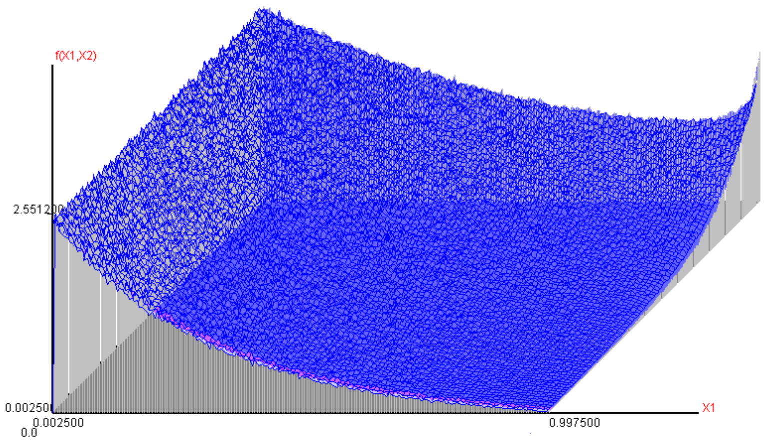
Continuous Bernoulli distribution
--- simulator and test statistic



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The computer software of this book, please download from [Google drive] or [Github].

Contents

Chapter 1, The Continuous Bernoulli distribution

1.The probability density function of Continuous Bernoulli distribution

The Bernoulli distribution and parameter= p ,

$f_X(x;p) = p^x(1-p)^{1-x}, x = 0,1, 0 < p < 1,$

X is discrete random variable,

Let X is continuous random variable and λ is the parameter which replaces p .

$$f_X(x;\lambda) = C(\lambda)\lambda^x(1-\lambda)^{1-x}, 0 \le x \le 1, 0 < \lambda < 1,$$

$$f_X(x;\lambda) = C(\lambda)(1-\lambda)\int_0^1\left(\frac{\lambda}{1-\lambda}\right)^x dx --- (1.1),$$

$$(i)\lambda \ne \frac{1}{2}, (1.1) = C(\lambda)(1-\lambda)\frac{\left(\frac{\lambda}{1-\lambda}\right)^x}{ln\left(\frac{\lambda}{1-\lambda}\right)}\Bigg|_0^1 = C(\lambda)\frac{2\lambda-1}{ln\left(\frac{\lambda}{1-\lambda}\right)} = 1,$$

$$C(\lambda) = \frac{ln(1-\lambda) - ln(\lambda)}{1-2\lambda},$$

$$(ii)\lambda = \frac{1}{2}, (1.1) = C(\lambda)\int_0^1 \frac{1}{2}dx = 2C(\lambda) = 1, C(\lambda) = 2,$$

Section 1, The Continuous Bernoulli distribution,

$X \sim CB(\lambda)$, this probability distribution for “machine learning”.

(1)The probability density function,

$$f_X(x;\lambda) = C(\lambda)\lambda^x(1-\lambda)^{1-x}, 0 \le x \le 1, 0 < \lambda < 1,$$

$$C(\lambda) = \begin{cases} \frac{2\,tanh^{-1}(1-2\lambda)}{1-2\lambda}, \lambda \ne \frac{1}{2} \\ 2, \lambda = \frac{1}{2} \end{cases}$$

$$tanh^{-1}(x) = \frac{1}{2}log_e\left(\frac{1+x}{1-x}\right) = \frac{1}{2}ln\left(\frac{1+x}{1-x}\right), -1 < x < 1,$$

(2)The distribution function,

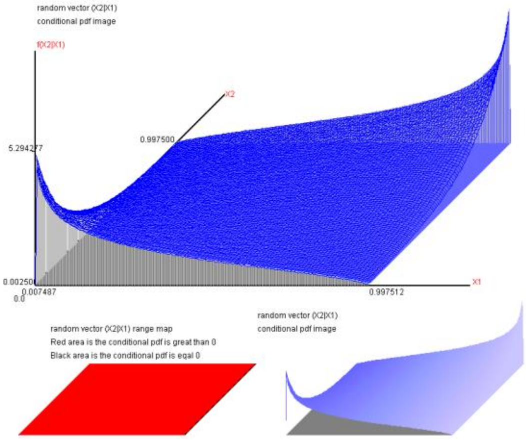
$$F_X(x;\lambda) = \begin{cases} \frac{\lambda^x(1-\lambda)^{1-x} + \lambda - 1}{2\lambda - 1}, \lambda \ne \frac{1}{2} \\ x, \lambda = \frac{1}{2} \end{cases}, 0 < x < 1$$

(3) The λ is the shape parameter,

Let X~Continous Bernoulli(λ), the λ is the shape parameter from the below diagram. The f(X|λ) is the conditional probability density in λ ,$0<\lambda<1$, but the E(X)=λ is the function of λ.

The following diagram, let X2=X, X1=λ, f(X2|X1)= f(X|λ),

the diagram is (X1=λ, f(X2|X1)).



The red area is the range of (X,λ).

