17

Quantitative Models for Centralised Supply Chain Coordination

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1. Introduction

A supply chain is defined as a network of facilities and distribution options that perform the functions of procurement of materials, transformation of these materials into intermediate and finished products, and the distribution of these finished products to customers. Managing such functions along the whole chain; that is, from the supplier's supplier to the customer's customer; requires a great deal of coordination among the players in the chain. The effectiveness of coordination in supply chains could be measured in two ways: reduction in total supply chain costs and enhanced coordination services provided to the end customer — and to all players in the supply chain.

Inventory is the highest cost in a supply chain accounting for almost 50% of the total logistics costs. Integrating order quantities models among players in a supply chain is a method of achieving coordination. For coordination to be successful, incentive schemes must be adopted. The literature on supply chain coordination have proposed several incentive schemes for coordination; such as quantity discounts, permissible delay in payments, price discounts, volume discount, common replenishment periods.

The available quantitative models in supply chain coordination consider up to four levels (i.e., tier-1 supplier, tier-2 supplier, manufacturer, and buyer), with the majority of studies investigating a two-level supply chain with varying assumptions (e.g., multiple buyers, stochastic demand, imperfect quality, etc). Coordination decisions in supply chains are either centralized or decentralized decision-making processes. A centralized decision making process assumes a unique decision-maker (a team) managing the whole supply chain with an objective to minimize (maximize) the total supply chain cost (profit), whereas a decentralized decision-making process involves multiple decision-makers who have conflicting objectives.

This chapter will review the literature for quantitative models for centralised supply chain coordination that emphasize inventory management for the period from 1990 to end of 2007. In this chapter, we will classify the models on the basis of incentive schemes, supply chain levels, and assumptions. This chapter will also provide a map indicative of the limitations of the available studies and steer readers to future directions along this line of research.

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2. Centralised supply chain coordination

A typical supply chain consists of multistage business entities where raw materials and components are pushed forward from the supplier's supplier to the customer's customer. During this forward push, value is gradually added at each entity in the supply chain transforming raw materials and components to take their final form as finished products at the customer's end, the buyer. These business entities may be owned by the same organization or by several organizations.

Goyal & Gupta (1989) suggested that coordination could be achieved by integrating lot-sizing models. However, coordinating orders among players in a supply chain might not be possible without trade credit options, where the most common mechanisms are quantity discounts and delay in payments.

There are available reviews in the literature on coordination in supply chains. Thomas & Griffin (1996) review the literature addressing coordinated planning between two or more stages of the supply chain, placing particular emphasis on models that would lend themselves to a total supply chain model. They defined three categories of operational coordination, which are vendor–buyer coordination, production-distribution coordination and inventory-distribution coordination. Thomas & Griffin (1996) reviewed models targeting selection of batch size, choice of transportation mode and choice of production quantity. Maloni & Benton (1997) provided a review of supply chain research from both the qualitative conceptual and analytical operations research perspectives. Recently, Sarmah et al. (2006) reviewed the literature dealing with vendor–buyer coordination models that have used quantity discount as coordination mechanism under deterministic environment and classified the various models. Most recently, Li & Wang (2007) provided a review of coordination mechanisms of supply chain systems in a framework that is based on supply chain decision structure and nature of demand. These studies lacked a survey of mathematical models so the reader may detect the similarities and differences between different models. This chapter does so and updates the literature.

The body of the literature on coordinating order quantities between entities (level) in a supply chain focused on a two-level supply chain for different assumptions. A two-level supply chain could consist of a single vendor and a single buyer, or of a single vendor and multiple buyers. Few works have investigated coordination of orders in a three-level (supplier→vendor→buyer) supply chain, and described by paucity those works that assumed four levels (tier-2 suppliers → tier-1 suppliers → vendor → buyer) or more.This chapter will classify the models by the number of levels, and therefore, there are three main sections. Section 3 reviews two-level supply chain models. Three-level models are discussed in section 4. Models with four or more levels are discussed in section 5.

3. Two-level supply chain models

The economic order quantity (EOQ) model has been the corner stone for almost all the available models in the literature. In a two-level chain, with coordination, the vendor (e.g., manufacturer, supplier) and the buyer optimize their joint costs.

The basics

Consider a vendor (manufacturer) and a buyer who each wishes to minimize its total cost. A basic model assumes the following: (1) instantaneous replenishment, (2) uniform and

constant demand, (3) single non-perishable product of perfect quality, (5) zero lead time, and (6) infinite planning horizon.
The buyer's unit time cost function is given as

$$TC_b(Q) = \frac{A_b D}{Q} + h_b \frac{Q}{2} \tag{1}$$

The optimal order quantity that minimizes (1) is $Q^* = \sqrt{2A_b D/h_b}$, where A_b is the buyer's order cost , h_b is the buyer's holding cost per unit per unit time, and D is the demand rate per unit time and assumed to be constant and uniform over time. Substituting Q^* in (1), then (1) reduces to $TC_b^* = \sqrt{2A_b D h_b}$. The vendor's unit time cost function is given as

$$TC_v(\lambda) = \frac{A_v D}{\lambda Q} + h_v \frac{Q}{2}(\lambda - 1) \tag{2}$$

Where A_v is the vendor's order (setup) cost, h_v is the vendor's holding cost per unit per unit time, and λ being the vendor lot-size multiplier (positive integer) of the buyer's order quantity Q.

