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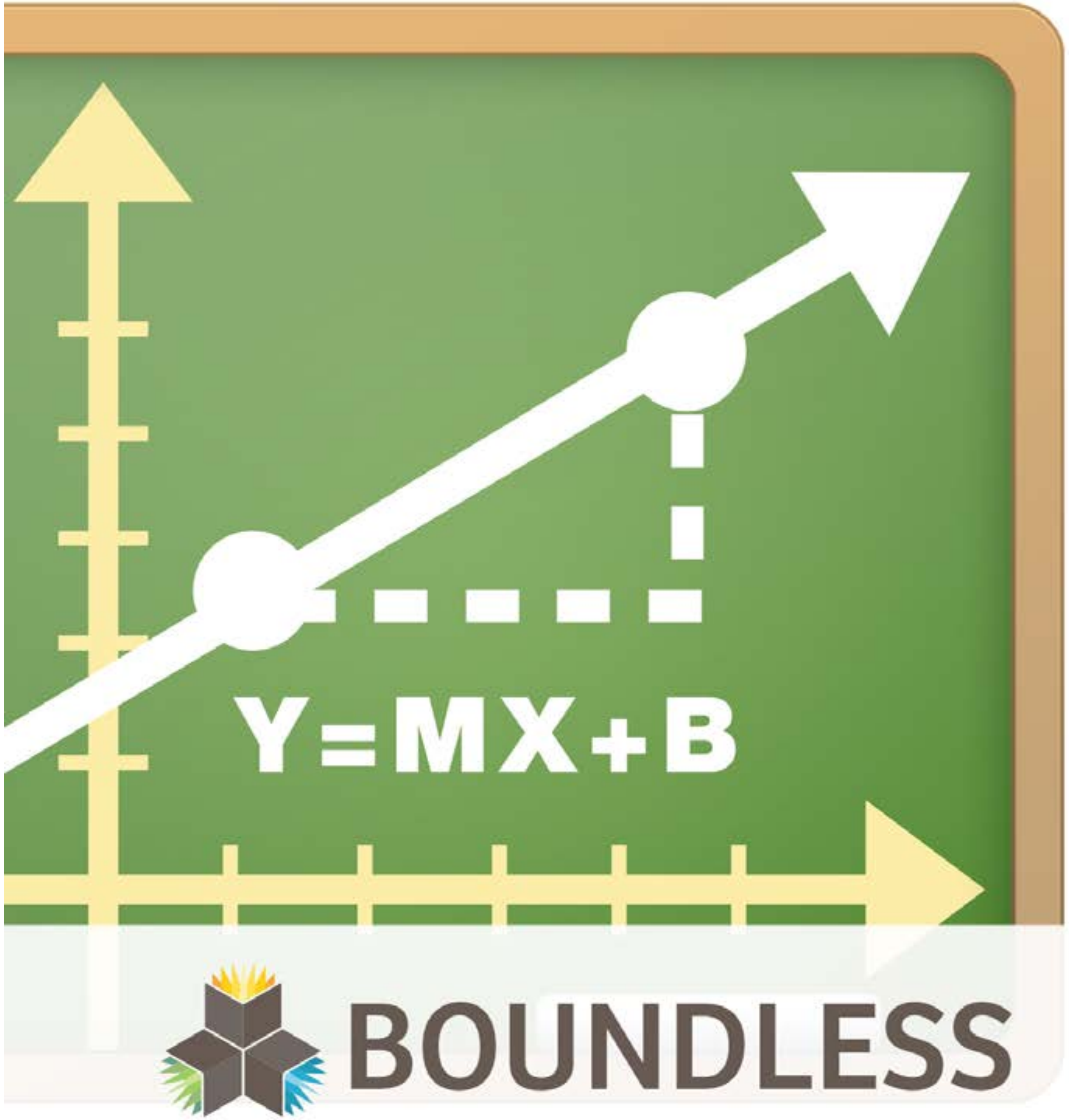
Algebra