Section 2, The simulator of Continuous Bernoulli distribution,

The inverse of $F_X(x;\lambda)$

$$x=\begin{cases}\dfrac{log_e\left(F_X(x;\lambda)\times(2\lambda-1)-(\lambda-1)\right)-log_e(1-\lambda)}{log_e\left(\dfrac{\lambda}{1-\lambda}\right)},\lambda\neq\dfrac{1}{2}\\ F_X(x;\lambda),\lambda=\dfrac{1}{2}\end{cases}$$

The random number= $RND=F_X(x;\lambda)\sim Uniform(0,1)$,

$$x\ \text{simulated value}=\begin{cases}\dfrac{log_e\left(RND\times(2\lambda-1)-(\lambda-1)\right)-log_e(1-\lambda)}{log_e\left(\dfrac{\lambda}{1-\lambda}\right)},\lambda\neq\dfrac{1}{2}\\ RND,\lambda=\dfrac{1}{2}\end{cases}$$

(1)The simulated data generator,

do

{

getting RND,

converting x simulated value,

}

(2)The probability distribution simulator,

The probability distribution simulated database,

do 100,000,000 times,

{

getting RND,

converting x simulated value and saving the database,

}

This frequency distribution is likely to the probability density function, the sample mean of database is closed to the population mean and the relative error is below 1/10000.

Note: The computer program is C:\C_Bernoulli\C_Bernoulli_01.exe, which can compute the simulated data of Continuous Bernoulli distribution.

Section 3, The expectation and variance,

(1) $E(X)=C(\lambda)(1-\lambda)\int_0^1 x\left(\frac{\lambda}{1-\lambda}\right)^x dx---(1.2),$

$$(i)\lambda \neq \frac{1}{2},(1.2)=C(\lambda)(1-\lambda)\left(x\times\frac{\left(\frac{\lambda}{1-\lambda}\right)^x}{ln\left(\frac{\lambda}{1-\lambda}\right)}\Bigg|_0^1-\int_0^1\frac{\left(\frac{\lambda}{1-\lambda}\right)^x}{ln\left(\frac{\lambda}{1-\lambda}\right)}dx\right)$$

$$=C(\lambda)(1-\lambda)\left(\frac{\frac{\lambda}{1-\lambda}}{ln\left(\frac{\lambda}{1-\lambda}\right)}-\frac{\left(\frac{\lambda}{1-\lambda}\right)^x}{\left(ln\left(\frac{\lambda}{1-\lambda}\right)\right)^2}\Bigg|_0^1\right)$$

$$=C(\lambda)\left(\frac{\lambda}{ln(\lambda)-ln(1-\lambda)}+\frac{1-2\lambda}{(ln(\lambda)-ln(1-\lambda))^2}\right)$$

$$(ii)\lambda=\frac{1}{2},(1.2)=\int_0^1 xdx=0.5,$$

$$\mu=E(X)=\begin{cases}\frac{\lambda}{2\lambda-1}+\frac{1}{2tan^{-1}(1-2\lambda)} & if\ \lambda\neq\frac{1}{2}\\ \frac{1}{2} & if\ \lambda=\frac{1}{2}\end{cases}$$

(2) $E(X^2)=C(\lambda)(1-\lambda)\int_0^1 x^2\left(\frac{\lambda}{1-\lambda}\right)^x dx---(1.3),$

$$(i)\lambda \neq \frac{1}{2},(1.3)=C(\lambda)(1-\lambda)\left(x^2\times\frac{\left(\frac{\lambda}{1-\lambda}\right)^x}{ln\left(\frac{\lambda}{1-\lambda}\right)}\Bigg|_0^1-2\int_0^1\frac{x\left(\frac{\lambda}{1-\lambda}\right)^x}{ln\left(\frac{\lambda}{1-\lambda}\right)}dx\right)$$

$$=C(\lambda)\left(\frac{\lambda}{ln(\lambda)-ln(1-\lambda)}\right)-2E(X)$$

$$(ii)\lambda=\frac{1}{2},(1.3)=\int_0^1 x^2dx=\frac{1}{3},$$

$$Var(X)=E(X^2)-E^2(X),$$

$$Var(X)=\begin{cases}\frac{(1-\lambda)\lambda}{(1-2\lambda)^2}+\frac{1}{(2tan^{-1}(1-2\lambda))^2} & if\ \lambda\neq\frac{1}{2}\\ \frac{1}{12} & if\ \lambda=\frac{1}{2}\end{cases}$$

The estimated equation of $E(X)$, $Var(X)$,

$$\gamma_1(X)=E\left[\left(\frac{X-E(X)}{\sqrt{Var(X)}}\right)^3\right],\gamma_2(X)=E\left[\left(\frac{X-E(X)}{\sqrt{Var(X)}}\right)^4\right],$$

$\gamma_1(X)$ is skewed coefficient and $\gamma_2(X)$ is kurtosis coefficient.

Continuous Bernoulli distribution computed $E(X)$, $Var(X),\gamma_1(X)$ and $\gamma_2(X)$ is complexity, the estimated those moments using λ is easy way.
The Curvi-linear analysis(Taylor's expansion and regression combined) getting the mathematical model and computing the coefficients, the result could be accurately.