From the buyer's perspective

If the buyer is the supply chain leader, then it orders Q^* every $T^* = Q^*/D$ units of time. Accordingly, the vendor treats Q^* as an input parameter and finds the optimal λ that minimizes its unit time cost, where $TC_v(\lambda^* - 1) > TC_v(\lambda^*) < TC_v(\lambda^* + 1)$. For this case, the vendor is the disadvantaged player. An approximate closed form expression is possible by assuming (2) to be differentiable over λ, then the optimal value of λ is given as

$$\lambda^* = \frac{1}{Q^*}\sqrt{\frac{2A_v D}{h_v}} = \sqrt{\frac{2A_v D}{h_v} \times \frac{h_b}{2A_b D}} = \sqrt{\frac{A_v h_b}{A_b h_v}} \tag{3}$$

For example, if the λ = 2.58, then λ^* =2 if $TC_v(\lambda^* = 2) < TC_v(\lambda^* + 1 = 3)$; otherwise, λ^* =3. The vendor may find the lot-for-lot (λ^* = 1) policy to be optimal if

$$\frac{A_v D}{Q} < \frac{A_v D}{\lambda Q} + h_v \frac{Q}{2}(\lambda - 1) \Rightarrow \frac{A_v D}{Q}\left(\frac{\lambda - 1}{\lambda}\right) < h_v \frac{Q}{2}(\lambda - 1) \Rightarrow \frac{A_v D}{Q\lambda} < h_v \frac{Q}{2} \Rightarrow \lambda > \frac{2A_v D}{h_v Q^2}.$$

From the vendor's perspective

The buyer's EOQ may not be optimal to the vendor. From a vendor's perspective, the optimal order quantity is given from differentiating (2) over Q and solving for Q to get

$$Q^{**} = \sqrt{\frac{2A_v D}{h_v \lambda(\lambda - 1)}}, \text{ where } \lambda > 1 \tag{4}$$

Then the optimal value of (2) as a function of $\lambda > 1$ is given as

$$TC_v^*(\lambda) = \sqrt{\frac{2A_vDh_v(\lambda-1)}{\lambda}} \tag{5}$$

The optimal cost occurs when $TC_v^*(\lambda^{**}-1)$ > $TC_v^*(\lambda^{**})$ < $TC_v^*(\lambda^{**}+1)$. For this case, the buyer is the disadvantaged player. The ideal case would occur when the EOQ of the buyer matches that of the vendor, i.e., $Q^* = Q^{**}$, where

$$\frac{A_v}{h_v\lambda(\lambda-1)} = \frac{A_b}{h_b} \Rightarrow \frac{A_vh_b}{A_bh_v} = \lambda(\lambda-1) \Rightarrow \lambda^2 - \lambda - \frac{A_vh_b}{A_bh_v} = 0 \Rightarrow \lambda^* = \frac{1+\sqrt{1+4A_vh_b/A_bh_v}}{2} \geq 2$$

$$\Rightarrow \sqrt{1+4A_vh_b/A_bh_v} \geq 3 \Rightarrow A_vh_b/A_bh_v \geq 2$$

Vendor-buyer coordination

In many cases, there is a mismatch between the quantity ordered by the buyer and the one that the vendor desires to sell to the buyer. A joint replenishment policy would be obtained by minimizing the joint supply chain cost which is given as

$$TC_{sc}(Q,\lambda) = TC_b(Q) + TC_v(\lambda) = \frac{A_bD}{Q} + h_b\frac{Q}{2} + \frac{A_vD}{\lambda Q} + h_v\frac{Q}{2}(\lambda-1) \tag{6}$$

Goyal (1977) is believed to be the first to develop a joint vendor-buyer cost function as the one described in (6). Differentiating (6) over Q and solving for Q to get

$$Q(\lambda) = \sqrt{\frac{2D(\lambda A_b + A_v)}{h_b\lambda + h_v\lambda(\lambda-1)}} \tag{7}$$

The order quantity in (7) is larger than the buyers EOQ for every $\lambda \geq 1$, which means higher cost to the buyer. This can be shown by setting $Q(\lambda)$ > Q^* to get $(\lambda A_b + A_v)/[h_b\lambda + h_v\lambda(\lambda-1)]$ > A_b/h_b. Some researchers added a third cost component to the cost function in (6). For example, Woo et al. (2000) studied the tradeoff between the expenditure needed to reduce the order processing time and the operating costs identified in Hill (1997), by examining the effects of investment in EDI on integrated vendor and buyer inventory systems. Another example is the work of Yang & Wee (2003) who incorporated a negotiation factor to balance the cost saving between the vendor and the buyer.

To make coordination possible, the vendor must compensate the buyer for its loss. This compensation may take the form of unit discounts and is computed as

$$d = \frac{TC_b(Q(\lambda)) - TC(Q^*)}{D}$$

$$= A_b\sqrt{\frac{h_b\lambda + h_v\lambda(\lambda-1)}{2D(\lambda A_b + A_v)}} + h_b\sqrt{\frac{(\lambda A_b + A_v)}{2D[h_b\lambda + h_v\lambda(\lambda-1)]}} - \sqrt{\frac{2h_bA_b}{D}} \tag{8}$$

Crowther (1964) is believed to be the first who focused on quantity discounts from the buyer-seller perspective. For a good understanding of the precise role of quantity discounts

and their design, readers may refer to the works of Dolan (1987) and Munson & Rosenblatt (1998).
Recently, Zhou & Wang (2007) developed a general production-inventory model for a single-vendor–single-buyer integrated system. Their model neither requires the buyer's unit holding cost be greater than the vendor's nor assumes the structure of shipment policy. Zhou & Wang (2007) extended their general model to consider shortages occurring only at the buyer's end. Following, their production-inventory model was extended to account for deteriorating items. Zhou & Wang (2007) identified three significant insights. First, no matter whether the buyer's unit holding cost is greater than the vendor's or not, they claimed that their always performs best in reducing the average total cost as compared to the existing models. Second, when the buyer's unit holding cost is less than that of the vendor's, the optimal shipment policy for the integrated system will only comprise of shipments increasing by a fixed factor for each successive shipment. Very recently, Sarmah et al. (2007) considered a coordination problem which involves a vendor (manufacturer) and a buyer where the target profits of both parties are known to each other. Considering a credit policy as a coordination mechanism between the two parties, the problem's objective was to divide the surplus equitably between the two parties.
In the following sections, we survey the studies that extended upon the basic vendor-buyer coordination problem (two-level supply chain) by relaxing some of its assumptions. The following sections are: (1) finite production rate, (2) non-uniform demand,(3) permissible delay in payments, (4) multiple buyers, (5) multiple Items, (6) product/process quality, (7) deterioration, (8) entropy cost and (9) stochastic models.

