The Age of Einstein

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PREFACE

This book had its origin in a one-year course that I taught at Yale throughout the decade of the 1970's. The course was for non-science majors who were interested in learning about the major branches of Physics. In the first semester, emphasis was placed on Newtonian and Einsteinian Relativity. The recent popularity of the Einstein exhibit at the American Museum of Natural History in New York City, prompted me to look again at my fading lecture notes. I found that they contained material that might be of interest to today's readers. I have therefore reproduced them with some additions, mostly of a graphical nature. I recall the books that were most influential in my approach to the subject at that time; they were Max Born's *The Special Theory of Relativity*, Robert Adair's *Concepts of Physics*, and Casper and Noer's *Revolutions in Physics*. These three books were written with the non-scientist in mind, and they showed what could be achieved in this important area of teaching and learning; I am greatly indebted to these authors.

1. INTRODUCTION

This brief book is for the inquisitive reader who wishes to gain an understanding of the immortal work of Einstein, the greatest scientist since Newton. The concepts that form the basis of Einstein's Theory of Special Relativity are discussed at a level suitable for Seniors in High School. Special Relativity deals with measurements of space, time and motion in inertial frames of reference (see chapter 4). An introduction to Einstein's Theory of General Relativity, a theory of space, time, and motion in the presence of gravity, is given at a popular level. A more formal account of Special Relativity, that requires a higher level of understanding of Mathematics, is given in an Appendix.

Historians in the future will, no doubt, choose a phrase that best characterizes the 20th-century. Several possible phrases, such as "the Atomic Age", "the Space Age" and "the Information Age", come to mind. I believe that a strong case will be made for the phrase "the Age of Einstein"; no other person in the 20th-century advanced our understanding of the physical universe in such a dramatic way. He introduced many original concepts, each one of a profound nature. His discovery of the universal equivalence of energy and mass has had, and continues to have, far-reaching consequences not only in Science and Technology but also in fields as diverse as World Politics, Economics, and Philosophy.

The topics covered include:

a) understanding the physical universe;

b) describing everyday motion;

relative motion,

Newton's Principle of Relativity,

problems with light,

c) Einstein's Theory of Special Relativity;

simultaneity and synchronizing clocks,

length contraction and time dilation,

examples of Einstein's world,

d) Newtonian and Einsteinian mass;

e) equivalence of energy and mass, $E = mc^2$;

f) Principle of Equivalence;

g) Einsteinian gravity;

gravity and the bending of light,

gravity and the flow of time, and

red shifts, blue shifts, and black holes.

2. UNDERSTANDING THE PHYSICAL UNIVERSE

We would be justified in thinking that any attempts to derive a small set of fundamental laws of Nature from a limited sample of all possible processes in the physical universe, would lead to a large set of unrelated facts. Remarkably, however, very few fundamental laws of Nature have been found to be necessary to account for all observations of basic physical phenomena. These phenomena range in scale from the motions of minute subatomic systems to the motions of the galaxies. The methods used, over the past five hundred years, to find the set of fundamental laws of Nature are clearly important; a random approach to the problem would have been of no use whatsoever. In the first place, it is necessary for the scientist to have a conviction that Nature can be understood in terms of a small set of fundamental laws, and that these laws should provide a quantitative account of all basic physical processes. It is axiomatic that the laws hold throughout the universe. In this respect, the methods of Physics belong to Philosophy. (In earlier times, Physics was referred to by the appropriate title, "Natural Philosophy").

