), the Tchebychev-Grübler equation is used:

$$M = 3N^* - 2C_5 \tag{2}$$

with: C_5 - number of simple rotary joints with one degree of mobility.

In this chapter only planar structures with simple rotary joints of class 5 shall be considered. If the value of M in Tchebychev-Grübler's equation (2) is M=0 it becomes:

$$3N^* = 2C_5 \tag{3}$$

Some of the most characteristic MSGs (called Assur groups) resulting from equation (3) are presented in Table 2 by their kinematic graphs or structural diagrams (Manolescu, 1964).

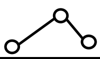
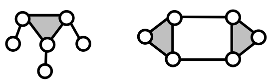
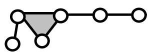
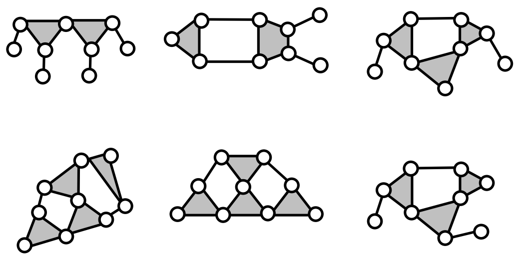
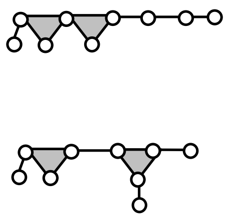
notation	N*	C_5	Assur groups	derivative groups
$A_{0,2}$	2	3		....
$A_{0,4}$	4	6		
$A_{0,6}$ ...	6 ...	9 ...		 ....

Table 2. Modular Structural Groups (MSGs) with zero degree of mobility

If the value of M in Tchebychev-Grübler's equation (2) is M=1 it becomes:

$$3N^* - 1 = 2C_5 \tag{4}$$

Some of its solutions are presented in table 3.

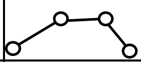
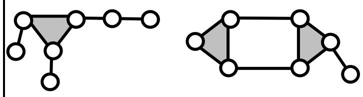
notation	N*	C_5	kinematic graphe (structural diagram)
$A_{1,3}$	3	4	
$A_{1,5}$ …	5 …	7 …	 …………………………

Table 3. SMGs with one degree of mobility

Finally for the MSGs adding two degrees of mobility to the structures (M=2), one obtains:

$$3N^* - 2 = 2C_5 \tag{5}$$

Some of its solutions are presented in table 4:

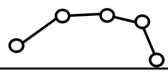
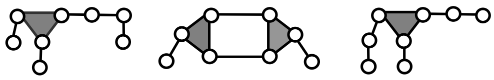
notation	N*	C_5	structural diagram
$A_{2,4}$	4	5	
$A_{2,6}$ …	6 …	8 …	

Table 4. MSGs with two degrees of mobility

3. Proposed method and results

Let us consider a plane kinematic structure comprising N links and C_5 kinematic joints in open, arborescent or closed chain. The first link S_1 is the presumed fixed frame of the robot. MMT, being part of the technological sciences, is at the base of mechanism design in robotics as has been previously mentioned. The question is: for a given degree of mobility and for a given number of links and joints, how many possibilities are there to join them in a mechanism suitable for application in kinematic chain design in robotics? The answer to this question is presented in this paragraph and in the following one. In order to answer the question above, let us consider a mechanism with a given degree of mobility (M). Gauhmann's generalised law (called *connectivity* law) giving the relationship between the number of the links and the joints is (Manolescu, 1987):

$$C_5 = I + N \tag{6}$$

with: I=R-2 -structural number representing the complexity level of the structure,

R - number of closed loops in the chain allowing to "read" (to pass through) twice each kinematic joint.

For the planar kinematic structures the *general law of mobility* giving the relationship between the number of degrees of mobility and the joints is:

$$C_5 = M + 3I + 3 \tag{7}$$

The equations (6) and (7) give:

$$N = M + 2I + 3 \tag{8}$$

Those two latter equations (7 and 8) lead to the *first manner* of calculating the number of joints C_5 and the number of links N for planar kinematic structures with different degrees of relative mobility M. The *general law of mobility* giving the relationship between the number of degrees of mobility and the joints is (Manolescu, 1987):

$$M + (1 + I)H = \sum_{k=f+1}^{5} (6 - k)C_k \tag{9}$$

with: - H=(6-F),
- F the number of imposed constraints in a chain.

