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Inconsistency Solution of Maxwell's Equations

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Annotation

A new solution of Maxwell equations for a vacuum, for wire with constant and alternating current, for the capacitor, for the sphere, etc. is presented. First it must be noted that the proof of the solution's uniqueness is based on the Law of energy conservation which is not observed (for instantaneous values) in the known solution.
The solution offered:

- Describes wave in vacuum and wave in wire;
- Complies with the energy conservation law in each moment of time, i.e. sets constant density of electromagnetic energy flux;
- Reveals phase shifting between electrical and magnetic intensities;
- Explains existence of energy flux along the wire that is equal to the power consumed.

The work offers some technical applications of the solution obtained. A detailed proof is given for interested readers.

Contents

Preface

Contents

1. Introduction

"To date, whatsoever effect that would request a modification of Maxwell's equations escaped detection" [36]. Nevertheless, recently criticism of validity of Maxwell equations is heard from all sides. Have a look at the Fig.1 that shows a wave being a known solution of Maxwell's equations. The confidence of critics is created first of all by the violation of the Law of energy conservation. And certainly "*the density of electromagnetic energy flow (the module of Umov-Pointing vector) pulsates harmonically. Doesn't it violate the Law of energy conservation?*" [1]. Certainly, it is violated, **if** the electromagnetic wave satisfies the **known solution** of Maxwell equations. But there is no other solution: "*The proof of solution's uniqueness in general is as follows. If there are two different solutions, then their difference due to the system's linearity, will also be a solution, but for zero charges and currents and for zero initial conditions. Hence, using the expression for electromagnetic field energy we must conclude that the difference between solutions is equal to zero, which means that the solutions are identical. Thus the uniqueness of Maxwell equations solution is proved*" [2]. So, <u>the uniqueness of solution is being proved on the base of using the law which is violated in this solution.</u>

Another result following from the existing solution of Maxwell equations is phase synchronism of electrical and magnetic components of intensities in an electromagnetic wave. This is contrary to the idea of constant transformation of electrical and magnetic components of energy in an electromagnetic wave. In [1[, for example, this fact is called "one of the vices of the classical electrodynamics".

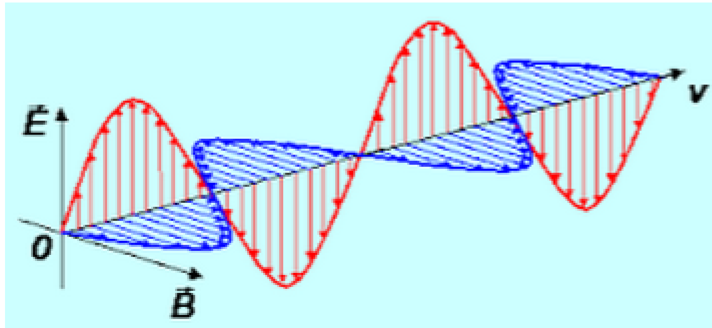

Рис. 1.

Such results following from the known solution of Maxwell equations allow doubting the authenticity of Maxwell equations. However, we must stress that these results follow **only from the found solution**. But this solution, as has been stated above, can be different (in their partial derivatives, equations generally have several solutions).

Further we shall deduct **another solution** of Maxwell equation, in which the density of electromagnetic energy flow remains constant in time, and electrical and magnetic components of intensities in the electromagnetic wave are shifted in in phase.

In addition, consider an electromagnetic wave in wire. With an assumed negligibly low voltage, Maxwell's equations for this wave literally coincide with those for the wave in vacuum. Yet, electrical engineering eludes any known solution and employs the one that connects an intensity of the circular magnetic field with the current in the wire (for brevity, it will be referred to as "electrical engineering solution"). This solution, too, satisfies the Maxwell's equations. However, firstly, it is one more solution of those equations (which invalidates the theorem of the only solution known). Secondly, and the most important, electrical engineering solution does not explain the famous experimental fact.

The case in point is skin-effect. Solution to explain skin-effect should contain a non-linear radius-to-displacement current (flowing along the wire) dependence. According to Maxwell's equations, such dependence should fit with radial and circular electrical and magnetic intensities that have non-linear dependence from the radius. Electrical engineering solution offers none of these. Explanation of skin-effect bases on the Maxwell's equations, yet it does not follow from electrical engineering solution. It allows the statement that electrical engineering solution does not explain the famous experimental fact.

