Amusements in Mathematics

by

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Units Abbreviation and Conversion

The most common units used were:

the Penny,	abbreviated: *d*. (from the Roman penny, denarius)
the Shilling,	abbreviated: *s*.
the Pound,	abbreviated: £

There was 12 Pennies to a Shilling and 20 Shillings to a Pound, so there was 240 Pennies in a Pound.

To further complicate things, there were many coins which were various fractional values of Pennies, Shillings or Pounds.

Farthing	¼*d*.
Half-penny	½*d*.
Penny	1*d*.
Three-penny	3*d*.
Sixpence (or tanner)	6*d*.
Shilling (or bob)	1*s*.
Florin or two shilling piece	2*s*.
Half-crown (or half-dollar)	2*s*. 6*d*.
Double-florin	4*s*.
Crown (or dollar)	5*s*.
Half-Sovereign	10*s*.
Sovereign (or Pound)	£1 or 20*s*.

Preface

In Mathematicks he was greater
Than Tycho Brahe or Erra Pater:
For he, by geometrick scale,
Could take the size of pots of ale;
Resolve, by sines and tangents, straight,
If bread or butter wanted weight;
And wisely tell what hour o' th' day
The clock does strike by algebra.
BUTLER'S *Hudibras*.

In issuing this volume of my Mathematical Puzzles, of which some have appeared in periodicals and others are given here for the first time, I must acknowledge the encouragement that I have received from many unknown correspondents, at home and abroad, who have expressed a desire to have the problems in a collected form, with some of the solutions given at greater length than is possible in magazines and newspapers. Though I have included a few old puzzles that have interested the world for generations, where I felt that there was something new to be said about them, the problems are in the main original. It is true that some of these have become widely known through the press, and it is possible that the reader may be glad to know their source.

On the question of Mathematical Puzzles in general there is, perhaps, little more to be said than I have written elsewhere. The history of the subject entails nothing short of the actual story of the beginnings and development of exact thinking in man. The historian must start from the time when man first succeeded in counting his ten fingers and in dividing an apple into two approximately equal parts. Every puzzle that is worthy of consideration can be referred to mathematics and logic. Every man, woman, and child who tries to "reason out" the answer to the simplest puzzle is working, though not of necessity consciously, on mathematical lines. Even those puzzles that we have no way of attacking except by haphazard attempts can be brought under a method of what has been called "glorified trial"—a system of shortening our labours by avoiding or eliminating what our reason tells us is useless. It is, in fact, not easy to say sometimes where the "empirical" begins and where it ends.

Arithmetical And Algebraical Problems

"And what was he?
Forsooth, a great arithmetician."
Othello, I. i.

The puzzles in this department are roughly thrown together in classes for the convenience of the reader. Some are very easy, others quite difficult. But they are not arranged in any order of difficulty—and this is intentional, for it is well that the solver should not be warned that a puzzle is just what it seems to be. It may, therefore, prove to be quite as simple as it looks, or it may contain some pitfall into which, through want of care or over-confidence, we may stumble.

Also, the arithmetical and algebraical puzzles are not separated in the manner adopted by some authors, who arbitrarily require certain problems to be solved by one method or the other. The reader is left to make his own choice and determine which puzzles are capable of being solved by him on purely arithmetical lines.

MONEY PUZZLES.

"Put not your trust in money, but put your money in trust."
OLIVER WENDELL HOLMES.

1.—A POST-OFFICE PERPLEXITY.

In every business of life we are occasionally perplexed by some chance question that for the moment staggers us. I quite pitied a young lady in a branch post-office when a gentleman entered and deposited a crown on the counter with this request: "Please give me some twopenny stamps, six times as many penny stamps, and make up the rest of the money in twopence-halfpenny stamps." For a moment she seemed bewildered, then her brain cleared, and with a smile she handed over stamps in exact fulfilment of the order. How long would it have taken you to think it out?

2.—YOUTHFUL PRECOCITY.

