BASIC BUSINESS STATISTICS

PREFACE

This book is the result of my working experience in the subject Basic Business Statistics for BBA and MBA students. It is designed to meet the requirements of Bachelors Masters and Ph D level students .The main highlight of the book is both theory instructor manual and solved problem approach & explanations added for numerical question problems framed by the author.This book has a large number of problems solved in all 11 chapters.

All theoretical concepts needed by students are covered by the author in this book.

I am extremely grateful to all my students who were attentive in the classes when I was conducting the classes.

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ABOUT THE BOOK

This book is on Basic Business Statistics is a compulsory subject for Commerce students.The higher level students and bachelor level students can also read it as it contains a lot of numerical problems framed by me with full Instructor Manual.

Introduction to Statistics,Concepts of probability,Sampling methods and sampling distribution,Random variables and Probability Distribution,Descriptive Statistics,Inferential Statistics,F-Test and Analysis of Variance,Chi-square applications,Simple linear regression and correlation,Time Series Analaysis and Business Forecasting,Index Numbersare the 11 chapters with various sub-topics covered in this book.

I feel that this is a unique book as there are many theoretical Instructor Manual concepts and numerical problems solved with explanation.

HAPPY READING.

THANKS

REGARDS

AUTHOR

(SRINIVAS R RAO)

ABOUT THE AUTHOR

Author's name is Srinivas R Rao, born and done his school level education in Mangalore,Karnataka in a reputed private school Canara High School and PUC(+2) from Canara PUC in Science stream with PCMB as main subjects.

Later, pursuing LL.B(5 Years) course passed the degree in 1999 and done Diploma in Export Management ,Diploma in Customs and Central Excise , Diploma in Business Administration and some important IT subjects like MS-Office,Internet/Email,Visual Basic 6.0,C,C++,Java,Advanced Java,Oracle with D2K,HTML with Javascript,VBscript and Active Server Pages.

Joined as a FACULTY for students in a small computer Institute in 2002 July and later after 4 months worked in a company by name CRP Technologies(I) .P.Ltd as Branch Manager(Risk Manager) for Mangalore,Udupi and Kasargod areas from January 26 2003 to June 11 2007.In the year 2005 pursued MBA distance education course. Currently working as a FACULTY in Sikkim Manipal University , Udupi centre for BBA & MBA students and teaching numerical subjects like Statistics/Operations Research(Mgt Science/Quant. Techniques for Mgt)/Accounting and several numerical and difficult oriented subjects for distance education students in their weekend contact classes from July 2010 till present day.

Thanks

Regards

Author

(SRINIVAS R RAO)

BASIC BUSINESS STATISTICS

FOR BACHELORS,MASTERS AND PH.D-STATISTICS UNDER COMMERCE,ENGINEERING & MANAGEMENT SUBJECTS

CHAPTERS:

INSTRUCTOR'S MANUAL

CHAPTER 1

Introduction to Statistics

Learning Objectives

The study of this chapter should enable you to:

- ❖ Define the meaning of Statistics and other popular terms widely used in statistics
- ❖ Describe the types of statistics—descriptive and inferential
- ❖ Describe the sources of data, the types of data and variables
- ❖ Understand the different levels of measurement
- ❖ Describe the various methods of collecting data

Key Teaching Points

1.1 WHAT IS STATISTICS?

- 'Statistics' is a science that involves the efficient use of numerical data relating to groups of individuals (or trials).
- Related to the collection, analysis and interpretation of data, including data collection design in the form of surveys and experiments.
- Defined as the science of:
  - ¤ Collecting
  - ¤ Organizing
  - ¤ Presenting
  - ¤ Analyzing
  - ¤ Interpreting numerical data to efficiently help the process of making decisions
- A person who works with the applied statistics (the practical application of statistics), and is particularly eloquent in the way of thinking for the successful implementation of statistical analysis is called a 'statistician'.
- The essence of the profession is to measure, interpret and describe the world and patterns of human activity in it both in the private and public sectors.
- Those involved in marketing, accounting, quality control and others, such as consumers, sports players, administrators, educators, political parties, doctors, etc. on the other hand, tend to widely use the outcomes of various statistical techniques to help make decisions.
- Population size refers to a very large amount of data where making a census or a complete sampling of all of the population would be impractical or impossible.
- A sample is a subset of the population.
- Samples are collected and statistics are calculated from the samples in order to make conclusions about the population.