2. On Energy Flux in Wire

Now, refer to energy flux in wire. The existing idea of energy transfer through the wires is that the energy in a certain way is spreading outside the wire [13]: *"… so our "crazy" theory says that the electrons are getting their energy to generate heat because of the energy flowing into the wire from the field outside. Intuition would seem to tell us that the electrons get their energy from being pushed along the wire, so the energy should be flowing down (or up) along the wire. But the theory says that the electrons are really being pushed by an electric field, which has come from some charges very far away, and that the electrons get their energy for generating heat from these fields. The energy somehow flows from the distant charges into a wide area of space and then inward to the wire."*

Such theory contradicts the Law of energy conservation. Indeed, the energy flow, travelling in the space must lose some part of the energy. But this fact was found neither experimentally, nor theoretically. But, most important, this theory contradicts the following experiment. Let us assume that through the central wire of coaxial cable runs constant current. This wire is isolated from the external energy flow. Then whence the energy flow compensating the heat losses in the wire comes? With the exception of loss in wire, the flux should penetrate into a load, e.g. winding of electrical motors covered with steel shrouds of the stator. This matter is omitted in the discussions of the existing theory.

So, the existing theory claims that the incoming (perpendicularly to the wire) electromagnetic flow permits the current to overcome the resistance to movement and performs work that turns into heat. This known conclusion veils the natural question: how can the current attract the flow, if the current appears due to the flow? It is natural to assume that the flow creates a certain emf which "moves the current". Meanwhile, energy flux of the electromagnetic wave exists in the wave itself and does not use space exterior towards the wave.

Solution of Maxwell's equations should model a structure of the electromagnetic wave with electromagnetic flux energy presenting in it.

The intuition Feynman speaks of has been well founded. The author proves it further while restricted himself to Maxwell's equations.

3. Requirements for Consistent Solution of Maxwell's Equations

Thus, the solution of Maxwell's equations must:

- describe wave in vacuum and wave in wire;

- comply with the energy conservation law in each moment of time, i.e. set constant density of electromagnetic energy flux;
- reveal phase shifting between electrical and magnetic intensities;
- explain existence of energy flux along the wire that is equal to power consumed.

What follows is an appropriate derivation of Maxwell's equations.

4. Variants of Maxwell's Equations

Further, we separate different special cases (alternatives) of Maxwell's equations system numbered for convenience of presentation.

Variant 1.

Maxwell's equations in the general case in the GHS system are of the form [3]:

$$\mathrm{rot}(E) + \frac{\mu}{c}\frac{\partial H}{\partial t} = 0, \tag{1}$$

$$\mathrm{rot}(H) - \frac{\varepsilon}{c}\frac{\partial E}{\partial t} - \frac{4\pi}{c} I = 0, \tag{2}$$

$$\mathrm{div}(E) = 0, \tag{3}$$

$$\mathrm{div}(H) = 0, \tag{4}$$

$$I = \sigma E, \tag{5}$$

where

I, H, E - conduction current, magnetic and electric intensitions respectively,

ε, μ, σ - dielectric constant, magnetic permeability, conductivity wire material.

Variant 2.

For the vacuum must be taken $\varepsilon = 1$, $\mu = 1$, $\sigma = 0$. When the system of equations (1-5) takes the form:

$$\mathrm{rot}(E) + \frac{1}{c}\frac{\partial H}{\partial t} = 0, \tag{6}$$

$$\mathrm{rot}(H) - \frac{1}{c}\frac{\partial E}{\partial t} = 0, \tag{7}$$

$$\mathrm{div}(E) = 0, \tag{8}$$

$$\mathrm{div}(H) = 0. \tag{9}$$

The solution to this system is offered in the **Chapter 1.**

Variant 3.

Consider the case 1 in the complex presentation:

$$\mathrm{rot}(E)+i\omega\frac{\mu}{c}H=0, \tag{10}$$

$$\mathrm{rot}(H)-i\omega\frac{\varepsilon}{c}E-\frac{4\pi}{c}\left(\mathrm{real}(I)+i\cdot\mathrm{imag}(I)\right)=0, \tag{11}$$

$$\mathrm{div}(E)=0, \tag{12}$$

$$\mathrm{div}(H)=0, \tag{13}$$

$$\mathrm{real}(I)=\sigma\cdot\mathrm{abs}(E). \tag{14}$$

It should be noted that instead of showing the whole current, (14) shows only its real component, i.e. conductivity current. Imaginary component formed by a displacement current does not depend on electrical charges.