Algebra

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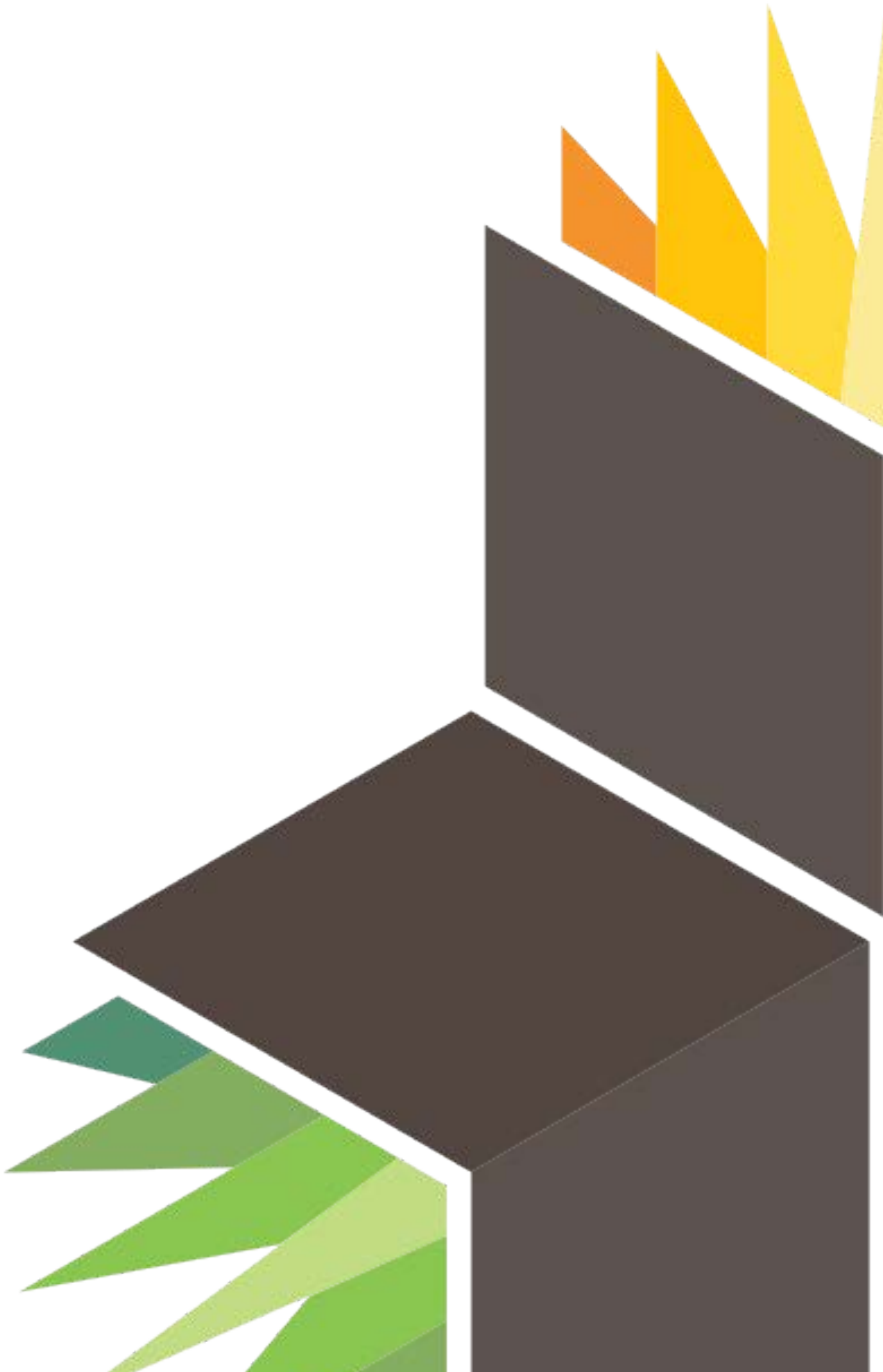
Chapter 1

The Building Blocks of Algebra



https://www.boundless.com/algebra/the-building-blocks-of-algebra/

Section 1

Real Numbers

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Real Numbers: Basic Operations

The basic arithmetic operations for real numbers are addition, subtraction, multiplication, and division.

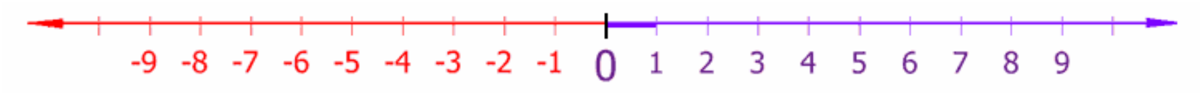
KEY POINTS

- A real number is a value that represents a quantity along a continuous line. Real numbers can be thought of as points on an infinitely long line called the number line or real line.
- The basic arithmetic operations for real numbers are addition, subtraction, multiplication, and division.
- Arithmetic operations are performed according to a specific hierarchy or order, not from left to right.

A real number is a value that represents a quantity along a continuous line. The real numbers include all the **rational numbers**, such as the integer −5 and the fraction 4/3, and all the irrational numbers, such as √2. Real numbers can be thought of as points on an infinitely long line called the number line (real line), where the points corresponding to integers are equally spaced as shown in *Figure 1.1*.

The basic arithmetic operations for real numbers are addition, subtraction, multiplication, and division. Arithmetic operations are performed according to a specific hierarchy or order, not from left to right.

Figure 1.1 Real Numbers



Real numbers can be thought of as points on an infinitely long line called the number line.

Addition and Subtraction

Addition is the basic operation of arithmetic. In its simplest form, addition combines two numbers into a single number. Adding more than two numbers can be viewed as repeated addition; this procedure is known as summation and includes ways to add infinitely many numbers in an infinite series. Addition is **commutative** and **associative**, so the order in which the terms are added does not affect their sum. The identity element of addition is 0; that is, adding zero to any number yields that same number.