Finite production rate

Banerjee (1986) assumed finite production rate rather than instantaneous replenishment. He also assumed a lot-for-lot ($\lambda = 1$) policy. Banerjee's cost function which is a modified form of (6) is given as

$$TC_{sc}(Q) = \frac{A_b D}{Q} + h_b \frac{Q}{2} + \frac{A_v D}{Q} + h_v \frac{D}{P} Q \tag{9}$$

Where $h_b = Ic_b$ and $h_v = Ic_v$ in which c_v is the vendor's unit purchase (production) cost, c_b is the buyer's unit purchase cost, I is the carrying cost dollar per dollar, and P is the manufacturer production rate (P>D). The optimal order quantity that minimizes (9) is given as

$$Q^* = \sqrt{\frac{2D(A_b + A_v)}{h_b + h_v \frac{D}{P}}} \tag{10}$$

Goyal (1988) extended the work of Banerjee (1986) by relaxing the assumption of lot-for-lot policy. He suggested that (9) should be written as

$$TC_{sc}(Q) = \frac{A_b D}{Q} + (h_b - h_v)\frac{Q}{2} + \frac{A_v D}{\lambda Q} + \frac{\lambda h_v Q}{2}\left(1 + \frac{D}{P}\right) \tag{11}$$

The optimal order quantity that minimizes (11) is given as

$$Q(\lambda)=\sqrt{\frac{2D\left(A_b+\frac{A_v}{\lambda}\right)}{h_b-h_v+\lambda h_v\left(1+\frac{D}{P}\right)}} \tag{12}$$

Joglekar & Tharthare (1990) presented the refined JELS model which relaxes the lot-for-lot assumption, and separates the traditional setup cost into two independent costs. They proposed a new approach to the problem which they claim will require minimal co-ordination between the vendor and purchasers. They believed this approach, known as the individually responsible and rational decision (IRRD) approach allows the vendor and the purchasers to carry out their individually rational decisions. Very recently, Ben-Daya et al. (2008) provided a comprehensive and up-to-date review of the JELS that also provides some extensions of this important problem. In particular, a detailed mathematical description of, and a unified framework for, the main JELP models was provided.

Wu & Ouyang (2003) determined the optimal replenishment policy for the integrated single-vendor single-buyer inventory system with shortage algebraically. This approach was developed by Grubbström & Erdem (1999) who showed that the formula for the EOQ with backlogging could be derived algebraically without reference to derivatives. Wu & Ouyang's (2003) integrated vendor–buyer total cost per year is given by

$$TC_{sc}=\frac{A_bD}{Q}+h_b\frac{(Q-B)^2}{2Q}+\pi_b\frac{B^2}{2Q}+\frac{A_vD}{\lambda Q}+h_v\frac{Q}{2}\left[\lambda\left(1-\frac{D}{P}\right)+\frac{2D}{P}-1\right]$$

Where B is the maximum shortage level for the buyer. The optimal solutions of Q and B are given as

$$Q(\lambda)=\sqrt{\frac{2D(\pi_b+h_b)\left(A_b+\frac{A_v}{\lambda}\right)}{h_b\pi_b+h_v(\pi_b+h_b)\left[\lambda\left(1-\frac{D}{P}\right)+2\frac{D}{P}-1\right]}}$$

$$B(\lambda)=\frac{h_b}{\pi_b+h_b}Q(\lambda)$$

Where π_b is the annual buyer's shortage cost per unit.

Ertogral et al. (2007) develop two new models that integrate the transportation cost explicitly in the single vendor single-buyer problem. The transportation cost was considered to be in an all-unit-discount format for the first model. Their supply chain cost function was of the form

$$TC_{sc}=\frac{(A_v+\lambda A_b)D}{\lambda q}+h_v\left[q\frac{D}{P}+\frac{\lambda q}{2}\left(1-\frac{D}{P}\right)\right]+(h_b-h_v)\frac{q}{2}+C_T$$

Where $C_T=c_{iD}$ is the transportation cost per unit of time and C_T is a step-form function, where $q\in[M_i,M_{i+1})$, i=0,1,2…,λ, and $M_0=0$, and q is the shipment lot size.

<u>*Non-uniform demand*</u>

Li et al. (1995) considered the case where the buyer is in monopolistic position with respect to the vendor. They assumed the demand, $D=\alpha_b p_b^{-\beta}$, by the buyer's customers is a decreasing function of the buyer's price p_b, where $\alpha_b > 0$ and $0 < \beta < 1$ that could be determined by some statistical technique from historical data. Li et al. (1995) assumed $p_b = kp$ where p is the buyer purchase price and $k > 0$, and rewriting the demand function as $D=\alpha p^{-\beta}$ where $\alpha = \alpha k^{-\beta}$. When the vendor and the buyer achieve full cooperation, the supply chain's total cost function is given

$$TC_{sc}(p,Q)=\alpha(1-G)p^{1-\beta}+\alpha(A_v+A_b)\frac{p^{-\beta}}{Q}+\frac{h_b}{2}pQ$$

Where G is the vendor's gross profit on sales. The above cost function was minimized subject to $\alpha p^{1-\beta}+\alpha A_b\, p^{-\beta}/Q + h_b pQ/2 \le C_0$, $p > 0$, and $Q > 0$, where C_0 is the maximum available annual investment. Then the equilibrium point of the co-operative game is

$$p^* = \left\{\frac{\eta^*}{\alpha G}\left(C_0 - \frac{\eta^* h_b A_b}{2\left(G-\eta^*\right)}\right)\right\}^{1/(1-\beta)}$$

$$Q^* = \frac{2\alpha\left(G-\eta^*\right)}{\eta^* h_b}\left(p^*\right)^{-\beta}$$

Where $\eta^* = \dfrac{G(h_b A_b + 2C_0) - \sqrt{G(h_b A_b + 2C_0)h_b(GA_b + A_v)}}{h_b A_b + 2C_0}$

Boyaci & Gallego (2002) analyzed coordination issues in a supply chain consisting of one vendor (wholesaler) and one or more buyers (retailers) under deterministic price-sensitive customer demand. They defined the total channel profits as

$$\Pi(w,\lambda,p,Q) = (p-c_v)D(p) - \left(\frac{A_v}{\lambda} + a_v + A_b\right)\frac{D(p)}{Q}$$

$$-\frac{1}{2}\left\{(I_v c_v + \theta_v)\lambda + \theta_b - \theta_v - |I_b - I_v| w\right\}Q$$

Where a_v is the vendor's fixed cost of processing a buyer's order, θ_v (θ_b) is the vendor's (buyer's) opportunity cost of the space required to store one unit of the product for one year, c_v is the vendor's unit ordering cost, and assumed to be known and constant, w is a decision variable selected by the wholesaler, $D(p)$ is the demand rate seen by the buyer when the Buyer (retailer) price is p, and I_v (I_b) the vendor's (buyer's) opportunity cost of capital per dollar per year. They investigated their model for the cases of inventory ownership ($I_v > I_b$ or $I_v < I_b$), equal ownership ($I_v = I_b$), and an arbitrage opportunity to make infinite profits ($I_v \neq I_b$).