The precocity of some youths is surprising. One is disposed to say on occasion, "That boy of yours is a genius, and he is certain to do great things when he grows up;" but past experience has taught us that he invariably becomes quite an ordinary citizen. It is so often the case, on the contrary, that the dull boy becomes a great man. You never can tell. Nature loves to present to us these queer paradoxes. It is well known that those wonderful "lightning calculators," who now and again surprise the world by their feats, lose all their mysterious powers directly they are taught the elementary rules of arithmetic.

A boy who was demolishing a choice banana was approached by a young friend, who, regarding him with envious eyes, asked, "How much did you pay for that banana, Fred?" The prompt answer was quite remarkable in its way: "The man what I bought it of receives just half as many sixpences for sixteen dozen dozen bananas as he gives bananas for a fiver."

Now, how long will it take the reader to say correctly just how much Fred paid for his rare and refreshing fruit?

3.—AT A CATTLE MARKET.

Three countrymen met at a cattle market. "Look here," said Hodge to Jakes, "I'll give you six of my pigs for one of your horses, and then you'll have twice as many animals here as I've got." "If that's your way of doing business," said Durrant to Hodge, "I'll give you fourteen of my sheep for a horse, and then you'll have three times as many animals as I." "Well, I'll go better than that," said Jakes to Durrant; "I'll give you four cows for a horse, and then you'll have six times as many animals as I've got here."

No doubt this was a very primitive way of bartering animals, but it is an interesting little puzzle to discover just how many animals Jakes, Hodge, and Durrant must have taken to the cattle market.

4.—THE BEANFEAST PUZZLE.

A number of men went out together on a bean-feast. There were four parties invited—namely, 25 cobblers, 20 tailors, 18 hatters, and 12 glovers. They spent altogether £6, 13*s*. It was found that five cobblers spent as much as four tailors; that twelve tailors spent as much as nine hatters; and that six hatters spent as much as eight glovers. The puzzle is to find out how much each of the four parties spent.

5.—A QUEER COINCIDENCE.

Seven men, whose names were Adams, Baker, Carter, Dobson, Edwards, Francis, and Gudgeon, were recently engaged in play. The name of the particular game is of no consequence. They had agreed that whenever a player won a game he should double the money of each of the other players—that is, he was to give the players just as much money as they had already in their pockets. They played seven games, and, strange to say, each won a game in turn, in the order in which their names are given. But a more curious coincidence is this—that when they had finished play each of the seven men had exactly the same amount—two shillings and eightpence—in his pocket. The puzzle is to find out how much money each man had with him before he sat down to play.

6.—A CHARITABLE BEQUEST.

A man left instructions to his executors to distribute once a year exactly fifty-five shillings among the poor of his parish; but they were only to continue the gift so long as they could make it in different ways, always giving eighteenpence each to a number of women and half a crown each to men. During how many years could the charity be administered? Of course, by "different ways" is meant a different number of men and women every time.

7.—THE WIDOW'S LEGACY.

A gentleman who recently died left the sum of £8,000 to be divided among his widow, five sons, and four daughters. He directed that every son should receive three times as much as a daughter, and that every daughter should have twice as much as their mother. What was the widow's share?

8.—INDISCRIMINATE CHARITY.

A charitable gentleman, on his way home one night, was appealed to by three needy persons in succession for assistance. To the first person he gave one penny more than half the money he had in his pocket; to the second person he gave twopence more than half the money he then had in his pocket; and to the third person he handed over threepence more than half of what he had left. On entering his house he had only one penny in his pocket. Now, can you say exactly how much money that gentleman had on him when he started for home?

9.—THE TWO AEROPLANES.

A man recently bought two aeroplanes, but afterwards found that they would not answer the purpose for which he wanted them. So he sold them for £600 each, making a loss of 20 per cent, on one machine and a profit of 20 per cent, on the other. Did he make a profit on the whole transaction, or a loss? And how much?

10.—BUYING PRESENTS.

"Whom do you think I met in town last week, Brother William?" said Uncle Benjamin. "That old skinflint Jorkins. His family had been taking him around buying Christmas presents. He said to me, 'Why cannot the government abolish Christmas, and make the giving of presents punishable by law? I came out this morning with a certain amount of money in my pocket, and I find I have spent just half of it. In fact, if you will believe me, I take home just as many shillings as I had pounds, and half as many pounds as I had shillings. It is monstrous!'" Can you say exactly how much money Jorkins had spent on those presents?