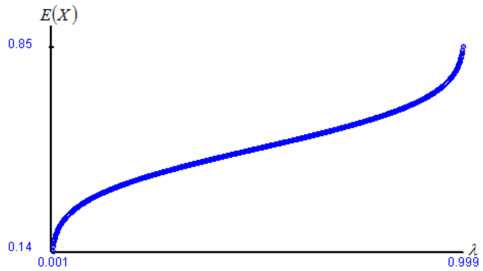
(1) $E(X)=G_1(\lambda)$, λ estimated $E(X)$,
The $E(X)$ estimated equation is $G_1(\lambda)$,
The $0.001\le\lambda\le 0.999$, $0.143853919\le\mu\le 0.856221427$,
The amount of paired data of (λ, $E(X)$) is 999, λ is setting value and $E(X)$ is computed by the simulator which has 100,000,000 data.

X= 0.279390+0.441311$\times\lambda$,
The estimated equation------
$G_1(\lambda)$ =0.50005887293491469+
0.77359483065083623*(X-0.50004573071171143)^1+
-0.01515211293008178500000000000*(X-0.50004573071171143)^2+
-27.27900934219360400*(X-0.50004573071171143)^3+
10.36370790004730200*(X-0.50004573071171143)^4+
15822.38842773437500000*(X-0.50004573071171143)^5+
-2817.42468261718750000*(X-0.50004573071171143)^6+
-3612752.6875*(X-0.50004573071171143)^7+
391281.72265625000000000*(X-0.50004573071171143)^8+
452401608.0000*(X-0.50004573071171143)^9+
-31440996.2500*(X-0.50004573071171143)^10+
-33874673664.0000*(X-0.50004573071171143)^11+
1540792624.0000*(X-0.50004573071171143)^12+
1582581137408.0000*(X-0.50004573071171143)^13+
-46642316288.0000*(X-0.50004573071171143)^14+
-46495537037312.0000*(X-0.50004573071171143)^15+
850124546048.0000*(X-0.50004573071171143)^16+
834533872107520.0000*(X-0.50004573071171143)^17+
-8542741594112.0000*(X-0.50004573071171143)^18+
-8357328558489600.0000*(X-0.50004573071171143)^19+
36339642531840.0000*(X-0.50004573071171143)^20+
35775834451083264.0000*(X-0.50004573071171143)^21
ANOVA

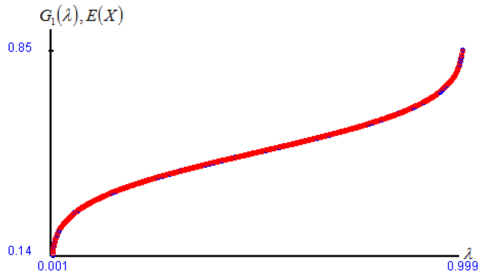
Source	df	SS	MS
Regression	21	16.7176990804	0.7960809086
Error	977	0.0001969542	0.0000002016
Total	998	16.7178960346	

H0:slope1=....=slope21=0, test statistic=3948994.157065,
sample size=999, R2=0.999988, R2(adj)=0.999988,MSE=0.000000,

(λ , $E(X)$) scatter diagram



(λ ,R= $G_1(\lambda)$,B= $E(X)$) scatter diagram



(2) $Var(X) = G_2(\lambda)$, λ estimated $Var(X)$,
The $Var(X)$ estimated equation is $G_2(\lambda)$,
The $0.001 \le \lambda \le 0.999$, $0.019960243 \le Var(X) \le 0.083352472$,
The amount of paired data of (λ , $Var(X)$) is 999, λ is setting value and $Var(X)$ is computed by the simulator which has 100,000,000 data.

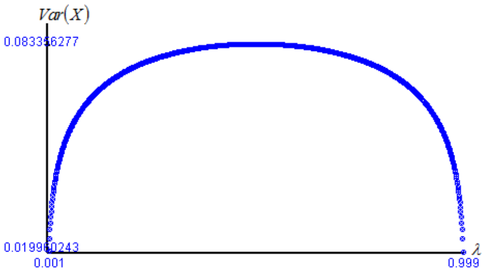
X=K(X1)=0.073806+-0.000019 $\times \lambda$,
The estimated equation ------
$G_2(\lambda)$=0.083298356117438743+
0.951844304800033570*(X-0.073795922003002973)^1+
-54413612.0*(X-0.073795922003002973)^2+
-200067416064.0*(X-0.073795922003002973)^3+
-508321342168110200000.0*(X-0.073795922003002973)^4+
723366691589871570000000.0*(X-0.073795922003002973)^5+
775849316051104270 0.0*(X-0.073795922003002973)^6+
-824069505565571400000 0.0*(X-0.073795922003002973)^7+
-609322451431830740.0*(X-0.073795922003002973)^8+
443071707403925570000.0*(X-0.073795922003002973)^9+
27276456959807344.0*(X-0.073795922003002973)^10+
-1314633807785993900 0.0*(X-0.073795922003002973)^11+
-73922949398858467000000000.0*(X-0.073795922003002973)^12+
228088785609802220.0*(X-0.073795922003002973)^13+
12339409252524324000000000.0*(X-0.073795922003002973)^14+
-230539976819978550000000000.0*(X-0.073795922003002973)^15+
-123962875241096120000000.0*(X-0.073795922003002973)^16+
125762656271838180000000000.0*(X-0.073795922003002973)^17+
687097336654666920000.0*(X-0.073795922003002973)^18+
-2862119022455184300000 0.0*(X-0.073795922003002973)^19+
-1614141452456421600.0*(X-0.073795922003002973)^20