The solution to this system is offered in the **Chapter 4.**

Variant 4.

For the wire with *sinusoidal current* I flowing out of an external source, $\mathrm{real}(I)$ may at times be excluded from equations (11-14). It is possible for a low-resistance wire and for a dielectric wire (for more details, refer to Chapter 2). As this takes place, the system (11-14) takes the form of

$$\mathrm{rot}(E)+\frac{\mu}{c}\frac{\partial H}{\partial t}=0, \tag{15}$$

$$\mathrm{rot}(H)-\frac{\varepsilon}{c}\frac{\partial E}{\partial t}-\frac{4\pi}{c}I=0, \tag{16}$$

$$\mathrm{div}(E)=0, \tag{17}$$

$$\mathrm{div}(H)=0. \tag{18}$$

It is significant that current I is not a conductivity current even when it flows along the conductor.

The solution for this system will be considered in the **Chapter 2.**

Variant 5.

For a constant current wire, system in alternative 1 simplifies due to lack of time derivative and takes the form of:

$$\mathrm{rot}(E)=0, \tag{21}$$

$$\mathrm{rot}(H)-\frac{4\pi}{c}I=0, \tag{22}$$

$$\mathrm{div}(E)=0, \tag{24}$$

$$\mathrm{div}(H)=0, \tag{25}$$

$$I = \sigma E \tag{26}$$

or

Variant 6.

$$\mathrm{rot}(I) = 0\,, \tag{27}$$

$$\mathrm{rot}(H) - \frac{4\pi}{c} I = 0\,, \tag{28}$$

$$\mathrm{div}(I) = 0\,, \tag{29}$$

$$\mathrm{div}(H) = 0\,. \tag{30}$$

The solution for this system will be considered in the **Chapter 3.**

We will be searching a monochromatic solution of the systems mentioned. A transition to polychromatic solution can be accomplished via Fourier transformation.

We will employ cylindrical system of coordinates $r,\ \varphi,\ z$ - see Appendix 1. Obviously, if solution exists in the cylindrical system of coordinates, it exists in any other system of coordinates, too.

Apppendix 1. Cylindrical Coordinates

As it is known to [4], in cylindrical coordinates scalar divergence of H vector, vector gradient of scalar function $a(x, y, z)$, vector rotor of H vector, accordingly, take the form of

$$\mathrm{div}(H) = \left(\frac{H_r}{r} + \frac{\partial H_r}{\partial r} + \frac{1}{r} \cdot \frac{\partial H_\varphi}{\partial \varphi} + \frac{\partial H_z}{\partial z} \right), \tag{a}$$

$$\mathrm{grad}_r(a) = \frac{\partial a}{\partial r},\quad \mathrm{grad}_\varphi(a) = \frac{1}{r} \cdot \frac{\partial a}{\partial \varphi},\quad \mathrm{grad}_z(a) = \frac{\partial a}{\partial z}, \tag{b}$$

$$\mathrm{rot}_r(H) = \left(\frac{1}{r} \cdot \frac{\partial H_z}{\partial \varphi} - \frac{\partial H_\varphi}{\partial z} \right), \tag{c}$$

$$\mathrm{rot}_\varphi(H) = \left(\frac{\partial H_r}{\partial z} - \frac{\partial H_z}{\partial r} \right), \tag{d}$$

$$\mathrm{rot}_z(H) = \left(\frac{H_\varphi}{r} + \frac{\partial H_\varphi}{\partial r} - \frac{1}{r} \cdot \frac{\partial H_r}{\partial \varphi} \right). \tag{e}$$

Apppendix 2. Spherical Coordinates

Fig. 1 shows a system of spherical coordinates ρ, θ, φ, and Table 1 contains expressions for rotor and divergence of vector **E** in these coordinates [4].

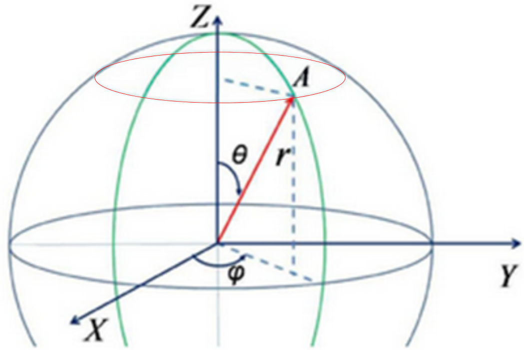

Fig. 1.