1.2 TYPES OF STATISTICS

- Two types of statistics:
  1. Descriptive statistics
  2. Inferential statistics
- Descriptive statistics explains the sample data whereas inferential statistics tries to reach conclusions that go beyond the existing data.

1.2.1 Descriptive Statistics

- Statistical methods used to describe the basic features of the data that have been collected in a study.
- Provide simple summaries about the data and the measures.
- Together with simple graphics analysis, they form the basis of virtually every quantitative analysis of data.
- Use descriptive statistics simply to describe what's going on in our data.
- Used to present quantitative descriptions in a manageable form.
- Help us to facilitate large data in a way that easily makes sense.
- Each statistic reduces large data into a simple summary.

1.2.2 Inferential Statistics

- Methods used to find out something about a population based on a sample taken from that population.
- Also called statistical inference or inductive statistics.
- Most of the major inferential statistics come from a general family of statistical models known as the General Linear Model
  - Includes the *t*-test
  - Analysis of variance (ANOVA)
  - Regression analysis
  - Multivariate methods like factor analysis, multidimensional scaling, cluster analysis, discriminant function analysis, etc.

1.3 SOURCES OF DATA

- Two sources of data: 'primary data' and 'secondary data'.
- Researchers conduct various research projects using questionnaires addressed directly to respondents, and their responses are known as the primary data.
- Other studies involving the use of data collected by others, such as information from census and earlier findings are also used by researchers—called secondary data.
- Primary data offer information tailored to specific studies, but are usually more expensive and takes a longer period to process.
- Secondary data are usually less expensive to be acquired and can be analyzed in a shorter period.

1.3.1 Primary Data

- 'Primary data' is the specific information collected by person who is doing research (researcher).
- Researchers collect data through surveys, interviews, direct observations and experiments.
- Essential to all areas of study because it is the original data of an experiment or observation that has not been processed or altered.
- Primary data can be prospective, retrospective, interventional or observational in nature.
- Prospective data is collected from subjects in real time
- Retrospective data is collected from archival records.
- Retrospective primary data provides information on past circumstances or behaviours.

- Interventional primary data can be gathered after the interventions of interest have been prospectively delivered, manipulated or managed.
- Observational primary data is collected by monitoring an intervention of interest without intervening in the delivery of the intervention.

Advantages:

1. Researchers can decide the type of method they will use in collecting the data and how long it will take them to gather that particular data.
2. Researchers can focus the data collection on specific issues of their research and enable them to collect more accurate information.
3. Researchers would know in detail how the data were gathered and hence, will be able to present original and unbiased data.

Disadvantages:

1. Primary data collection consumes a lot of time, effort and cost; the researchers will not only need to make certain preparations, in addition, they will need to manage both their time and cost effectively
2. Researchers will have to collect large volumes of data since they will interact with different people and environments; also they will need to spend a lot of time checking, analyzing and evaluating their findings before using such data.

1.3.2 Secondary Data

- Any material that has been collected from published records, such as newspapers, journals, research papers and so on.
- Sources of secondary data may include information from the census, records of employees of a company, or government statistical information such as Malaysia gross national income (GNI) in different sectors and many others.
- Easily available and cheap.
- Available for a longer period of time.

Advantages:

1. Using data from secondary sources is more convenient as it requires less time, effort and cost.
2. Secondary data helps to decide what further researches need to be done.

Disadvantages:

1. Secondary data may have transcription errors (reproduction errors).
2. Data from secondary sources may not meet the user's specific needs.
3. Not all secondary data is readily available or inexpensive.
4. The accuracy of the secondary data can be questionable.

1.4 TYPES OF DATA AND VARIABLES

- 'Data' refers to qualitative or quantitative attributes of a variable or set of variables.
- A variable is defined as any measured attribute that varies for different subjects.
- Two basic types of data
  1. *Quantitative data*
  2. *Qualitative data*

1.4.1 Quantitative Data

- Data that measures or identifies based on a numerical scale.
- Can be analyzed using statistical methods
  - Values obtained can be illustrated using diagrams such as tables, graphs and histograms.