ANOVA

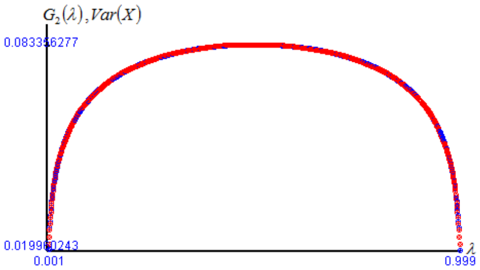
Source	df	SS	MS
Regression	20	0.1398193120	0.0069909656
Error	978	0.0000154000	0.0000000157
Total	998	0.1398347119	

H0:slope1=....=slope20=0, test statistic=443972.489429,
sample size=999, R2=0.999890, R2(adj)=0.999888,MSE=0.000000,

(λ, $Var(X)$) scatter diagram



(λ, R=$G_2(\lambda)$, B=$Var(X)$) scatter diagram



(3) $\gamma_1(X)=G_3(\lambda)$, λ estimated $\gamma_1(X)$,

The $\gamma_1(X)$ estimated equation is $G_3(\lambda)$,

The $0.001 \le \lambda \le 0.999$, $-1.7961485553 \le \gamma_1(X) \le 1.795827056$,

The amount of paired data of (λ, $\gamma_1(X)$) is 999, λ is setting value and $\gamma_1(X)$ is computed by the simulator which has 100,000,000 data.

X=0.984739+-1.969753$\times\lambda$,

The estimated equation ------

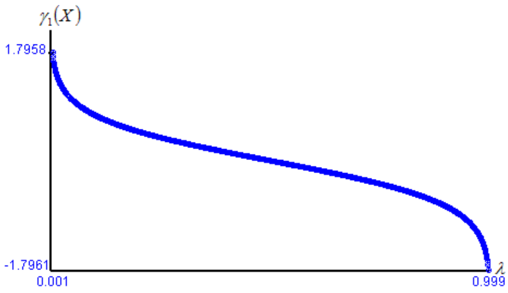
$G_3(\lambda)$=0.000152371816199092792+
0.72288572564741571000*(X--0.000137542062061679 14)^1+
-0.07771367823443142700*(X--0.00013754206206167914)^2+
-1.48555698631025730000*(X--0.00013754206206167914)^3+
3.23668327310588210000*(X--0.00013754206206167914)^4+
44.19691285805311100000*(X--0.00013754206206167914)^5+
-52.74214139766991100000*(X--0.00013754206206167914)^6+
-514.35292186448351000000*(X--0.00013754206206167914)^7+
441.66157603263855000000*(X--0.00013754206206167914)^8+
3275.48317032307390000000*(X--0.00013754206206167914)^9+
-2160.62375265359880000000*(X--0.00013754206206167914)^10+
-12449.11081837862700000000*(X--0.00013754206206167914)^11+
6596.01762938499450000000*(X--0.00013754206206167914)^12+
29480.76403187215300000000*(X--0.00013754206206167914)^13+
-12939.83110857009900000000*(X--0.00013754206206167914)^14+
-43855.79631179571200000000*(X--0.00013754206206167914)^15+
16311.62740564346300000000*(X--0.00013754206206167914)^16+
39823.57315185666100000000*(X--0.00013754206206167914)^17+
-12768.25018835067700000000*(X--0.00013754206206167914)^18+
-20163.34744052588900000000*(X--0.00013754206206167914)^19+
5647.26117467880250000000*(X--0.00013754206206167914)^20+
4361.87453491799530000000*(X--0.00013754206206167914)^21+
-1078.29322034120560000000*(X--0.00013754206206167914)^22

ANOVA

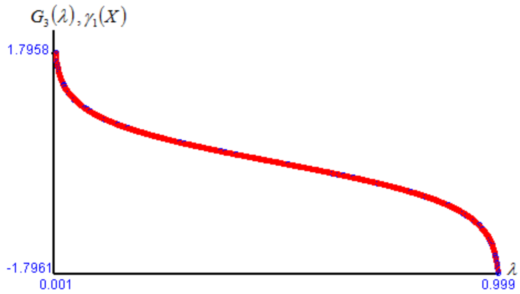
Source	df	SS	MS
Regression	22	340.2086189293	15.4640281332
Error	976	0.0059924144	0.0000061398
Total	998	340.2146113437	

H0:slope1=....=slope22=0, test statistic=2518666.166276,
sample size=999, R2=0.999982, R2(adj)=0.999982,MSE=0.000006,

$(\lambda, \gamma_1(X))$ scatter diagram



$(\lambda$,R=$G_3(\lambda)$,B=$\gamma_1(X))$ scatter diagram



(4) $\gamma_2(X)=G_4(\lambda)$, λ estimated $\gamma_2(X)$,
The $\gamma_2(X)$ estimated equation is $G_4(\lambda)$,
The $0.001 \le \lambda \le 0.999$, $1.799857270 \le \gamma_2(X) \le 7.0808074006$,
The amount of paired data of $(\lambda, \gamma_2(X))$ is 999, λ is setting value and $\gamma_2(X)$ is computed by the simulator which has 100,000,000 data.