Table 1.

1	2	3
1	$\mathrm{rot}_\rho(E)$	$\frac{E_\varphi}{\rho \mathrm{tg}(\theta)} + \frac{\partial E_\varphi}{\rho \partial \theta} - \frac{\partial E_\theta}{\rho \sin(\theta) \partial \varphi}$
2	$\mathrm{rot}_\theta(E)$	$\frac{\partial E_\rho}{\rho \sin(\theta) \partial \varphi} - \frac{E_\varphi}{\rho} - \frac{\partial E_\varphi}{\partial \rho}$
3	$\mathrm{rot}_\varphi(E)$	$\frac{E_\theta}{\rho} + \frac{\partial E_\theta}{\partial \rho} - \frac{\partial E_\rho}{\rho \partial \varphi}$
4	$\mathrm{div}(E)$	$\frac{E_\rho}{\rho} + \frac{\partial E_\rho}{\partial \rho} + \frac{E_\theta}{\rho \mathrm{tg}(\theta)} + \frac{\partial E_\theta}{\rho \partial \theta} + \frac{\partial E_\varphi}{\rho \sin(\theta) \partial \varphi}$

Apppendix 3. Some Correlations Between GHS and SI Systems

Further, formulas appear in GHS system, yet, for illustration, some examples are shown in SI system. This is why, for reader's convenience, Table 1 contains correlations between some measurement units of these systems.

Table 1.

Name	GHS	SI
electric current	1 GHS	$3{,}33 \cdot 10^{-10}$ A
voltage	1 GHS	$3 \cdot 10^{2}$ V
power, energy flux density	1 GHS	10^{-7} Wt
energy flux density per unit length of wire	1 GHS	10^{-5} Wt/m
electric current density	1 GHS	$3.33 \cdot 10^{-6}$ A/m^2 $3.33 \cdot 10^{-12}$ A/mm^2
electric field intensity	1 GHS	$3 \cdot 10^{4}$ V/m
magnetic field intensity	1 GHS	80 A/m
magnetic induction	1 GHS	10^{-4}T
absolute dielectric permittivity	1 GHS	$8.85 \cdot 10^{-12}$ F/m
absolute magnetic permeability	1 GHS	$1.26 \cdot 10^{-8}$ H/m
capacitance	1 GHS	$1.1 \cdot 10^{-12}$ F
inductance	1 GHS	10^{-9} H
electrical resistance	1 GHS	$9 \cdot 10^{11}$ Om
electrical conductivity	1 GHS	$1.1 \cdot 10^{-12}$ sm
specific electrical resistance	1 GHS	$9 \cdot 10^{9}$ Om·m
specific electrical conductivity	1 GHS	$1.1 \cdot 10^{-10}$ sm/m

Chapter 1. The Second Solution of Maxwell's Equations for vacuum

Contents

1. Introduction

In Chapter "Introduction" inconsistency of well-known solution of Maxwell's equations was demonstrated. A new solution Maxwell's equations for vacuum is proposed below [5].

2. Solution of Maxwell's Equations

First we shall consider the solution of Maxwell equation for vacuum, which is shown in Chapter "Introduction" as variant 1, and takes the following form

$$\mathrm{rot}(E)+\frac{1}{c}\frac{\partial H}{\partial t}=0,$$

$$\mathrm{rot}(H)-\frac{1}{c}\frac{\partial E}{\partial t}=0,$$

$$\mathrm{div}(E)=0,$$

$$\mathrm{div}(H)=0.$$

In cylindrical coordinates system $r,\ \varphi,\ z$ these equations look as follows:

$$\frac{E_r}{r}+\frac{\partial E_r}{\partial r}+\frac{1}{r}\cdot\frac{\partial E_\varphi}{\partial \varphi}+\frac{\partial E_z}{\partial z}=0, \tag{1}$$

$$\frac{1}{r}\cdot\frac{\partial E_z}{\partial \varphi}-\frac{\partial E_\varphi}{\partial z}=M_r, \tag{2}$$

$$\frac{\partial E_r}{\partial z} - \frac{\partial E_z}{\partial r} = M_\varphi, \tag{3}$$

$$\frac{E_\varphi}{r} + \frac{\partial E_\varphi}{\partial r} - \frac{1}{r} \cdot \frac{\partial E_r}{\partial \varphi} = M_z, \tag{4}$$