- Variable being studied can be reported numerically and is called a quantitative variable while the population is called a quantitative population.
- Quantitative variables can be further classified as either discrete or continuous.
- Discrete variables can assume only integer values (whole number such as 0, 1, 2, 3, 4, 5, 6, etc.).
- Discrete variables result from counting.
- Continuous variable can assume any value over a continuous range of possibilities.
  - For example:
    - ✓ Time (05:31:24 a.m)
    - ✓ Temperature (35.5 °C)
    - ✓ Weight (85.6 kilograms)
    - ✓ Height (167.5 cm)
    - ✓ Speed (183.7 km/h), etc.
- Continuous variables result from measuring something.

1.4.2 Qualitative Data

- Provide items in a variety of qualities or categories that may be 'informal' or even using features that is relatively obscure, such as warmth and taste.
- Although, the data that was originally collected as qualitative information, it can be quantitative if it is further simplified using the method of counting.
- Can include the obvious aspects such as gender, age or occupation.
- Can also be in the form of pass-fail, yes-no, or various other categories.
- If qualitative data uses categories based on ideas of subjective or non-existent, it is generally less valuable for scientific study than quantitative data.
- Sometimes it is possible to obtain quantitative estimates of the qualitative data.
  - For example:
    - ✓ People can be asked to rate their perceptions about their interest in a sport based on the Likert scale, that is, a rating or a psychometric scale commonly used in questionnaires.
    - ✓ If a 10-point scale is used, '1' would signify 'strongly agree' and '10' would indicate 'strongly disagree'.
- When the characteristics or *variable* being studied is non-numeric (categorical), it is called a *qualitative variable* or an *attribute*, while the population is called a qualitative population.
- When the data are qualitative, we are usually interested in:
  - How many?
  - What proportion fall in each category?
- Qualitative variables are measured according to their specific categories and are often summarized in charts.
  - For example:
    - ✓ Gender is measured as 'male' or 'female'.
    - ✓ Marital status is measured as 'single' or 'married', and so on.

1.5 LEVELS OF MEASUREMENT

- Can be classified into four categories:
  - Nominal
  - Ordinal
  - Interval
  - Ratio

1.5.1 Nominal Level

- The most 'primitive', 'the lowest', or the most limited type of measurement.

- In this level of measurement,
  - ¤ Numbers or even words and letters are used to categorize the data.
- Suppose there are data about students who sat for an examination.
  - ¤ Hence, in a nominal level of measurement,
    - ✓ Students who passed the examination are classified as 'P'
    - ✓ Students who failed can be classified as 'F'

1.5.2 Ordinal Level

- Describes the relationship within a group of items in some specified order.
- For example,
  - ¤ For a student with the highest marks in a class—he will be placed as the first rank.
  - ¤ Then, a student who received the second highest marks will be placed as the second rank, and so on.
- This level of measurement indicates an approximate ordering of the measurements. The difference or the ratio between any two types of rankings is not always the same along the scale.

1.5.3 Interval Level

- Includes all the features of ordinal level (classification and direction).
- States that the distances between intervals are the same along the interval scale from low to high (constant size).
- A popular example of this level of measurement is temperature in Celsius.

1.5.4 Ratio Level

- Is the 'highest' level of measurement
- Has all the characteristics of interval level.
- Major differences between interval and ratio levels of measurement are:
  (1) Ratio-level data has a meaningful zero point
  (2) Ratio between any given two numbers is meaningful
- Divisions between the points on the scale have an equivalent distance between them
- Rankings assigned to the items are according to their size.
- Money is a good illustration,
  - ¤ Having zero ringgit means 'you have none'
- Weight is another ratio-level measurement.
  - ¤ If the dial on a scale is zero, there is a complete absence of weight.
  - ¤ If you earn $40 000 a year and Abu earns $10 000, you earn four times what he does.