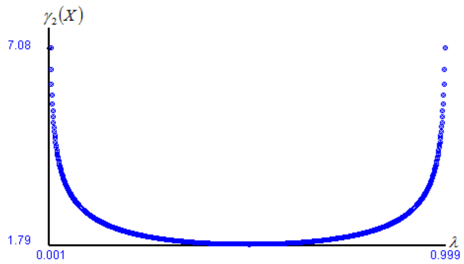
X=2.292589+0.000951 $\times\lambda$,
The estimated equation ------
$G_4(\lambda)$=1.8082038890859193+
9.0944448420777917*(X-2.293064877314313400)^1+
-5649327.2372012138000000*(X-2.293064877314313400)^2+
-2840484322.50*(X-2.293064877314313400)^3+
1454772784505248.00*(X-2.293064877314313400)^4+
282173067709382660.00*(X-2.293064877314313400)^5+
-93623181371148578000000.00*(X-2.293064877314313400)^6+
-12843445897786422000000000.00*(X-2.293064877314313400)^7+
30545377164991993.00*(X-2.293064877314313400)^8+
3212971560766148400.00*(X-2.293064877314313400)^9+
-568216426784795810000000.00*(X-2.293064877314313400)^10+
-48295690587336284000000000.00*(X-2.293064877314313400)^11+
63968562608824166.00*(X-2.293064877314313400)^12+
4544885501268294000.00*(X-2.293064877314313400)^13+
-443419149014227060000000.00*(X-2.293064877314313400)^14+
-26959294213922125000000000.00*(X-2.293064877314313400)^15+
18493181124335300.00*(X-2.293064877314313400)^16+
978467103510877170.00*(X-2.293064877314313400)^17+
-425413014879464930000000.00*(X-2.293064877314313400)^18+
-19833682514142766000000000.00*(X-2.293064877314313400)^19+
4146315834826265700000000000.00*(X-2.293064877314313400)^20+
17195292699711689.00*(X-2.293064877314313400)^21

ANOVA

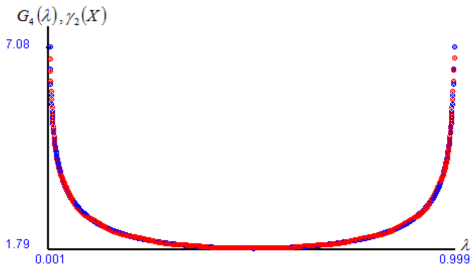
Source	df	SS	MS
Regression	21	553.4887357077	26.3566064623
Error	977	0.4692730413	0.0004803204
Total	998	553.9580087490	

H0:slope1=....=slope21=0, test statistic=54872.967861,
sample size=999, R2=0.999153, R2(adj)=0.999135,MSE=0.000480,

(λ, $\gamma_2(X)$) scatter diagram



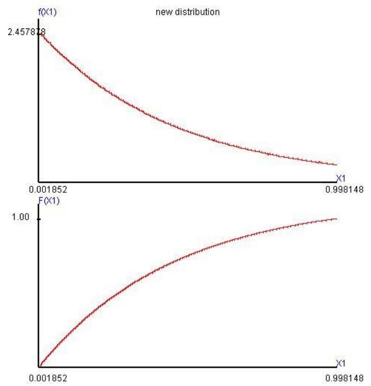
(λ, R=$G_4(\lambda)$, B=$\gamma_2(X)$) scatter diagram



Note: The computer program is C:\C_Bernoulli\C_Bernoulli_02.exe, which can compute the $E(X)$, $Var(X), \gamma_1(X), \gamma_2(X)$ and frequency table when Continuous Bernoulli distribution(λ). The simulated data amount=100,000,000, the sample mean, sample variance, sample skewed coefficient and sample kurtosis coefficient is closed to $E(X)$, $Var(X), \gamma_1(X), \gamma_2(X)$ and the frequency distribution is similar to Continuous Bernoulli distribution (λ).

example 3-1, λ=0.1,

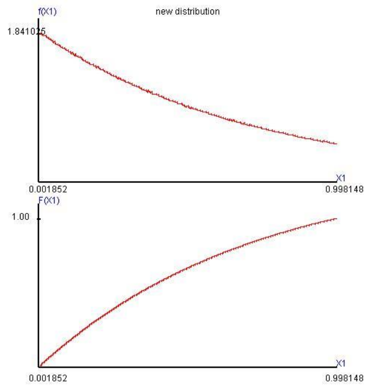
X1 pdf and df



Coefficient		
Mathematical Mean	:	0.33015
Geometrical Mean	:	0.20663
Harmonic Mean	:	0.01882
Variance	:	0.06652
S.D.	:	0.25791
Skewed Coef.	:	0.74382
Kurtosis Coef.	:	2.58122
MAD	:	0.21455
Range	:	1.00000
Mid_range	:	0.50000
Median	:	0.26754
Q1	:	0.11441
Q2	:	0.26754
Q3	:	0.50003
IQR	:	0.38562
C.V.	:	0.78118