$$\frac{H_r}{r} + \frac{\partial H_r}{\partial r} + \frac{1}{r} \cdot \frac{\partial H_\varphi}{\partial \varphi} + \frac{\partial H_z}{\partial z} = 0, \tag{5}$$

$$\frac{1}{r} \cdot \frac{\partial H_z}{\partial \varphi} - \frac{\partial H_\varphi}{\partial z} = J_r, \tag{6}$$

$$\frac{\partial H_r}{\partial z} - \frac{\partial H_z}{\partial r} = J_\varphi, \tag{7}$$

$$\frac{H_\varphi}{r} + \frac{\partial H_\varphi}{\partial r} - \frac{1}{r} \cdot \frac{\partial H_r}{\partial \varphi} = J_z, \tag{8}$$

$$J = \frac{1}{c}\frac{\partial E}{\partial t}, \tag{9}$$

$$M = -\frac{1}{c}\frac{\partial H}{\partial t}. \tag{10}$$

For the sake of brevity further we shall use the following notations:

$$co = \cos(\alpha\varphi + \chi z + \omega t), \tag{11}$$

$$si = \sin(\alpha\varphi + \chi z + \omega t), \tag{12}$$

where α, χ, ω – are certain constants. Let us present the unknown functions in the following form:

$$J_r. = j_r(r)co, \tag{13}$$

$$J_\varphi. = j_\varphi(r)si, \tag{14}$$

$$J_z. = j_z(r)si, \tag{15}$$

$$H_r. = h_r(r)co, \tag{16}$$

$$H_\varphi. = h_\varphi(r)si, \tag{17}$$

$$H_z. = h_z(r)si, \tag{18}$$

$$E_r. = e_r(r)si, \tag{19}$$

$$E_\varphi. = e_\varphi(r)co, \tag{20}$$

$$E_z. = e_z(r)co, \tag{21}$$

$$M_r. = m_r(r)co, \tag{21}$$

$$M_\varphi. = m_\varphi(r)si, \tag{22}$$

$$M_z. = m_z(r)si, \tag{23}$$

where $j(r)$, $h(r)$, $e(r)$, $m(r)$ - certain function of the coordinate r.

By direct substitution we can verify that the functions (13-23) transform the equations system (1-10) with three arguments $r,\ \varphi,\ z$ into equations system with one argument r and unknown functions $j(r),\ h(r),\ e(r),\ m(r)$.

In Appendix 1 it is shown that for such a system there **exists** a solution of the following form (in Appendix 1 see (24, 27, 18, 31, 33, 34, 32) respectively):

$$h_z(r) = 0\,,\ e_z(r) = 0\,. \tag{24}$$

$$e_r = e_\varphi = \frac{A}{2} r^{-(1-\alpha)}, \tag{25}$$

$$h_\varphi(r) = e_r(r). \tag{26}$$

$$h_r(r) = -e_\varphi(r), \tag{27}$$

$$\chi = \omega/c\,. \tag{28}$$

where $A, c, \alpha, \chi, \omega$ – constants.

Thus we have got a monochromatic solution of the equation system (1-10). A transition to polychromatic solution can be achieved with the aid of Fourier transform.

If it exists in cylindrical coordinate system, then it exists in any other coordinate system. It means that we have got a common solution of Maxwell equations in vacuum.

3. Intensities

We consider (2.25):

$$e_r = e_\varphi = 0.5A \cdot r^{\alpha-1}, \tag{1}$$

where ($A\backslash 2$) - the amplitude of the intensities. From (1) it follows that

$$\left(e_r^2 + e_\varphi^2\right) = A \cdot r^{2(\alpha-1)}. \tag{2}$$

Fig. 1 shows, for example, the graphics functions (1, 2) for $A = -1,\ \alpha = 0.8$.

Fig. 2 shows the vectors of intensities originating from the point $A(r, \varphi)$. Let us remind that $h_\varphi(r) = e_r(r)$ and $h_r(r) = -e_\varphi(r)$ - see (2.28, 2.29). The directions of vectors $e_r(r)$ and $e_\varphi(r)$ are chosen as: $e_r(r) > 0$, $e_\varphi(r) < 0$. Note that the vectors $E,\ H$ are always orthogonal. The sum of the modules of these vectors is determined from (2.17, 2.18, 2.20, 2.21, 2.26, 2.27) and is equal to

$$W = E^2 + H^2 = \left(e_r(r)si\right)^2 + \left(e_\varphi(r)si\right)^2 + \left(h_r(r)co\right)^2 + \left(h_\varphi(r)co\right)^2$$

or

$$W = (e_r(r))^2 + (e_\varphi(r))^2 \qquad (3)$$

- see also (10) and Fig. 1. Thus, the density of electromagnetic wave energy is constant in all points of a circle of this radius.