Permissible delay in payments

Besides quantity discounts, permissible delay in payments is a common mechanism of trade credit that facilitates coordinating orders among players in a supply chain.

Jamal et al. (2000) assumed that the buyer can pay the vendor either at time some time *M* to avoid the interest payment or afterwards with interest on the unpaid balance due at *M*. Typically, the buyer may not pay fully the wholesaler by time *M* for lack of cash. On the other hand, his cost will be higher the longer the buyer waits beyond *M*. Therefore, the buyer will gradually pay the wholesaler until the payment is complete. Since the selling price is higher than the unit cost, and interest earned during the credit period *M* may also be used to payoff the vendor, the payment will be complete at time *P* before the end of each cycle *T* (i.e., $M \le P \le T$). Jamal et al. (2000) modelled the vendor-buyer system as a cost minimization problem to determine the optimal payment time P^* under various system parameters.

$$TC_{sc}(P,T) = \frac{A_v + A_b}{T} + \frac{cD}{\theta^2 T}\left(e^{\theta T} - 1\right)(\theta + I) - cD - \frac{IcD}{\theta} - \frac{cI_pD}{\theta^2 T}\left(e^{\theta(T-P)} - e^{\theta(T-M)}\right)$$
$$- \frac{cI_pD}{\theta T}(P-M) - I_pD(s-c)\left(P^2 - M^2\right)/2T$$
$$- I_p s I_e D M^2 (P-M)/2T - sI_eD\left(M^2 + (T-P)^2\right)/2T$$

Where I_e is the interest earned per dollar per unit time, I_p the interest paid per dollar per unit time dollars/dollar-year, *I* is the inventory carrying cost rate, *c* is the unit cost, *s* is the unit selling price, and θ is the deterioration rate, a fraction of the on-hand inventory. No closed form solution was developed, and an iterative search approach is employed simultaneously to obtain solutions for *P* and *T*. Recently, Yang & Wee (2006a) proposed a collaborative inventory model for deteriorating items with permissible delay in payment with finite replenishment rate and price-sensitive demand. A negotiation factor is incorporated to balance the extra profit sharing between the two players.

Abad & Jaggi (2003) considered a vendor–buyer channel in which the end demand is price sensitive and the seller may offer trade credit to the buyer. The unit price seller charged by the seller and the length of the credit period offered by the vendor to the buyer both influence the final demand for the product. The paper provides procedures for determining the vendor's and the buyer's policies under non-cooperative as well as cooperative relationships. Here, we present the model for the cooperative case. Abad & Jaggi (2003) used Pareto efficient solutions that can be characterized by maximizing (Friedman, 1986)

$$Z = \mu\left[Kp^{-e}\left(c_b - c_v - I_v c_b M - \frac{A_v}{Q}\right)\right] + (1-\mu)\left[Kp^{-e}\left(p - c_b(1 - I_c M) - \frac{A_b}{Q}\right) - \frac{Ic_bQ}{2}\right]$$

Where $D(p) = Kp^{-e}$ is annual demand rate as a function of the buyer's price, *e* the index of price elasticity, *M* is the credit period (vendor's decision variable), c_b the price charged by the vendor to the buyer, c_v is the seller's unit purchase cost, I_{cb} vendor's opportunity cost of capital, I_c short-term capital cost for the buyer, I_b inventory carrying charge per year excluding the cost of financing inventory, and $I = I_c + I_b$. The first order necessary condition for maximizing Z with respect to c_b yields

$$0 \le \mu = \frac{1 - I_c M + IQ/\left(2Kp^{-e}\right)}{2 - I_v M - I_c M + IQ/\left(2Kp^{-e}\right)} \le 1$$

First order conditions with respect to Q and M yield

$$Q = \sqrt{\frac{2Kp^{-e}\left[\mu A_v + (1-\mu)A_b\right]}{(1-\mu)Ic_b}}$$

$$M = \frac{(1-\mu)I_c - a\mu}{2b\mu}$$

where $I_v = a + bM$, $a > 0$, $b > 0$. Abad & Jaggi (2003) cautioned that not all μ in the interval [0,1] may yield feasible solutions.

Jaber & Osman (2006) proposed a centralized model where players in a two-level (vendor–buyer) supply chain coordinate their orders to minimize their local costs and that of the chain. In the proposed supply chain model the permissible delay in payments is considered as a decision variable and it is adopted as a trade credit scenario to coordinate the order quantity between the two-levels. They presented the buyer and vendor unit time cost functions respectively as

$$TC_b(Q,t,\tau) = \frac{A_b D}{Q} + c_b D + \frac{D}{Q} H_b(Q,t,\tau) + \frac{s_b}{2} Q + c_b D(e^{k_v(\tau - t)} - e^{k_b \tau})$$

where $H_r(Q,t,\tau) = h_b(Q - Dt)^2 / 2D$ (Case I), or $h_b(Q - D\tau)^2 / 2D$ (Case II), or 0 (Case III). It should be clarified that the retailer must settle his/her balance, $c_b Q$, with the supplier either by time t or by time τ, which are respectively the *interest-free* and the interest permissible delay in payment periods, where $0 \le H_b(Q,t,\tau) \le h_b Q^2 / 2D$

$$TC_v(Q,\lambda,t,\tau) = \frac{A_v D}{\lambda Q} + \frac{h_v + s_v}{2} Q(\lambda - 1) + h_v \tau D + (c_b - c_v) D e^{k_v t} - c_b D e^{k_v(\tau - t)} + c_v D$$