For a planar kinematic structures, F=3 hence H=3. Equation (9) becomes:

$$M + 3(1 + I) = C_5 + 2C_4 \tag{10}$$

This latter equation allows the *second manner* of calculating the number of links and joints for planar kinematic structures with different degrees of mobility.

3.1 Logical equation

3.1.1 Notations

We note by:

- $A_{i,j}$ a MSG (Assur group or its derivative) which may be open or closed. There are always some exterior (free) joints which only become active when the group is in connection with a joint shared with a chain or a mechanism (Manolescu, 1964),
- $G_{k,l}$ a closed group (structure) without exterior joints.

The first mark represents the degree of mobility of the group and the second one represents the number of its links (cf. Table 5).

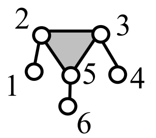
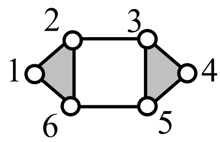
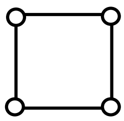
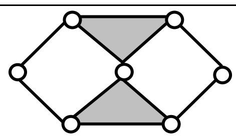
groupes	structural diagram	joints	
		exterior	interior
Ai,j		1,4,6	2,3,5
		1,4	2,3,5,6
Gk,*l*	 , 	interior only	

Table 5. Structural diagram and types of joints

A *logical equation* is defined as the association (combination) of two initial terms (marks in lower case letters). The result of this association is a final closed kinematic structure of type $G_{M,N}$ (marks in capital letters) of upper level. The two initial terms may be (Mitrouchev, 2001):

1. Two identical MSG groups, for example: $A_{i,j} + A_{i,j} = G_{M,N}$

with: M=2i+1; for M=1 and I=0,2
for M=3 and I=0
N=2j.

2. Two different MSG groups, for example: $A_{i,j} + A_{i',j'} = G_{M,N}$

with: M=i+i'+1; for M=1, and I=1,2,3
for M=2, and I=0
N=j+j'.

3. One MSG group and one closed group of type $G_{k,l}$: $A_{i,j} + G_{k,l} = G_{M,N}$

with: M=i+k; for M=1, and I=1,2,3
for M=2,3 and I=1,2
N=j+*l*

It can be noted that the association of two closed structures is not possible.

3.1.2 Rules for associations

The association of two groups may be done by two operations: direct *junction* and *transformation* of links. They may be executed separately or together. The aim is to use all possible combinations to join or transform the links in order to obtain as many structures as possible (for given N and C_5) by avoiding those, which are isomorphic. Some examples are shown in the Table 6:

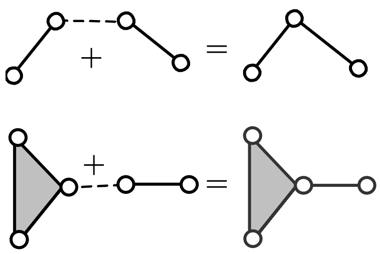
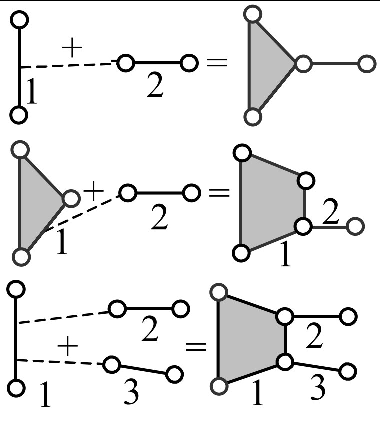
by direct junction	
by transformation of links: - the binary link 1 becomes ternary - the ternary link 1 becomes quaternary - the binary link 1 becomes quaternary and so on	

Table 6. Rules for associations of groups

3.2 Generation of chains with one relative degree of mobility (M=1)

Fof M=1 the equations (7) and (8) give:

$$C_5 = 4 + 3I \tag{11}$$

and

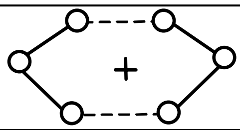
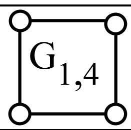
$$N = 4 + 2I \tag{12}$$