1.6 METHODS OF COLLECTING DATA

- Data collection is an important aspect of any type of research study as inaccurate data collection can impact the results of a study and ultimately lead to invalid results.
- Investigator (researcher) must first of all, define the scope of his inquiry in every detail.
- The probable cost, time and labour required must next be estimated.
- If a complete coverage of information is not possible, for example, in market research, the sample size and method of sampling will have to be determined.
- Investigators collect primary data directly from the original sources.
- They can collect the necessary data appropriate for specific research needs, in the form they need.
- In most cases, primary data collection is costly and time-consuming.
- For some areas within social science research, such as socio-economic surveys, studies of social anthropology, market research, etc., necessary data are not always available from secondary sources, and they must be directly collected from the original or primary sources.

- In cases where the available secondary data are not suitable, again, the primary data should be collected.

1.6.1 Methods of Primary Data Collection

- 'Method' refers to a data collection mode or method
- 'Tool' is an instrument used to carry out the method.
- Some important methods of data collection:
  1. Observations
  2. Experimentation
  3. Simulation
  4. Interviewing
  5. Panel Method
  6. Mail Survey
  7. Projective Techniques
  8. Sociometry

1.6.2 Tools for Data Collection

- A number of different types of instruments or tools are used for data collection depending on the nature of the information to be gathered.

1. Types of Tools

✓ Observation schedule
✓ Interview guide and schedule
✓ Questionnaires
✓ Rating scale
✓ Checklists
✓ Data sheet
✓ Institution's schedule

2. Constructing Schedule and Questionnaire

✓ Schedule and questionnaire are the most common tools of data collection.
✓ These tools have many similarities and contain a set of questions related to the problem under study.
✓ Both these tools aim at retrieving information from the respondents.
✓ The content, structure, question words, question order, etc. are the same for all respondents.
✓ Each may use a different method; schedule is used for interviewing (the interviewer fills the schedule) and questionnaire is used for mailing (the respondents fill out questionnaires by themselves).
✓ Schedule and questionnaire are constructed almost in the same way.
✓ It consists of some main steps as below:
  (i) Identifying the research data
  (ii) Prepare 'dummy' tables
  (iii) Determine the level of the respondents
  (iv) Decide methods of data collection
  (v) Design instrument/tool
  (vi) Assessment of the design instrument
  (vii) Pre-testing
  (viii) Specification of procedures
  (ix) Planning format

3. Pilot Studies and Pre-Tests

- It is often difficult to design a large study without adequate knowledge of the problem; population to cover, level of knowledge, and so on.
  - What are the issues and the concepts related to the problem under study?
  - What is the best method of study?
  - How long will it take and what is the cost?
  - These and other related questions require a lot of knowledge about the subject matter.
- To obtain such pre-knowledge, a preliminary or pilot study should be conducted.
- Pilot study is a full-fledged miniature study of a problem
- Pre-test is a trial test of a specific aspect of the study such as method of data collection or instrument.
- Instrument of data collection is designed with reference to the data requirements of the study.
  - It cannot be perfected purely on the basis of a critical scrutiny by the designer and other researchers.
  - It should be empirically tested (should be tested using a collection of data). Hence, pre-testing of a draft instrument is rather indispensable.
- Pre-testing has several beneficial functions:
  - To test whether the instrument will get the responses needed to realize the objectives of the study.
  - To examine whether the content of the instrument is relevant and sufficient.
  - To test the questions whether the words are clear and in accordance with the understanding of the respondents.
  - To examine other qualitative aspects of the instrument such as the question structure and the sequence of questions.
  - To develop appropriate procedure to deal with the instrument in the field.

Teaching Notes

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INSTRUCTOR'S MANUAL

CHAPTER

2 Concepts of Probability

Learning Objectives

The study of this chapter should enable you to:

- ❖ Define the meaning of probability and other key terms
- ❖ Understand the concepts of sample space and events
- ❖ Apply permutations and combinations rules to count sample points
- ❖ Calculate the probability of an event including conditional probability
- ❖ Define and apply additive rules, multiplicative rules, law of total probability and Bayes' rule

Key Teaching Points

2.1 INTRODUCTION

- Probability is a branch of mathematics that studies the possible outcomes of events with its possible likelihoods and relative distributions.
- The word 'probability' refers to the chance that a particular event (or series of events) will occur on a linear scale from 0 (impossibility) to 1 (certainty), or as a percentage (0 to 100%).
- Frequentists (classic approaches) see probability as a measure of the frequency of an event.
- Bayesians (evidential probabilities) considers probability as an estimating parameter for a set of observed distributions.