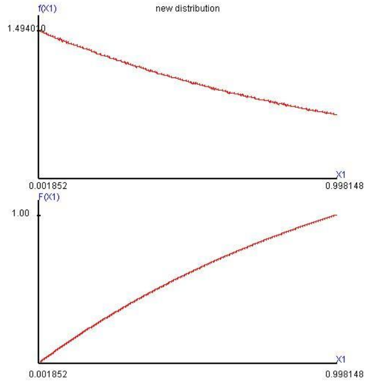
example 3-2, λ=0.2,

X1 pdf and df

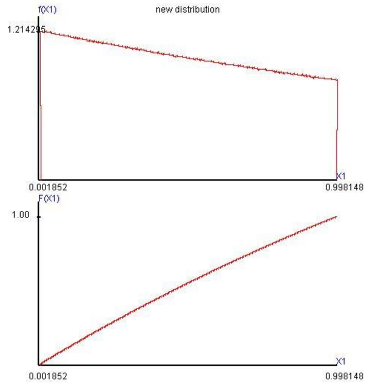


Coefficient		
Mathematical Mean	:	0.38814
Geometrical Mean	:	0.25589
Harmonic Mean	:	0.03197
Variance	:	0.07595
S.D.	:	0.27558
Skewed Coef.	:	0.47578
Kurtosis Coef.	:	2.11516
MAD	:	0.23452
Range	:	1.00000
Mid_range	:	0.50000
Median	:	0.33913
Q1	:	0.14981
Q2	:	0.33913
Q3	:	0.59652
IQR	:	0.44671
C.V.	:	0.71000

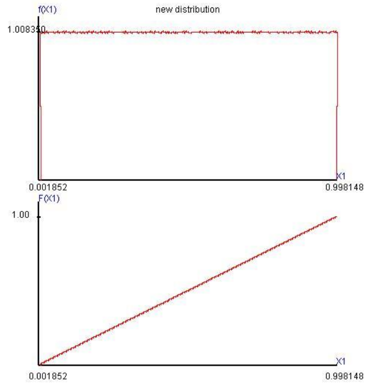
example 3-3, λ =0.3,

X1 pdf and df	Coefficient	
	Mathematical Mean:	0.43033
	Geometrical Mean :	0.29538
	Harmonic Mean :	0.03728
	Variance :	0.08046
	S.D. :	0.28365
	Skewed Coef. :	0.29223
	Kurtosis Coef. :	1.91812
	MAD :	0.24399
	Range :	1.00000
	Mid_range :	0.50000
	Median :	0.39722
	Q1 :	0.18196
	Q2 :	0.39722
	Q3 :	0.66073
	IQR :	0.47877
	C.V. :	0.65914

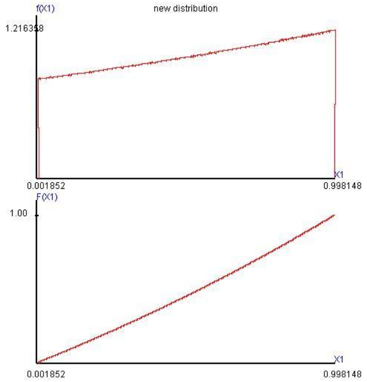
example 3-4, λ =0.4,

X1 pdf and df	Coefficient	
	Mathematical Mean:	0.46633
	Geometrical Mean :	0.33176
	Harmonic Mean :	0.03856
	Variance :	0.08266
	S.D. :	0.28751
	Skewed Coef. :	0.14031
	Kurtosis Coef. :	1.82714
	MAD :	0.24860
	Range :	1.00000
	Mid_range :	0.50000
	Median :	0.44968
	Q1 :	0.21460
	Q2 :	0.44968
	Q3 :	0.70952
	IQR :	0.49492
	C.V. :	0.61654

example 3-5, λ =0.5,此為 Uniform(0,1)。

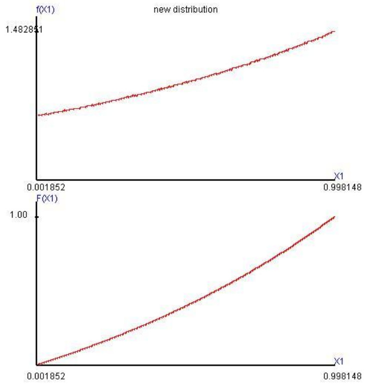
X1 pdf and df	Coefficient	
	Mathematical Mean:	0.50002
	Geometrical Mean :	0.36791
	Harmonic Mean :	0.04653
	Variance :	0.08334
	S.D. :	0.28869
	Skewed Coef. :	-0.00004
	Kurtosis Coef. :	1.79990
	MAD :	0.25002
	Range :	1.00000
	Mid_range :	0.50000
	Median :	0.50002
	Q1 :	0.25001
	Q2 :	0.50002
	Q3 :	0.75001
	IQR :	0.50000
	C.V. :	0.57735