3.2.1 First level of abstraction, (I=0)

For I=0, equations (11) and (12) give C_5=4 and N=4. Therefore only one closed planar structure of type $G_{1,4}$ can be synthesised from the following logical equation:

$$A_{0,2} + A_{0,2} = G_{1,4} \tag{13}$$

as follows:

logical equation	schematic	closed structure
$A_{0,2} + A_{0,2} = G_{1,4}$		

The equation (6) gives C_5=N. For C_5=N=4 there is a mono-contour mobile structure called a group of type G4. For M=1, the equation (10) gives:

$$C_5 + 2C_4 = 4 \tag{14}$$

The possible solutions of this equation are: a) C_4=0, C_5=4, b) C_4=1, C_5=2 and c) C_4=2, C_5=0.

From the solution a) only one closed planar structure of type G4 and noted $G_{1,4}$ can be synthesised by the logical equation (13) above.

3.2.2 Second level of abstraction, (I=1)

For I=1, equations (11) and (12) give C_5=7 and N=6. The law for generation a group of elementary modules allows the calculation of the number of links, which are ternary and more (quaternary, quintary and so on) in a closed chain (Manolescu et al, 1972).

$$2I = \sum_{j=3}^{R} (j-2)N_j \tag{15}$$

For I=1 equation (15) gives N_3=2. From equation (10) one deduces:

$$7 = C_5 + 2C_4 \tag{16}$$

The possible solutions of this equation are: a) C_4=0, C_5=7; b) C_4=1, C_5=5; c) C_4=2, C_5=3 and d) C_4=3, C_5=1. The number of the links is calculated by equation (6). It becomes:

$$N = C_5 - I = 6 \tag{17}$$

Consequently the number of binary links is:

$$N_2 = N - N_3 \tag{18}$$

The second level planar structures, of type $G_{1,6}$ are obtained from the following logical equations:

$$G_{1,4} + A_{0,2} = G_{1,6} \tag{19}$$

$$A_{0,4} + A_{0,2} = G_{1,6} \tag{20}$$

Both equations (19) and (20) give two solutions: Watt's and Stephenson's structures. These latter are described by their kinematic graphs (structural diagram) in table 7.

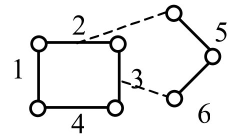
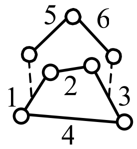
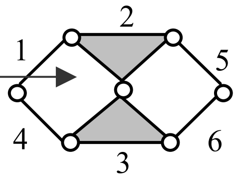
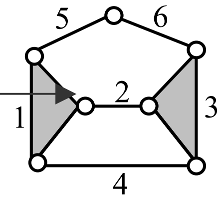
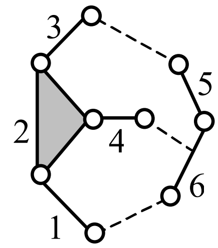
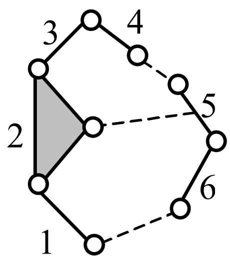
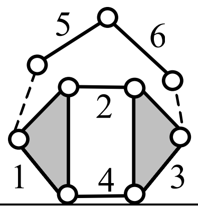
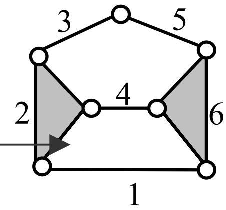
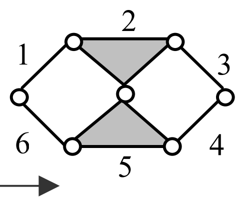
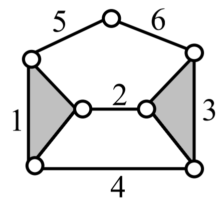
logical equation	groups	method for association	structural diagram
$G_{1,4} + A_{0,2} = G_{1,6}$	 	by transformation of links	 Watt  Stephenson
$A_{0,4} + A_{0,2} = G_{1,6}$	  	by junction and transformation of links by junction and transformation of links by junction	  

Table 7. Watt's and Stephenson's structures

It may be noted that there are three isomorphic structures, which restrict their number to two. Watt's and Stephenson's structures are generated from *closed immobile structure* composed of two ternary links by adding to it four binary links as presented in figure 4.