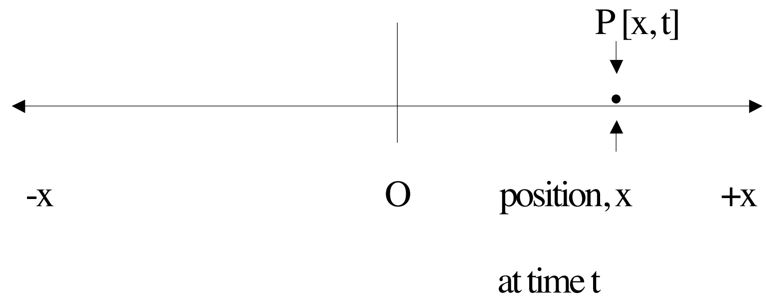
2.1 Reality and Pure Thought

In one of his writings entitled "On the Method of Theoretical Physics", Einstein stated: "If, then, experience is the alpha and the omega of all our knowledge of reality, what then is the function of pure reason in science?" He continued, "Newton, the first creator of a comprehensive, workable system of theoretical physics, still believed that the basic concepts and laws of his system could be derived from experience." Einstein then wrote "But the tremendous practical success of his (Newton's) doctrines may well have prevented him, and the physicists of the eighteenth and nineteenth centuries, from recognizing the fictitious character of the foundations of his system". It was Einstein's view that "..the concepts and fundamental principles which underlie a theoretical system of physics are *free inventions of the human intellect*, which cannot be justified either by the nature of that intellect or in any other fashion *a priori*." He continued, "If, then, it is true that the axiomatic basis of theoretical physics cannot be extracted from experience but must be freely invented, can we ever hope to find the right way? ... Can we hope to be guided safely by experience at all when there exist theories (such as Classical (Newtonian) Mechanics) which to a large extent do justice to experience, without getting to the root of the matter? I answer without hesitation that there is, in my opinion, a right way, and that we are capable of finding it." Einstein then stated "Experience remains, of course, the sole criterion of the physical utility of a mathematical construction. But the creative principle resides in Mathematics. ... I hold it true that pure thought can grasp reality, as the ancients dreamed."

3. DESCRIBING EVERYDAY MOTION

3.1 Motion in a straight line (the absence of forces)

The simplest motion is that of a point, P, moving in a straight line. Let the line be labeled the "x-axis", and let the position of P be measured from a fixed point on the line, the origin, O. Let the motion begin (time $t = 0$) when P is at the origin ($x = 0$). At an arbitrary time, t, P is at the distance x:



If successive positions of P are plotted, together with their corresponding times, we can generate what is called the "world line" of P.

Let us observe a racing car moving at high speed along the straight part of a race track (the x–axis), and let us signal the instant that it passes our position, x = 0, by lowering a flag: An observer, standing at a measured distance D, from x = 0, starts his clock at the instant, t = 0, when he sees the flag lowered, and stops his clock at the instant t = T, as the car passes by. We can obtain the average speed of the car, v, during the interval T, in the standard way; it is

$v = D/T$ (in units of velocity).

If, for example, D = 1 mile, and T = 20 seconds (1/180 hour), then

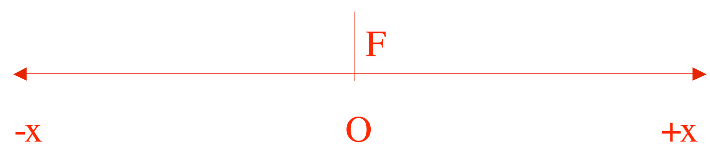
$v =$ 1 (mile)/(1/180) (hour) = 180 miles per hour.

This is such a standard procedure that we have no doubt concerning the validity of the result.

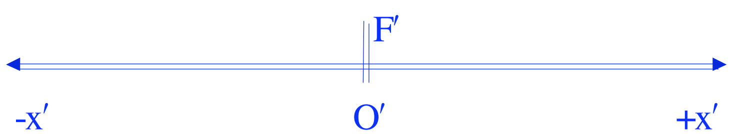
3.2 The relativity of everyday events

Events, the description of when and where happenings occur, are part of the physical world; they involve finite extensions in both time and space. From the point of view of a theory of motion, it is useful to consider "point-like" events that have vanishingly small extensions in time and space. They then can be represented as "points" in a space-time geometry. We shall label events by giving the time and space coordinates: event $E \rightarrow E[t, x]$, or in three space dimensions, $E[t, x, y, z]$, where x, y, z are the Cartesian components of the position of the event. There is nothing special about a Cartesian coordinate system, it is a mathematical construct; *any* suitable coordinate mesh with a metrical property (measured distances defined in terms of coordinates) can be used to describe the spatial locations of events. A familiar non-Cartesian system is the spherical polar coordinate system of the lines of latitude and longitude on the surface of the earth. The time t can be given by any device that is capable of producing a stable, repetitive motion such as a pendulum, or a quartz-controlled crystal oscillator or, for high precision, an atomic clock.