2.2 SAMPLE SPACE AND EVENTS

- A *sample space* is defined as a list of all possible outcomes of a random experiment or trial.
- An *event* is a set of outcomes or sample points.

2.2.1 Sample Space

- The word *experiment* is used by Statistician to describe any process that generates a set of data.
- A simple example of a statistical experiment is the tossing of a coin several times.
- There are only two possible outcomes, 'heads' or 'tails' and we are particularly interested in the uncertain observations every time the coin is tossed.
- The set of all possible outcomes of a statistical experiment is called the sample space and is usually represented by the symbol *S*.
- Each outcome in a sample space is called an element or a member of the sample space, or simply known as a *sample point*.
- The sample space *S*, of possible outcomes when a coin is tossed, may be written as $S = \{H, T\}$, where *H* and *T* correspond to 'heads' and 'tails', respectively.

2.2.2 Tree Diagram

- In some experiments it is helpful to list the elements of the sample space systematically through a tree diagram.
- The term 'tree diagram' refers to a graphic organizer used to list all possibilities of a sequence of events in a systematic way.

2.2.3 Events

- For any given experiment we may be interested in the occurrence of certain *events* rather than in the outcome of a specific element in the sample space.
- For instance, we may be interested in the event A, the outcome that is divisible by 3 when a dice is tossed.
  - This will occur if the outcome is an element of the subset $A = \{3, 6\}$.
  - To each event we assign a collection of sample points, which constitutes to a subset of the sample space.
  - Subset represents all of the elements for which the event is true while an *event* is a subset of a sample space.

2.3 COUNTING SAMPLE POINTS

- In many cases we shall be able to solve a probability problem by counting the number of sample points without actually listing each element.

2.3.1 Multiplication Rule

- The fundamental principle of counting is often referred to as the multiplication rule.

Theorem 2.3.1
If an operation can be performed in n_1 ways, and if for each of these a second operation can be performed in n_2 ways, then the two operations can be performed in $n_1 n_2$ ways.

Theorem 2.3.2
Suppose that an operation can be performed in n_1 ways, and if for each one of these (from the first operation) a second operation can be performed in n_2 ways, and then for each one of these (from the second operation) a third operation can be performed in n_3 ways, and so on, then it can be shown that the sequence of k operations can be performed in $n_1 n_2, \ldots, n_k$ ways.

2.3.2 Permutations

- Frequently, we are interested in a sample space that contains elements with all possible orders or arrangements of a group of objects.
- The different arrangements are called *permutations*.
- Consider the letters a, b and c.
  - The six possible, distinct permutations are *abc*, *acb*, *bac*, *bca*, *cab* and *cba*.
  - Using Theorem 2.3.2, we could arrive at the answer 6 without actually listing the different orders.
  - n distinct objects can be arranged in $n(n-1)(n-2)\ldots(3)(2)(1) = n!$ ways.

Theorem 2.3.3
The number of permutations of n distinct objects is $n!$.
In general, n distinct objects taken r at a time can be arranged in

$$n(n-1)(n-2)\ldots(n-r+1) \text{ ways} = {}^{n}P_{r} = \frac{n!}{(n-r)!}.$$

Theorem 2.3.4

The number of permutations of n distinct objects arranged in a circle is $(n - 1)!$. So far we have considered permutations of distinct objects. Obviously, if the letters b and c are both equal to x, then the 6 permutations of the letters a, b, c become axx, axx, xax, xax, xxa and xxa, of which only 3 are distinct. Therefore, with 3 letters, 2 being the same, we have $3!/2! = 3$ distinct permutations.

Theorem 2.3.5

The number of different permutations of n objects of which n_1 objects of type 1, n_2 objects of type 2, ..., and n_k objects of type k is $\frac{n!}{n_1!\, n_2! \ldots n_k!}$.