example 3-6, λ =0.6,



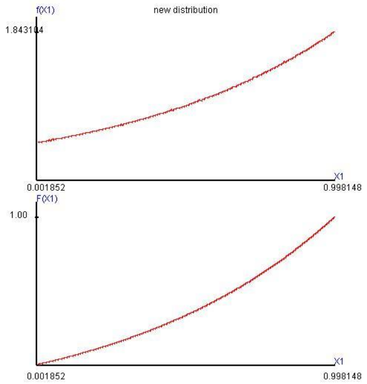
X1 pdf and df	Coefficient	
	Mathematical Mean:	0.53377
	Geometrical Mean :	0.40612
	Harmonic Mean :	0.06289
	Variance :	0.08267
	S.D. :	0.28752
	Skewed Coef. :	-0.14060
	Kurtosis Coef. :	1.82720
	MAD :	0.24861
	Range :	1.00000
	Mid_range :	0.50000
	Median :	0.55043
	Q1 :	0.29050
	Q2 :	0.55043
	Q3 :	0.78554
	IQR :	0.49504
	C.V. :	0.53867

example 3-7, λ =0.7,



X1 pdf and df	Coefficient	
	Mathematical Mean:	0.56986
	Geometrical Mean :	0.44932
	Harmonic Mean :	0.08201
	Variance :	0.08044
	S.D. :	0.28362
	Skewed Coef. :	-0.29288
	Kurtosis Coef. :	1.91890
	MAD :	0.24395
	Range :	1.00000
	Mid_range :	0.50000
	Median :	0.60297
	Q1 :	0.33959
	Q2 :	0.60297
	Q3 :	0.81822
	IQR :	0.47863
	C.V. :	0.49770

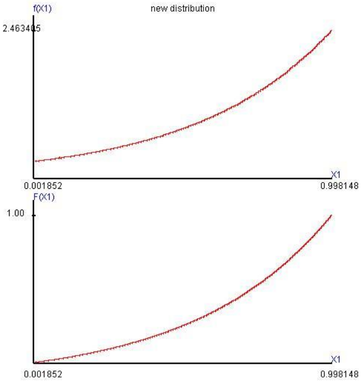
example 3-8, λ =0.8,



X1 pdf and df	Coefficient	
	Mathematical Mean:	0.61200
	Geometrical Mean :	0.50263
	Harmonic Mean :	0.09574
	Variance :	0.07590
	S.D. :	0.27551
	Skewed Coef. :	-0.47608
	Kurtosis Coef. :	2.11563
	MAD :	0.23446
	Range :	1.00000
	Mid_range :	0.50000
	Median :	0.66100
	Q1 :	0.40365
	Q2 :	0.66100
	Q3 :	0.85024
	IQR :	0.44659
	C.V. :	0.45018

example 3-9, λ =0.9,

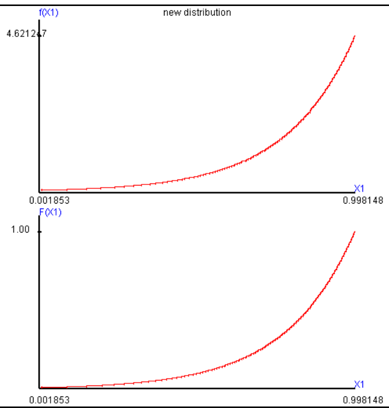
X1 pdf and df



Coefficient	
Mathematical Mean:	0.66987
Geometrical Mean :	0.58009
Harmonic Mean :	0.14364
Variance :	0.06651
S.D. :	0.25790
Skewed Coef. :	-0.74372
Kurtosis Coef. :	2.58089
MAD :	0.21455
Range :	1.00000
Mid_range :	0.50000
Median :	0.73250
Q1 :	0.49996
Q2 :	0.73250
Q3 :	0.88561
IQR :	0.38565
C.V. :	0.38499

example 3-10, λ =0.99,

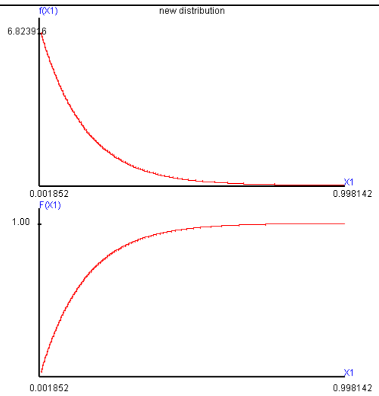
X1 pdf and df



Coefficient	
Mathematical Mean:	0.79258
Geometrical Mean :	0.75294
Harmonic Mean :	0.51282
Variance :	0.03707
S.D. :	0.19253
Skewed Coef. :	-1.41514
Kurtosis Coef. :	4.82773
MAD :	0.14894
Range :	1.00000
Mid_range :	0.50000
Median :	0.85137
Q1 :	0.70480
Q2 :	0.85137
Q3 :	0.93816
IQR :	0.23336
C.V. :	0.24292

example 3-11, λ =0.001,

X1 pdf and df



Coefficient	
Mathematical Mean:	0.14384
Geometrical Mean :	0.08110
Harmonic Mean :	0.00953
Variance :	0.01999
S.D. :	0.14138
Skewed Coef. :	1.79668
Kurtosis Coef. :	7.08231
MAD :	0.10543
Range :	0.99999
Mid_range :	0.50000
Median :	0.10020
Q1 :	0.04161
Q2 :	0.10020
Q3 :	0.20031
IQR :	0.15870
C.V. :	0.98292

Chapter 2, The sufficient statistic of Continuous Bernoulli distribution

The sufficient statistic of parameter is bassis on the parameter point estimator and the test statistic and confidence interval statistic.

$X_1, X_2, ..., X_n \overset{iid}{\sim} CB(\lambda)$, there are n independent random variables and same Continuous Bernoulli distribution (λ).