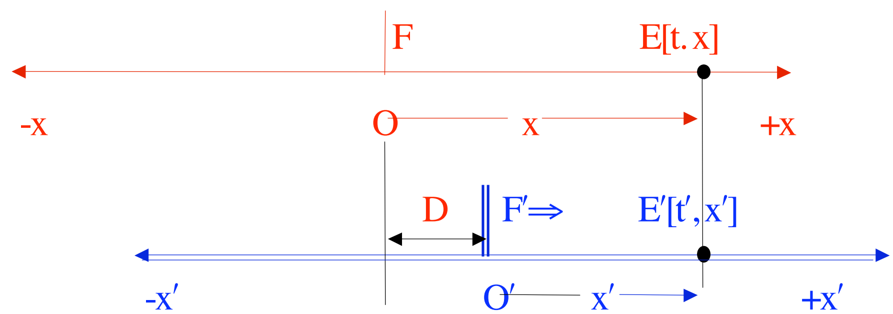
Suppose we have an observer, O, at rest at the origin of an x–axis, in the F-frame. O has assistants with measuring rods and clocks to record events occurring on the x-axis:



We introduce a second observer, O′, at rest at the origin of his frame of reference, F′. O′ has his assistants with their measuring rods (to measure distances, x′) and clocks (to measure times, t′) to record events on the x′-axis. (The F′-clocks are identical in construction and performance to the clocks in the F-frame). Let O′ coincide with O at a common origin $O = O'$ ($x = x' = 0$), at the synchronized time zero $t = t' = 0$. At $t = t' = 0$, we have



Suppose that the observer O′, and his assistants with rods and clocks, move to the right with *constant speed* V along the common x, x′–axis. At some later time t, the two sets of observers, represented by O and O′, record a common event that they write as E[t, x] and E′[t′, x′], respectively. The relative positions of the two observers at time t is:



where D = Vt is the distance that O′ moves at constant speed V, in the time t.

We therefore write the relationship between the two measurements by the plausible equations (based on everyday experience):

$$t' = t \text{ (everyday identical clocks tick at the same rate)}$$

and

$$x' = x - D = x - Vt.$$

These are the basic equations of relative motion according to the concepts first put forward by Galileo and Newton. They are fully consistent with measurements made in our real world (the world of experience). They are not, however, *internally consistent*. In the equation that relates the measurement of distance x′ in the F′-frame to the measurements in the F-frame, we see that the space part, x', in the F′-frame, is related to the space part, x, *and* the time part, t, in the F-frame: space-time in one frame is not related to space-time in the other frame! Furthermore, the time equation makes no mention of space in either frame. *We see that there is a fundamental lack of symmetry in the equations of relative motion, based on everyday experience*. The question of the "symmetry of space-time" will lead us to Einstein's philosophy of the "free invention of the intellect".

3.3 Relative velocities

We have seen that the position of an event, E[t, x], measured by an observer O, is related to the position of the same event, E′[t′, x′], measured by an observer O′, moving with constant speed V along the common x, x′–axis of the two frames, by the equation

$$x' = x - Vt\,.$$

The speed v of a point P[t, x], moving along the x–axis, is given by the ratio of the finite distance the point moves, Δx, in a given finite time interval, Δt:

$$v = \Delta x/\Delta t\,.$$

We can obtain the speeds v, and v′ of the same moving point, as measured in the two frames, by calculating $v = \Delta x/\Delta t$ and $v' = \Delta x'/\Delta t'$, as follows:

$\Delta x'/\Delta t' = v' = \Delta x/\Delta t - V\Delta t/\Delta t$ (where we have used $\Delta t' = \Delta t$ because $t = t'$ in everyday experience).