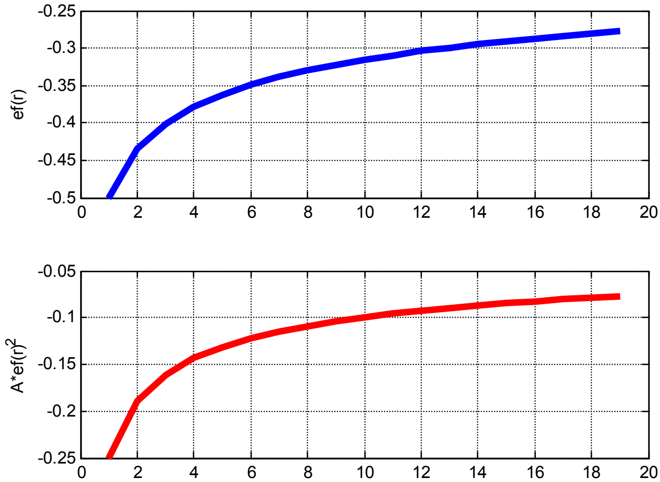

Fig.1. SecondSolMax.m

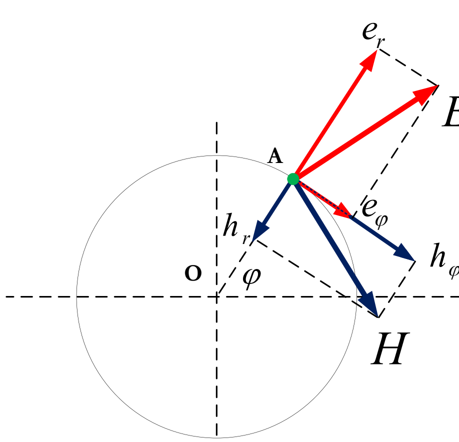

Fig. 2.

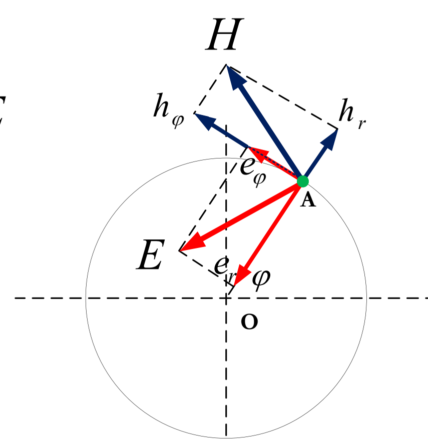

Fig. 3.

The solution exists also for changed signs of the functions (2.11, 2.21). This case is shown on Fig 3. Fig. 2 and Fig. 3 illustrate the fact that there are two possible type of electromagnetic wave circular polarization.

In order to demonstrate phase shift between the wave components let's consider the functions (2.11, 2.12) and (2.16-2.21). It can be seen, that at each point with coordinates r, φ, z intensities H, E are shifted in phase by a quarter-period.

Let's consider the functions (2.11, 2.12) and (2.28). Then, we can find

$$co = \cos(\alpha\varphi + \frac{\omega}{c} z + \omega t), \quad si = \sin(\alpha\varphi + \frac{\omega}{c} z + \omega t). \tag{4}$$

Let's consider a point moving along a cylinder of constant radius r, at which the value of intensity depends on time as follows:

$$H_{r}. = h_r(r)\cos(\omega t) \tag{5}$$

Comparing this equation with (2.16) and taking (4) into account, we can notice that equation (7) is the same as (2.16), if at any moment of time

$$\alpha\varphi + \frac{\omega}{c} z = 0 \tag{6}$$

or

$$\varphi = -\frac{\omega}{\alpha \cdot c} z. \tag{7}$$

Path of the point described by equations (4, 7, 2.28) is a helix. Thus, the line, along which the point moves in such a way, that its intensity varies in a sinusoidal manner, is determined by the equation describing a helix. The same conclusion can be repeated for other intensities (2.17-2.21). Thus,

> path of the point, which moves along a cylinder of given radius in such a manner, that each intensity value varies harmonically with time, is described by a helix.