Section 1, The sufficient statistic of λ ,

(1)The likelihood function of λ ,

$X_1, X_2, ..., X_n \overset{iid}{\sim} CB(\lambda)$,

$$f_{X_1, X_2, ..., X_n}(x_1, x_2, ..., x_n; \lambda) = \prod_{i=1}^{n} f_{X_i}(x_i; \lambda) = (C(\lambda))^n \lambda^{\sum_{i=1}^{n} x_i} (1-\lambda)^{n-\sum_{i=1}^{n} x_i},$$

(2) The sufficient statistic of λ ,

$$f_{X_1, X_2, ..., X_n}(x_1, x_2, ..., x_n; \lambda) = ((1-\lambda)C(\lambda))^n \left(\frac{\lambda}{1-\lambda}\right)^{\sum_{i=1}^{n} x_i},$$

Let $T = \sum_{i=1}^{n} X_i$, $0 < x_n = t - \sum_{i=1}^{n-1} x_i < 1$, $\sum_{i=1}^{n-1} x_i < t$ and $min(0, t-1) < \sum_{i=1}^{n-1} x_i$,

$$f_T(t; \lambda) = \int_0^1 \int_0^1 .... \int_0^1 (C(\lambda))^n \lambda^t (1-\lambda)^{n-t} dx_1 dx_2 ... dx_{n-1},$$

$$f_{X_1, X_2, ..., X_n | T=t}(x_1, x_2, ..., x_n | T = t) = \frac{((1-\lambda)C(\lambda))^n \left(\frac{\lambda}{1-\lambda}\right)^{\sum_{i=1}^{n} x_i}}{\int \int .... \int (C(\lambda))^n \lambda^t (1-\lambda)^{n-t} dx_1 dx_2 ... dx_{n-1}}$$

$$= \frac{1}{\int \int .... \int 1 dx_1 dx_2 ... dx_{n-1}}$$ is independent with λ ,

$\sum_{i=1}^{n} X_i$ is the sufficient statistic of λ ,(Fisher-Neymana factorization theorem).

Section 2, The sampling distribution of $\sum_{i=1}^{n} X_i$ is Continuous Binomial(n, λ),

$X_1, X_2, ..., X_n \overset{iid}{\sim}$ Continuous Bernoulli(λ),

1. The $X = X_1 + X_2 + ... + X_n$ pdf,

(1) n=2,

The probability density function,

$f_{X_1}(x_1; \lambda, n) = C(\lambda)\lambda^{x_1}(1-\lambda)^{1-x_1}, 0 \le x_1 \le 1, 0 < \lambda < 1,$

$f_{X_2}(x_2; \lambda, n) = C(\lambda)\lambda^{x_2}(1-\lambda)^{1-x_2}, 0 \le x_2 \le 1, 0 < \lambda < 1,$

$X_{,1}, X_2$ are independent random variables,

$f_{X_1, X_2}(x_1, x_2; \lambda, n) = f_{X_1}(x_1; \lambda, n) f_{X_2}(x_2; \lambda, n)$

$= (C(\lambda))^2 \lambda^{x_1 + x_2}(1-\lambda)^{2 - x_1 - x_2}, 0 \le x_1 \le, 0 \le x_2 \le 1,$

$f_{X_1, X}(x_1, x; \lambda, n) = f_{X_1, X_2}(x_1, x_2 = x - x_1; \lambda, n),$

$= (C(\lambda))^2 \lambda^{x}(1-\lambda)^{2-x} \times \frac{\partial(x_1, x_2)}{\partial(x_1, x)}, \frac{\partial(x_1, x_2)}{\partial(x_1, x)} = 1,$

$X = X_{,1} + X_2, 0 < x_2 = x - x_1 < 1,$

$max(0, x-1) < x_1 < min(1, x), 0 \le x \le 2,$

$$\begin{cases} 0 < x_1 < x & if\ 0 \le x \le 1, \\ x - 1 < x_1 < 1 & if\ 1 \le x \le 2, \end{cases}$$

$f_X(x; \lambda, n) = \int_{max(0, x-1)}^{min(1, x)} (C(\lambda))^2 \lambda^x (1-\lambda)^{2-x} dx_1$

$$\begin{cases} f_X(x; \lambda, n) = (C(\lambda))^2 \lambda^x (1-\lambda)^{2-x} \int_0^x 1 dx_1 & if\ 0 \le x \le 1, \\ f_X(x; \lambda, n) = (C(\lambda))^2 \lambda^x (1-\lambda)^{2-x} \int_{x-1}^1 1 dx_1 & if\ 1 \le x \le 2, \end{cases}$$

$$f_X(x; \lambda, n) = \begin{cases} (C(\lambda))^2 x \lambda^x (1-\lambda)^{2-x} & if\ 0 \le x \le 1 \\ (C(\lambda))^2 (2-x) \lambda^x (1-\lambda)^{2-x} & if\ 1 \le x < 2 \end{cases}$$

for example, $\lambda = \frac{1}{2}$,

$$f_X(x; \lambda, n) = \begin{cases} x & if\ 0 \le x \le 1 \\ 2 - x & if\ 1 \le x \le 2 \end{cases}$$