Define t as the permissible delay in payment in time units, (interest-free period), and τ is the buyer's time to settle its account with the vendor. If $\tau > t$ the supplier charges interest for the period of $\tau - t$ (interest period). The other parameters are defined as follows (where i = v, b): k_i , the return on investment, h_i is holding cost per unit of time, representing the cost of capital excluding the storage cost, s_i the storage cost per unit of time at level i excluding the holding cost, and c_i = Procurement unit cost for level $i = v, b$. With coordination, the buyer and the vendor need to agree on the following decision variables Q, λ, t, and τ, that minimizes the total supply chain cost by solving the following mathematical programming model

Minimize $\quad TC_{sc}(Q,\lambda,t,\tau) = TC_v(Q,\lambda,t,\tau) + TC_b(Q,t,\tau)$

Subject to:

$\tau - t \geq 0$

$\lambda \geq 1$

$Q/D - t \geq 0$ (Case I), $Q/D - \tau \geq 0$ (Case II), $\tau - Q/D \geq 0$ (Case III)

$t \geq 0$, $\tau \geq 0$, λ=1, 2, 3,.., and $Q \geq 1$

Jaber & Osman (2006) assumed profits (savings) from coordination to be shared between the buyer and the vendor in accordance with some prearranged agreement.

Chen & Kang (2007) considered a similar model to that of Jaber & Osman (2006), where they investigated their model for predetermined and extended periods of delay in payments. However, and unlike the work of Jaber & Osman (2006), Chen & Kang (2007) have not treated the length of delay in payment as a decision variable. Sheen & Tsao (2007) consider vendor-buyer channels subject to trade credit and quantity discounts for freight cost. Their work determined the vendor's credit period, the buyer's retail price and order quantity while still maximizing profits. Sheen & Tsao (2007) focused on how channel coordination can be achieved using trade credit and how trade credit can be affected by quantity discounts for freight cost. Like Chen & Kang (2007), they set an upper and lower bounds on the length of the permissible delay in payments. They search for the optimal length of this credit from the vendor's perspective and not from that of the supply chain coordination.

Multiple buyers

Affisco et al. (1993) provided a comparative analysis of two sets of alternative joint lot-sizing models for the general one-vendor, many-nonidentical buyers' case. Specifically, the basic joint economic lot size (JELS) and individually responsible and rational decision (IRDD) models, and the simultaneous setup cost and order cost reduction versions are explored. The authors considered co-operation is required of the parties regardless of which model they choose to implement, it is worthwhile to investigate the possible impact of such efforts on the model. The joint total relevant cost on all buyers and the vendor is given by

$$TC_{sc} = \sum_{i=1}^{n} \left\{ \left(\frac{D_i}{Q_i} \right) \left(A_{b,i} + \alpha \right) + h_{b,i} \frac{Q_i}{2} \right\} + h_v \frac{Q_v}{2} \left(1 - \frac{D}{P} \right) + A_v \frac{D}{Q_v}$$

Where α is the vendor's cost of handling and processing an order from a purchaser. This included such costs as inspection, packing and shipping of an order, and the cost of any related paperwork, but not the cost of manufacturing setup to produce a production quantity. The refined JELS model results from minimizing TC which yields the following relationships for the vendor's and ith buyer's joint optimal lot sizes are $Q_v^* = \sqrt{2DA_v / \left(h_v (1 - D/P) \right)}$, and $Q_i^* = \sqrt{2D_i (A_{b,i} + \alpha)/h_{b,i}}$ respectively, where $D = \sum_{i=1}^{n} D_i$.

Under the IRRD model, since a purchaser must pay for the vendor's handling costs every time it orders $O_i = \left(D_i / Q_i \right) \left(A_{b,i} + \alpha \right)$.The holding cost per unit per unit time is also reduced due to the transferred handling costs.

Lu (1995) considered an integrated inventory model with a vendor and multiple buyers. Lu assumed the case where the vendor minimizes its total annual cost subject to the maximum cost that the buyer may be prepared to incur. They presented a mixed integer programming problem of the form

$$\text{Minimize} \quad TC_{sc}(T, k_i \mid i = 1,\ldots,n) = \frac{1}{T}\left(A_v + \sum_{i=1}^{n} \frac{A_{b,i}}{\max\{1, k_i\}}\right)$$

$$+\frac{T}{2}\sum_{i=1}^{n} \max\{1, k_i\} h_{b,i} D_i \left(1 + \min\{1, k_i\} - \frac{D_i}{P_i} - \frac{2m_i}{k_i}\right)$$

Subject to

$$T \ge 0,$$

$$\frac{1}{2}\left(\frac{T_i^*}{k_i T} + \frac{k_i T}{T_i^*}\right) \le B_i$$

$$k_i \in \{1,2,3,\ldots\} \cup \left\{\frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \ldots\right\}$$

$$m_i = \lfloor k_i (1 - D_i / P_i) \rfloor, \; i = 1,2,\ldots,n$$

Where $T_i^* = \sqrt{2A_{b,i}/(h_{b,i}D_i)}$, $A_{b,i}$, $h_{b,i}$, and D_i are respectively the optimal cycle time, order cost, holding cost, and demand for buyer *i*. *T* is the order interval suggested by the vendor and $B_i > 1$ is some threshold value. Lu (1995) considered a quantity discount schedules to maximize the vendor's total profit subject to the maximum cost that the buyer may be prepared to incur. Yao & Chiou (2004) proposed an efficient heuristic which solves Lu's model by exploring its optimality structure. They observed that the vendor's optimal annual total cost function is a piece-wise convex curve with respect to the vendor's production setup interval. Yao & Chiou (2004) proposed an effective heuristic that out-performs Lu's heuristic.

Goyal (1995) commented on the work of Lu (1995) and suggested a joint inventory cost function of the form

$$TC_{sc}(k) = \frac{D(A_v + kA_b)(n-1)}{q_1(n^k - 1)} + \frac{q_1}{2}\left(h_b + \frac{h_v}{k}\right)\frac{n^k + 1}{n+1}$$

Where *k* is the number of shipments in which the entire lot of size $Q = q_1(n^k - 1)/(n-1)$ is transported by the vendor to the buyer in shipments of size q_i, where *i* = 1, 2, …, *k*. Assuming that the ratio between the $(i+1)$-*st* shipment and the *i*-th shipment is equal to *n*. For a particular value of *k*, the economic value of $q_1 = q(k)$ and the minimum joint total annual costs are given respectively as

$$q(k) = \sqrt{\frac{2D(A_v + kA_b)(n^2 - 1)}{(n^{2k} - 1)\left(h_b + \dfrac{h_v}{2}\right)}}$$

$$TC_{sc}(q(k)) = \sqrt{\frac{2D(h_b + h_v/2)(n-1)(n^k+1)(A_v + kA_b)}{(n+1)(n^k-1)}}$$

The works of Lu (1995) and Goyal (1995) are further analyzed in Hill (1997) and Viswanathan (1998).