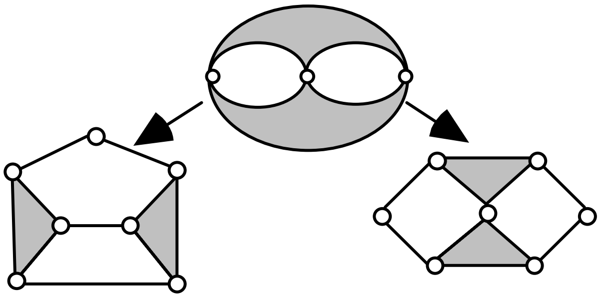


Figure 4. Closed immobile structure (collapse) and its derivatives

The closed immobile structures have no degree of mobility. These are hyperstatic systems, thus they are invariant concerning the mobility parameter. The theory of closed structures is applied just as much in robotics as in civil engineering, concerning bridges and roadways, seismic structures etc. They are equally applicable for planar structures as for spatial ones.

3.2.3 Third level of abstraction, (I=2)

According to expressions (11) and (12) for I=2 one obtains C_5=10 and N=8 in other words the structure has 8 links and 10 joints. According to equation (15) the number of links is:

$$N_3 + 2N_4 = 4 \tag{21}$$

The possible solutions of this equation and the associated closed structures are given in table 8:

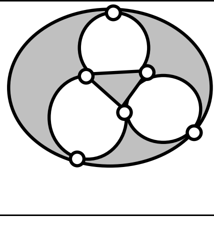
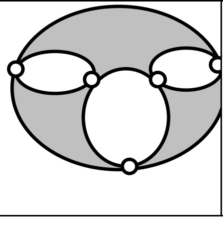
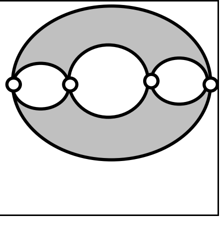
solution	a) N_3=4, N_4=0	b) N_3=2, N_4=1	c) N_3=0, N_4=2
associated closed structure			

Table 8. Third level closed structures

For the case of planar structure, the number of the binary links is:

$$N_2 = N_0 + \sum_{j=4}^{R=I+2} (j-3)N_j \tag{22}$$

where N_0=M+H is the number of links in the main Crossley's chain (N_0=1+3=4) hence:

$$N_2 = 4 + \sum_{j=4}^{R} (j-3)N_j \tag{23}$$

For I=0 and I=1 the equation (23) has no physical meaning. For j≤3, the number of binary links is N_2=4, so to have a mobile mechanism one needs four binary links, hence for j=4:

$$N_2 = 4 + N_4 \tag{24}$$

The possible solutions of this equation are: a) N_2=4, N_4=0, b) N_2=5, N_4=1, c) N_2=6, N_4=2 and d) N_2=7, N_4=3. The three solutions of (24) which respect the solutions of (21) are a), b) and c).
The third level planar structures of type $G_{1,8}$ are obtained from the following logical equations:

$$G_{1,6} + A_{0,2} = G_{1,8} \tag{25}$$

$$G_{1,4} + A_{0,4} = G_{1,8} \tag{26}$$

$$A_{0,4} + A_{0,4} = G_{1,8} \tag{27}$$

$$A_{0,2} + A_{0,6} = G_{1,8} \tag{28}$$

Equation (25) gives 12 structures; equation (26) gives two structures and a double (isomorphic); equation (27) gives one structure, one defect and one double. Finally, equation (28) gives one structure and two doubles. Table 14 (cf. § 4.) recapitulates the sixteen structures solutions of these four logical equations.