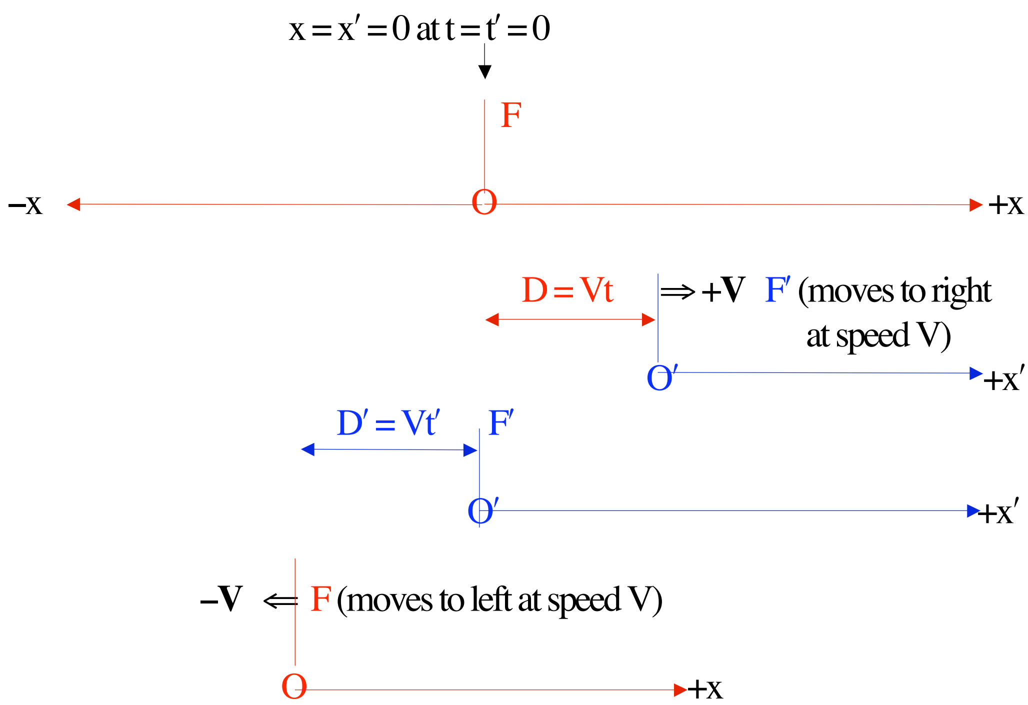
We therefore find

$$v' = v - V,$$

the speeds differ by the relative speed of the two frames. This is consistent with experience: if a car moves along a straight road at a constant speed of v = 60 mph, relative to a stationary observer O, and an observer O′ follows in a car at a constant speed of V = 40 mph relative to O, then the speed of the first car relative to the occupant of the second car is v′ = 20 mph.

3.4 The Newtonian Principle of Relativity

The Newtonian *Principle of Relativity* asserts that, in the inertial frames F, F′, the following two situations



cannot be distinguished by experiments that involve mechanical systems (classical systems that obey Newton's Laws of Motion).

The speed V has been written in **bold face** to remind us that here we are dealing with a **vector** quantity that has both magnitude (the speed in mph) and a sense of direction: **+V** in the +x-direction and **–V** in the –x-direction.

3.5 Problems with light

We are accustomed to the notion that waves propagate through a medium, required to support the waves. For example, sound waves propagate as pressure variations in air, and water waves propagate as coupled displacements of the water molecules, perpendicular to the direction of the wave motion. In the 19th-century, Maxwell discovered that light waves are electromagnetic phenomena. This great work was based on theoretical arguments, motivated by the experimental results of Faraday and Henry. One of the most pressing questions facing scientists of the day was:

"what is waving when a beam of light propagates through empty space?"

It was proposed that the universe is filled with a medium called the *aether* with the property of supporting light waves, and having no other physical attributes. (For example, it would have no effect on the motion of celestial bodies). In the latter part of the 19th-century, Michelson and Morley carried out a famous experiment at the Case Institute in Cleveland that showed there is *no experimental evidence for the aether*. Light travels through the void, and that is that. Implicit in their work was the counter-intuitive notion that the speed of light does not depend on the speed of the source of the light.

The Aether Theory was popular for many years. Non-traditional theories were proposed to account for the null-result of the Michelson-Morley experiment. Fitzgerald (Trinity College, Dublin) proposed that the Michelson-Morley result could be explained, and the Aether Hypothesis retained, if the lengths of components in their apparatus were "velocity-dependent" – lengths contract in the direction of motion, and lengths remain unchanged when perpendicular to the direction of motion. He obtained the result

$$L_0 \quad = \quad [1/\sqrt{(1 - (v/c)^2)}]\, L \quad \equiv \quad \gamma L$$

⇑ ⇑

(length of rod at rest) (length of rod moving at speed v)

Here, c is the constant speed of light ($2.99 \ldots \times 10^8$ meters/second).

All experiments are consistent with the statement that the ratio v/c is always less than 1, and therefore γ is always greater than 1. This means that the measured length of the rod L_0, in its rest frame, is always greater than its measured length when moving.

At the end of the 19th-century, Larmor introduced yet another radical idea: a moving clock is observed to tick more slowly than an identical clock at rest. Furthermore, the relationship between the clock rates in the moving and rest frames is given by the same factor, γ, introduced previously by Fitzgerald. Specifically,

$$\Delta t \quad = \quad \gamma \Delta t_0$$

⇑ ⇑

(an interval on a moving clock) (an interval on a clock at rest)