Chen et al. (2001) proposed a coordination model for a centralized two-echelon system whose profit function is given as

$$\Pi = \sum_{i=1}^{n}\left[(p_i(D_i) - c_v - c_{b,i})D_i - \Psi(D_i) - \frac{A_{b,i}}{T_i} - \frac{1}{2}h_v D_i \max\{T_v, T_i\} - \frac{1}{2}h_{b,i}D_i T_i\right] - \frac{A_v}{T_v}$$

Where p_i retail price charged by buyer i, $p_i(D_i)$ annual demand a decreasing function of the retail price, $c_{b,i}$ unit shipping cost to from the vendor to the buyer $\Psi(D_i)$ is the annual cost incurred by the vendor for managing buyer i's account with $\Psi(\cdot)$ being a nondecreasing and concave where $\Psi(0)=0$, T_i is the replenishment interval for buyer i, and T_v is the replenishment interval for the vendor.

Viswanathan & Piplani (2001) proposed a supply chain model of coordinating supply chain inventories through the use of common replenishment epochs (CRE) or time periods. They considered a vendor and multiple buyers with a single product. With the CRE strategy, the vendor specifies that the buyers can only place orders at specific points in time. The vendor was assumed to insist that the replenishment interval for each buyers i T_i^* should be an integer multiple of the common replenishment period $T = \lambda_i T_i^*$, where λ_i is a positive integer. With the specification of the CRE, the buyers' flexibility is reduced and inventory costs increased. The vendor will need to provide a price discount Z_i to compensate buyer i for inventory cost increase. The problem of determining the T and Z for the vendor can then be formulated as follows

$$\text{Minimize } TC_{sc} = \frac{A_v}{T} + \sum_{i=1}^{n}\left(D_i Z + \frac{a_i}{\lambda_i T}\right)$$

Subject to:

$$D_i Z \geq \frac{A_{b,i}}{\lambda_i T} + h_{b,i}\lambda_i T - 2(1-S)\sqrt{A_{b,i}h_{b,i}}, \; i = 1, \ldots, n$$

$$T \in \mathbf{X}$$

$$\lambda_i \geq 1 \text{ and integer, } i = 1, \ldots, n$$

Where $\mathbf{X} = \{1/365, 1/52, 2/52, 1/12, 2/12, 1/4\}$, a_i is the cost of processing the order of buyer i, S being the percentage savings, and $D_i Z$ is the total dollar discount offered to buyer i. Further investigation of the work of Viswanathan & Piplani (2001) is provided in Piplani & Viswanathan (2004).

Woo et al. (2001) extended upon the work of Woo et al. (2000) to account for the case of multiple buyers. They assumed that vendor and all buyers are willing to invest in reducing the ordering cost (e.g., establishing an electronic data interchange based inventory control system) in order to decrease their joint total cost. Woo et al. (2001) stressed that a major managerial implication from this ordering cost reduction is that the efforts to streamline and speedup transactions via the application of information technologies may result in a higher degree of coordination and automation among allied trading parties. Woo et al. (2001) also assume that shortages are not allowed for the vendor and that the information of buyers' replenishment decision parameters is available to the vendor. The joint total cost for the vendor and all the buyers per unit time is

$$TC_{sc} = K + \frac{1}{T}\left[\frac{A_v}{\lambda} + S_v + \sum_{i=1}^{n} T_i(K)\right] + \frac{T}{2}\left[uh_{v,m}\sum_{i=1}^{n} D_i\left(\lambda - 1 + \frac{\sum_{i=1}^{n} D_i}{P}\right)\right]$$

$$+\frac{h_{v,p}}{P}\sum_{i=1}^{n} D_i^2 + \sum_{i=1}^{n} h_{b,i}(1-f_i)^2 D_i + \sum_{i=1}^{n} L_i f_i^2 D_i$$

Where K is expenditure per unit time to operate the planned ordering system between vendor and all buyers, which is a decision variable, and $T_i(K)$ is the planned ordering cost per buyer i's order, which is a strictly decreasing function of K with $T_i(0) = T_{0,i}$ and $T_i(K_0) = 0$, T is the common cycle time for buyers, which is a decision variable, u is usage rate of raw materials for producing each finished item, $h_{v,m}$ and $h_{v,p}$ are respectively the vendor's carrying cost per unit of raw materials and finished products, $h_{b,i}$ is the carrying cost per unit held per unit time for buyer i, f_i is the fraction of backlogging time in a cycle for buyer i, which is a decision variable, and L_i is the backlogging cost per unit backlogged per unit time for buyer i. Note that this paper assumes the vendor incurs ordering cost for raw material A_v and a setup cost per production run for vendor S_v.

Recently, Yu et al. (2006) improved upon the work of Woo et al. (2001) by providing a lower or equal joint total cost as compared to the relaxation of their integral multiple material ordering cycle policy to a fractional-integral multiple material ordering cycle policy. More recently, Zhang et al. (2007) extended the work of Woo et al. (2001) by relaxing the assumption of a common cycle time for all buyers and the vendor.

Siajadi et al. (2006a,b) presented a methodology to obtain the Joint Economic Lot size in the case where multiple buyers are demanding one type of item from a single vendor. The shipment policy is found and a new model is proposed to minimize the joint total relevant cost (JTRC) for both vendor and buyer(s). Further it is shown that a multiple shipment policy is more beneficial than a single shipment policy considered by Banerjee (1986). The incurred saving is increasing as the total demand rate approaches the production rate. This means that as long as the first assumption is still satisfied, the better the production capacity is utilized, the greater the saving will be. Conversely, when the dominating cost is the transportation cost, the saving is decreasing as the numbers of shipment approach to one.