Theorem 2.3.6

The number of ways of partitioning a set of n objects into r cells with n_1 elements in the first cell, n_2 elements in the second, and so forth, is

$$\binom{n}{n_1, n_1, \ldots, n_r} = \frac{n!}{n_1!\, n_2! \ldots n_r!}, \text{ where } n_1 + n_2 + \ldots + n_r = n.$$

2.3.3 Combinations

- In many problems we are interested in,
  - The number of ways of selecting r objects from n without considering its order.
- These selections are called *combinations*.
- A combination is actually a partition with two cells,
  - The one cell containing the r objects selected
  - The other cell containing the $(n - r)$ objects that are left
- The number of such combinations, denoted by,

  $\binom{n}{r, n-r}$, is shortened to, $\binom{n}{r}$

  since the number of elements in the second cell must be $n - r$.

Theorem 2.3.7

The number of combinations of n distinct objects taken r at a time is

$$\binom{n}{r} = \frac{n!}{r_1(n - r)}.$$

2.4 CALCULATING THE PROBABILITY OF AN EVENT

- The probability of an event is calculated as
  - The ratio of the outcomes (sample points) of the event divided by the total number of possible outcomes (all points in the sample space).

2.4.1 Probability of an Event

- In many experiments, such as tossing a coin or a dice,
  - All the sample points have the same chance of occurring and are assigned equal probabilities.
  - For points outside the sample space, that is, for simple events that cannot possibly occur, we assign a probability of zero.
- To find the probability of an event A, we sum all the probabilities assigned to the sample points in A.
  - The sum is called the probability of A and is denoted $P(A)$

    $0 \le P(A) \le 1$,

    $P(\varnothing) = 0$ and $P(S) = 1$

Theorem 2.4.1
If an experiment can result in any one of N different equally likely outcomes, and if exactly n of these outcomes correspond to event A, then the probability of event A is:
$P(A) = n/N$.

2.4.2 Additives Rule

- Several important laws that frequently simplify the computation of probabilities are as follows. The first, called the additive rules, applies to unions of events.

Theorem 2.4.2
If A and B are any two events, then
$$P(A \cup B) = P(A) + P(B) - P(A \cap B).$$
If A and B are mutually exclusive, then
$$P(A \cup B) = P(A) + P(B)$$
If $A_1, A_2, A_3, \ldots, A_n$ are mutually exclusive, then
$$P(A_1 \cup A_2 \cup \ldots \cup A_n) = P(A_1) + P(A_2) + \ldots + P(A_n).$$
If $A_1, A_2, A_3, \ldots, A_n$ is a partition of a sample space S, then
$$P(A_1 \cup A_2 \cup \ldots \cup A_n) = P(A_1) + P(A_2) + \ldots + P(A_n) = P(S) = 1.$$

Theorem 2.4.3
For three events A, B and C,
$$P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - \mathrm{P}(A \cap C) - P(B \cap C) + P(A \cap B \cap C).$$

Theorem 2.4.4
If A and A' are complementary events, then:
$$P(A) + P(A') = 1.$$

2.4.3 Conditional Probability

- The probability of an event B occurring when it is known that some event A has occurred is called a conditional probability and is denoted by $P(B|A)$.
- The conditional probability of B, given the occurrence of A, denoted by $P(B|A)$, is defined by

$$P(B|A) = \frac{P(B \cap A)}{P(A)}, \text{ if } P(A) > 0.$$

Independent Events

- $P(A|D)$ differs from $P(A)$.
- This suggests that the occurrence of A influenced D.
- However, consider the situation where we have events A and B and $P(A|B) = P(A)$;
  - In other words the occurrence of B had no impact on the odds of occurrence of A.
  - Here the occurrence of A is independent of the occurrence of B.
- Two events A and B are independent if and only if $P(B|A) = P(B)$ and $P(A|B) = P(A)$.
  - Otherwise, A and B are dependent.

2.4.4 Multiplicative Rules

- Multiplicative rules apply to intersections of events.

Theorem 2.4.5
If in an experiment the events A and B can both occur, then
$$P(A \cap B) = P(B \cap A) = P(A)P(B|A) = P(B)P(A|B).$$