3.2.4 Problem of defect, degeneration and isomorphism

The problem of degenerated structures is central in synthesis of structures. It does not concern the chains with one degree of mobility (M=1). But if the chains have two, three or more degrees of mobility there are two degenerate forms of chains called *partial* and *fractionated* mobilities (Manolescu, 1964). If any link of the chain is selected to be the frame of a mechanism and if one chooses any M members to be the independent driving links (called "motor" links), then the position of every remaining member is dependent on the position of all links, such a mechanism is said to have a *total degree of mobility* (Davies & Crossley, 1966). This is the case of structures: 2 of Table 11, and 2, 3, 4 and 6 of Table 12. A mechanism may possess a *partial degree of mobility* if it cannot fulfil the conditions of total degree of mobility mechanism. In all or some of the derived mechanisms, the movement of certain of the driven links depends on the movements of only a number Mp of the motor links, where Mp<M (Manolescu, 1964). This is the case of structures: 1 and 3 of Table 11, and 1, of Table 12. Finally, a mechanism has a *fractionated degree of mobility* if it does not satisfy the conditions for a total degree of mobility. Such a mechanism possesses at least one link (with at least four joints), which may be decomposed into two or more parts, with the result that each sub-mechanism forms a closed chain. The mobility of the whole mechanism is equal to the sum of the mobilities of all sub-mechanisms (Davies & Crossley, 1966). This is the case of structures 4 of table 11; and 5 of table 12.

The rules for association of two groups allow the elaboration of different closed chains. But there are some defect structures, which do not respect the connectivity and mobility laws. Therefore great care has been taken to avoid them. For instance a defect structure is the one among the three solutions of the logical equation (27) shown in Table 9.

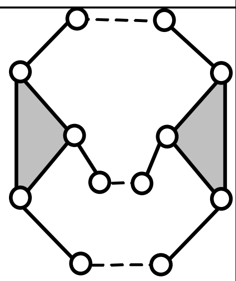
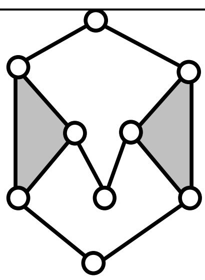
logical equation	groups	method for association	structural diagram
$A_{0,4} + A_{0,4} = G_{1,8}$		by junction	

Table 9. Defect structure

By junction of the six binary links, which is the simplest way, the resulting structure is a defect because it has I=9-8=1 and M=21-18=3. Nevertheless the structural number I is equal to 2 and the degree of mobility M is equal to 1. Finally there are some isomorphic structures too. Consequently the defect structures and the isomorphic ones are systematically moved away from the possible solutions.

3.2.5 Fourth level of abstraction, I=3

According to expressions (11) and (12) for I=3, one obtains: C_5=13 and N=10. The number of links, which are ternary or greater, is calculated from equation (15). It becomes:

$$6 = N_3 + 2N_4 + 3N_5 \tag{29}$$

The possible solutions of this equation and their associated closed structures are given in the table 10:

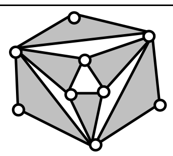
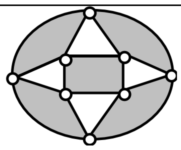
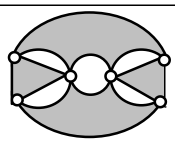
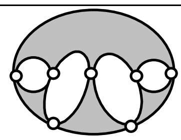
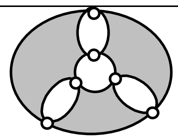
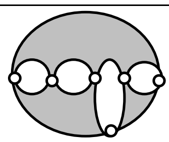
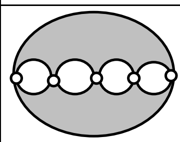
solution	a)N_3=6, N_4=0, N_5=0	b)N_3=4, N_4=1, N_5=0	c)N_3=2, N_4=2, N_5=0	d)N_3=3, N_4=0, N_5=1
associated closed structure				
solution	e)N_3=0, N_4=3, N_5=0	f)N_3=1, $_4$=1, N_5=1	g)N_3=0, N_4=0, N_5=2	
associated closed structure				

Table 10. Fourth level closed structures

The number of binary or greater links is calculated by equation (23):

$$N_2 = 4 + N_4 + 2N_5 \tag{30}$$

The possible solutions of this equation are:

a) N_2=4, N_4=0, N_5=0, b) N_2=5, N_4=1, N_5=0, c) N_2=6, N_4=2, N_5=3, d) N_2=6, N_4=0, N_5=1, e) N_2=7, N_4=3, N_5=0, f) N_2=7, N_4=1, N_5=1, g) N_2=8, N_4=0, N_5=2 and h) N_2=8, N_4=4, N_5=0.