11.—THE CYCLISTS' FEAST.

'Twas last Bank Holiday, so I've been told,Some cyclists rode abroad in glorious weather.Resting at noon within a tavern old,They all agreed to have a feast together."Put it all in one bill, mine host," they said,"For every man an equal share will pay."The bill was promptly on the table laid,And four pounds was the reckoning that day.But, sad to state, when they prepared to square,'Twas found that two had sneaked outside and fled.So, for two shillings more than his due shareEach honest man who had remained was bled.They settled later with those rogues, no doubt.How many were they when they first set out?

12.—A QUEER THING IN MONEY.

It will be found that £66, 6*s*. 6*d*. equals 15,918 pence. Now, the four 6's added together make 24, and the figures in 15,918 also add to 24. It is a curious fact that there is only one other sum of money, in pounds, shillings, and pence (all similarly repetitions of one figure), of which the digits shall add up the same as the digits of the amount in pence. What is the other sum of money?

13.—A NEW MONEY PUZZLE.

The largest sum of money that can be written in pounds, shillings, pence, and farthings, using each of the nine digits once and only once, is £98,765, 4*s*. 3½*d*. Now, try to discover the smallest sum of money that can be written down under precisely the same conditions. There must be some value given for each denomination—pounds, shillings, pence, and farthings—and the nought may not be used. It requires just a little judgment and thought.

14.—SQUARE MONEY.

"This is queer," said McCrank to his friend. "Twopence added to twopence is fourpence, and twopence multiplied by twopence is also fourpence." Of course, he was wrong in thinking you can multiply money by money. The multiplier must be regarded as an abstract number. It is true that two feet multiplied by two feet will make four square feet. Similarly, two pence multiplied by two pence will produce four square pence! And it will perplex the reader to say what a "square penny" is. But we will assume for the purposes of our puzzle that twopence multiplied by twopence is fourpence. Now, what two amounts of money will produce the next smallest possible result, the same in both cases, when added or multiplied in this manner? The two amounts need not be alike, but they must be those that can be paid in current coins of the realm.

15.—POCKET MONEY.

What is the largest sum of money—all in current silver coins and no four-shilling piece—that I could have in my pocket without being able to give change for a half-sovereign?

16.—THE MILLIONAIRE'S PERPLEXITY.

Mr. Morgan G. Bloomgarten, the millionaire, known in the States as the Clam King, had, for his sins, more money than he knew what to do with. It bored him. So he determined to persecute some of his poor but happy friends with it. They had never done him any harm, but he resolved to inoculate them with the "source of all evil." He therefore proposed to distribute a million dollars among them and watch them go rapidly to the bad. But he was a man of strange fancies and superstitions, and it was an inviolable rule with him never to make a gift that was not either one dollar or some power of seven—such as 7, 49, 343, 2,401, which numbers of dollars are produced by simply multiplying sevens together. Another rule of his was that he would never give more than six persons exactly the same sum. Now, how was he to distribute the 1,000,000 dollars? You may distribute the money among as many people as you like, under the conditions given.

17.—THE PUZZLING MONEY-BOXES.

Four brothers—named John, William, Charles, and Thomas—had each a money-box. The boxes were all given to them on the same day, and they at once put what money they had into them; only, as the boxes were not very large, they first changed the money into as few coins as possible. After they had done this, they told one another how much money they had saved, and it was found that if John had had 2*s*. more in his box than at present, if William had had 2*s*. less, if Charles had had twice as much, and if Thomas had had half as much, they would all have had exactly the same amount.

Now, when I add that all four boxes together contained 45*s*., and that there were only six coins in all in them, it becomes an entertaining puzzle to discover just what coins were in each box.

18.—THE MARKET WOMEN.

A number of market women sold their various products at a certain price per pound (different in every case), and each received the same amount—2*s*. 2½d. What is the greatest number of women there could have been? The price per pound in every case must be such as could be paid in current money.