Consequently, the new model becomes identical with the traditional model, as the numbers of shipment are equal to one.
Yang & Wee (2006b) considered a pricing policy for a two-level supply chain with a vendor and multiple buyers. Three scenarios are discussed. The first scenario neglects integration and quantity discount. The second scenario considers the integration of all players without considering quantity discount. The last scenario considers the integration and the quantity discount of all players simultaneously. The total supply chain cost for scenario i =1,2,3 was of the form

$$TC_{sc,i} = \frac{D(A_v + \lambda_i a_v)}{\lambda_i \sum_{j=1}^{n} Q_{i,j}} + \frac{h_v}{2} \sum_{j=1}^{n} Q_{i,j} \left[(\lambda_i - 1)\left(1 - \frac{D}{P}\right) + \frac{D}{P} \right] + \sum_{j=1}^{n} \left(c_{b,1,j} - c_{b,i,j} \right) D_j$$

$$+ \sum_{j=1}^{n} \frac{D_j A_{b,j}}{Q_{i,j}} + \sum_{j=1}^{n} \frac{Q_{i,j} h_{b,i,j}}{2} - \sum_{j=1}^{n} \left(c_{b,1,j} - c_{b,i,j} \right) D_j$$

Where $D = \sum_{j=1}^{n} D_j$ total demand rate of all buyers with D_j being the demand for buyer j, A_v and $A_{b,j}$ are as defined earlier respectively the vendor's and buyer's j order/setup costs, a_v a fixed cost to process buyer's order of any size, λ_i the number of deliveries from vendor to each buyer per cycle for scenario i, $Q_{i,j}$, the order quantity for buyer j for scenario i, h_v is the vendor's holding cost, $h_{b,i,j}$ is the buyer's holding cost for buyer j for scenario i, and $c_{b,i,j}$ being the unit purchase cost for buyer j for scenario i. Recently, Wee & Yang (2007) proposed a very similar work to that of Yang and Wee (2006b), where they extended the work of Yang et al. (2007) to consider multiple buyers rather than a single buyer. They developed an optimal pricing and replenishment policy in a "leagile" (lean and agile) supply chain system for an integrated vendor-buyers system considering JIT concept and price reduction to the buyers for ordering larger quantity.
Yugang et al. (2006) considered a Vendor-Managed-Inventory (VMI) supply chain, which consists of one vendor (manufacturer) and multiple different buyers (retailers) with a single product. The vendor produces a single product with a limited production capacity and distributes it to its buyers. Each buyer buys the product from the manufacturer at wholesale price, and then sells it to the consumer market at a retail price. The buyer' markets are assumed to be dispersed and independent of each other. In the proposed supply chain, the vendor, as a leader, determines the wholesale price and inventory policy for the supply chain to maximize its own profit, and each retailer, as a follower, in turn takes the vendor's decision results as given inputs to determine the optimal retail prices to maximise its own profits. Along this line of research, Nachiappan et al. (2006) proposed a methodology to determine the common optimal price (contract and selling prices) that protects the profit of the buyer which is the main reason for the existence of partnership, for maximum channel profit in a two-echelon SC to implement VMI.

Multiple Items

Kohli & Park (1994) examined joint ordering policy in a vendor-buyer system as a method for reducing the transactions cost for multiple products sold by a seller to a homogeneous group of buyers. They found that efficient joint lot-sizes are independent of prices, and are supported by a range of average-unit prices that permit every possible allocation of the transactions-cost saving between the buyer and the seller. Kohli & Park (1994) also found that product bundling supports efficient joint orders across products, just as a quantity discount supports efficient transactions for a single product.

Chen & Chen (2005a) proposed both centralized and decentralized decision policies to analyze the interplay and investigate the joint effects of two-echelon coordination and multi-product replenishment on reduction of total costs. The total joint cost was given as

$$TC_{sc}(\{\lambda_i\},T)=\frac{A_b}{T}+\sum_{i=1}^{k}\left[\frac{a_{b,i}}{T}+\frac{T}{2}\left(h_{b,i}D_i\right)\right]$$

$$+\frac{A_v}{T}+\sum_{i=1}^{k}\left[\frac{a_{v,i}}{T}+\frac{T}{2}\left(\frac{h_{v,i}D_i^2}{P_i}\right)+\frac{a_{r,i}}{\lambda_i T}+\frac{u_i h_{r,i}T}{2}\left(\frac{D_i^2}{P_i}+(\lambda_i-1)D_i\right)\right]$$

Where T is the common cycle, $T>0$, D_i the demand rate of finished item i, P_i the production rate of finished item i produced by the vendor ($P_i > D_i$), $h_{b,i}$ is the inventory holding cost of finished item i per unit time for the buyer, $h_{v,i}$ is the inventory holding cost of finished item i per unit time for the vendor, $h_{r,i}$ the inventory holding cost of raw material for finished item i per unit time for the vendor, $a_{b,i}$ the minor setup cost for adding finished item i into the order for the buyer, $a_{v,i}$ the minor setup cost for adding finished item i into the production schedule for the vendor, $a_{r,i}$ the ordering cost of raw material for finished item i per lot for the vendor, u_i usage rate of raw material for the end item i produced by the manufacturer, and k is the total number of items. The optimal integer multiple of the common replenishment cycle for the raw material, the optimal common replenishment cycle, and the optimal order quantity for each item are given respectively as

$$\lambda_i^*=\left\lceil -\frac{1}{2}+\frac{1}{2}\sqrt{1+\frac{8a_{r,i}}{T^2u_ih_{r,i}D_i}}\right\rceil$$

$$T^*=\sqrt{\frac{2\left[A_b+A_v+\sum_{i=1}^{k}\left(a_{b,i}+a_{v,i}+\frac{a_{r,i}}{\lambda_i^*}\right)\right]}{\sum_{i=1}^{k}\left[h_{b,i}D_i+\frac{h_{v,i}D_i^2}{P_i}+u_ih_{r,i}\left(\frac{D_i^2}{P_i}+(\lambda_i^*-1)D_i\right)\right]}}$$

$$Q_i^* = \sqrt{\frac{2D_i^2\left[A_b + A_v + \sum_{i=1}^{k}\left(a_{b,i} + a_{v,i} + \frac{a_{r,i}}{\lambda_i^*}\right)\right]}{\sum_{i=1}^{k}\left[h_{b,i}D_i + \frac{h_{v,i}D_i^2}{P_i} + u_i h_{r,i}\left(\frac{D_i^2}{P_i} + \left(\lambda_i^* - 1\right)D_i\right)\right]}}$$

Chen & Chen (2005 b,c) proposed several optimization models adopting the joint replenishment program and channel coordination practice for a three level inventory system. The main purpose behind these models is to investigate how they influence possible supply chain improvements. The works of Chen & Chen (2005b,c) neither considered marketing stimulus into account, nor they assumed that the goods being imperishable for the period of production and selling. Furthermore, they dealt with cost-minimization supply chain design.