For example, Fig. 4 shows a helix, for which
$r = 1,\ c = 300000,\ \omega = 3000,\ \alpha = 3,\ \varphi = [0 \div 2\pi]$.

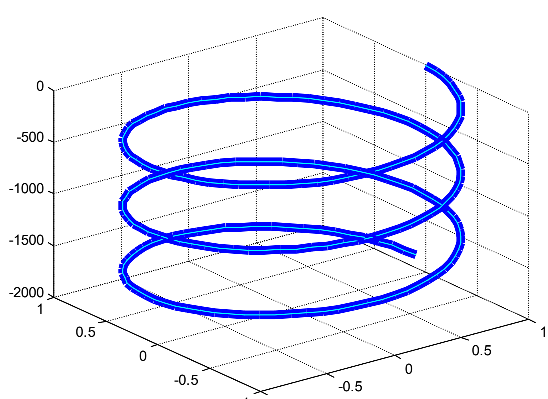

Fig. 4.

The last means that at point T, moving along this helix the vectors of intensities (2.16-2.21) can be written as follows:

$$H_{r}. = h_{r}(r)\cos(\omega t),\ H_{\varphi}. = h_{\varphi}(r)\sin(\omega t),\ H_{z}. = h_{z}(r)\sin(\omega t),$$
$$E_{r}. = e_{r}(r)\sin(\omega t),\ \ E_{\varphi}. = e_{\varphi}(r)\cos(\omega t),\ \ E_{z}. = e_{z}(r)\cos(\omega t).$$

It was shown above (see 2.24-2.27), that
$h_z(r)=0$, $e_z(r)=0$, $e_r(r)=e_\varphi(r)=e_{r\varphi}(r)$, $h_\varphi(r)=e_{r\varphi}(r)$, $h_r(r)=-e_{r\varphi}(r)$.
Therefore, at each point there are only vectors

$$H_{r}. = -e_{r\varphi}(r)\cos(\omega t),\ H_{\varphi}. = e_{r\varphi}(r)\sin(\omega t),$$
$$E_{r}. = e_{r\varphi}(r)\sin(\omega t),\quad E_{\varphi}. = e_{r\varphi}(r)\cos(\omega t).$$

In this case resultant vectors $\mathrm{H}_{r\varphi}=\mathrm{H}_r+\mathrm{H}_\varphi$ and $\mathrm{E}_{r\varphi}=\mathrm{E}_r+\mathrm{E}_\varphi$ lay in plane r, φ, and their moduli are $|H_{r\varphi}|=e_{r\varphi}(r)$ and $|E_{r\varphi}|=e_{r\varphi}(r)$. Fig. 4a shows all these vectors. It can be seen, that when the point T moves along the helix, resultant vectors $\mathrm{H}_{r\varphi}$ and $\mathrm{E}_{r\varphi}$ rotate in plane r, φ. Their moduli are constant and equal one to the other. These vectors $\mathrm{H}_{r\varphi}$ and $\mathrm{E}_{r\varphi}$ are always orthogonal.

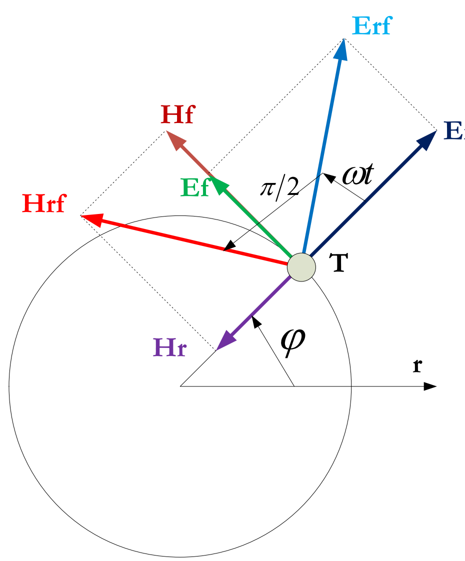

Fig. 4a.

So, at each point T, which moves along this helix, vectors of magnetic and electric intensities:

- exist only in the plane which is perpendicular to the helix axis, i.e. there only two projections of these vectors exist,
- vary in a sinusoidal manner,
- are shifted in phase by a quarter-period.