Chen & Chen (2007) focused on an area of emerging research: managing a multi-product and multi-echelon supply chain which produces and sells deteriorating goods in the marketplace. They formulated four profit-maximization models by considering the effects of channel coordination and a joint replenishment program on the supply-side cost control, taking into account the effect of the pricing scheme on demand and revenue increment. In addition, a profit-sharing mechanism via target rebates has been proposed, leading to Pareto improvements among channel participants.

Product/process Quality

Huang (2002) investigated the model of Salameh and Jaber (2000) in an integrated vendor-buyer context, where imperfect items at the buyer's end are withdrawn from inventory as a single batch and sold at a discounted price.The total annual cost of the vendor-buyer

$$TC_{sc}(\lambda, Q) = \left\{\frac{(A_v + A_b)D}{\lambda Q} + \frac{FD}{Q} + (d + W)D + \frac{DQ(h_b - h_v)}{x}\right\}E\left[\frac{1}{1-\gamma}\right] - WD$$

$$+\left\{\frac{DQ}{P} + \frac{\lambda Q}{2}\left(1 - \frac{D}{P}\right)\right\}h_v - \frac{DQ}{x}(h_b - h_v) + \frac{Q(1 - E[\gamma])}{2}(h_b - h_v)$$

Where F is the transportation cost per delivery, γ is the percentage of defective items whose probability density function is $f(y)$, x is the screening rate per unit ($x > D$), d is the unit screening cost, and W is the vendor's unit warranty cost of a defective item. The optimal order quantity that minimizes the above equation was given as

$$Q(\lambda) = \sqrt{\frac{2D\left[\frac{A_v + A_b}{\lambda} + F\right]E\left[\frac{1}{1-\gamma}\right]}{\left[\frac{2D(h_b - h_v)}{x}\right]\left(E\left[\frac{1}{1-\gamma}\right] - 1\right) + \left[\frac{2D}{P} + \lambda\left(1 - \frac{D}{P}\right)\right]h_v + (1 - E[\gamma])(h_b - h_v)}}$$

Khouja (2003a) considers a simple supply chain consisting of a vendor who produces a product and delivers it to a buyer who in turn sells it to the final customer. He assumed the lot size quality relationship to follow that of Porteus (1986). Porteus assumed the production process to be functioning perfectly at the start of production. With the production of each unit, the process may shift out-of-control with a constant known transition probability, and start producing all defective units. Once the process is out of control, it stays that way while the remainder of the lot is produced. The production system is restored to perfect quality when it is set up again. Porteus (1986) estimated the expected defectives per lot to be $\rho Q^2/2$, where ρ is the probability of the process going out of control and ρ is very small (Khouja, 2005). The expected total annual cost for the vendor and the buyer is

$$TC_{sc} = \left(A_b + \frac{A_v}{\lambda}\right)\frac{D}{Q} + \left[h_b - h_v + h_v\lambda\left(1 + \frac{D}{P}\right)\right]\frac{Q}{2} + \frac{1}{2}\rho\lambda QDw$$

Where λ is, as defined earlier, the vendor's lot-size multiplier (positive integer) of the buyer's order quantity Q, and w being the cost to rework a defective unit. Minimizing the expected total annual cost for the whole supply chain (i.e. joint optimization), then the optimality conditions are given by

$$\lambda^*\left(\lambda^* - 1\right) \le \frac{(h_b - h_v)A_v}{A_b\left(Dw\rho + (1 + D/P)h_v\right)} \le \lambda^*\left(\lambda^* + 1\right)$$

$$Q^*(\lambda) = \sqrt{\frac{2(\lambda A_b + A_v)}{\lambda\left[\lambda w\rho + h_b/D + (\lambda/P + (\lambda - 1)/D)h_v\right]}}$$

Khouja (2003a) also investigated his model for the cases when the vendor has a constant failure rate, and when demand is stochastic.

Similar to Huang (2002), Goyal et al. (2003) considered a two-level supply chain where there is a vendor and a buyer for a single product, where the number of perfect units is at least equal to the demand during the screening time and that the defective units are sold as a single batch at the end of the screening period (Salameh & Jaber, 2000). Their expected annual cost was given as

$$TC_{sc}(\lambda, Q) = (A_v + A_b + \lambda F)\frac{D}{Q} + \frac{Q}{2\lambda}\left\{\left[1 + (\lambda - 2)(1 - D/P)\right]h_v + \frac{h_b}{E\left[1/(1-\gamma)\right]}\right\}$$

Where F is the transportation cost per shipment, γ is the percentage of defective items, a random variable, and $E\left[1/(1-\gamma)\right] = \int_0^\infty f(y)dy$ is the expected value with $f(y)$ being the probability density function of γ. The optimal order quantity that minimizes the above equation is

$$Q(\lambda) = \sqrt{\frac{2\lambda(A_v + A_b + \lambda F)D}{\left[1 + (\lambda - 2)(1 - D/P)\right]h_v + h_b/E\left[1/(1-\gamma)\right]}}$$

A very similar model to that of Goyal et al (2003) was developed in Huang (2004). Siajadi et al. (2005) analysed scenario is that a single buyer (or a group of buyers), demand(s) a particular type of end/finished item where back-order is not allowed. The delivery of the finished item to the customer is based on multiple small deliveries of equal size, Q, instead of a lot-for-lot basis. They assumed that the production of the finished item will include the production of imperfect quality items, where 100% inspection is performed for each lot at a