Subtraction is the inverse of addition; it finds the difference between two numbers. As such, taking a number x, adding b to it and subsequently subtracting b from it affords the same number x. Subtraction is neither commutative nor associative.

Multiplication and Division

Multiplication also combines two numbers into a single number, the product. Multiplication is best viewed as a simplification of many additions. For example the product of x and y is the sum of x written out y times.

Multiplication is commutative and associative, and its identity is 1. That is, multiplying any number by 1 yields that same number. Division is the inverse of multiplication. Thus, taking a number x and multiplying it by b and then dividing it by b results in the same number x. Division is neither commutative nor associative.

Interval Notation

Intervals notation uses parentheses and brackets to describe sets of real numbers and their endpoints.

KEY POINTS

- A real interval is a set of real numbers with the property that any number that lies between two numbers in the set is also included in the set.
- The interval of numbers between a and b, including a and b, is often denoted [a, b]. The two numbers are called the endpoints of the interval.
- To indicate that one of the endpoints is to be excluded from the set, the corresponding square bracket can be either replaced with a parenthesis, or reversed. One may use an infinite endpoint to indicate that there is no bound in that direction.
- An open interval does not include its endpoints, and is indicated with parentheses. A closed interval includes its endpoints, and is denoted with square brackets.

A real **interval** is a set of real numbers with the property that any number that lies between two numbers in the set is also included in the set. For example, the set of all numbers x satisfying $0 \leq x \leq 1$ is an interval which contains 0 and 1, as well as all numbers between them. Other examples of intervals are the set of all real numbers and the set of all negative real numbers.

The interval of numbers between a and b, including a and b, is often denoted [a, b]. The two numbers are called the **endpoints** of the interval. To indicate that one of the endpoints is to be excluded from the set, the corresponding square bracket can be either replaced with a parenthesis, or reversed.

One may use an infinite endpoint to indicate that there is no bound in that direction. For example, $(0, +\infty)$ is the set of all positive real numbers, and $(-\infty, +\infty)$ is the set of real numbers.

An **open interval** does not include its endpoints, and is indicated with parentheses. For example (0,1) means greater than 0 and less than 1. A closed interval includes its endpoints, and is denoted with square brackets. For example [0,1] means greater than or equal to 0 and less than or equal to 1 (*Figure 1.2*).

An interval is said to be left-bounded or right-bounded if there is some real number that is, respectively, smaller than or larger than all its elements. An interval is said to be bounded if it is both left- and right-bounded, and is said to be unbounded otherwise. Intervals that are bounded at only one end are said to be half-bounded. The empty set is bounded, and the set of all reals is the only interval that is unbounded at both ends. Bounded intervals are also commonly known as finite intervals.

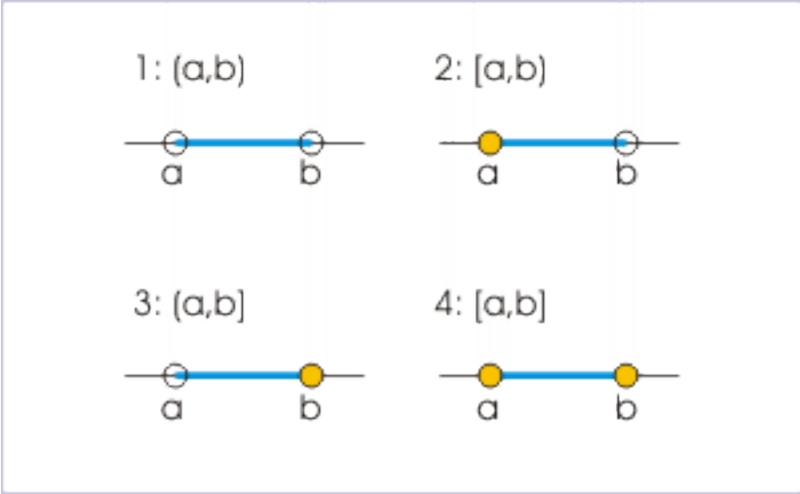


Figure 1.2 Intervals
Representation on real number line.

Source: https://www.boundless.com/algebra/the-building-blocks-of-algebra/real-numbers/interval-notation/

Equations, Inequalities, Properties

An equation states that two expressions are equal, while an inequality relates two different values.

KEY POINTS

- An equation is a mathematical statement that asserts the equality of two expressions.
- An inequality is a relation that holds between two values when they are different.
- The notation $a \neq b$ means that a is not equal to b. It does not say that one is greater than the other, or even that they can be compared in size. If one were to compare the size of the values, the notation $a < b$ means that a is less than b, while the notation $a > b$ means that a is greater than b.

Equations

An **equation** is a mathematical statement that asserts the equality of two expressions. This is written by placing the expressions on either side of an equals sign (=), for example:

$x + 3 = 5$

asserts that x + 3 is equal to 5 (*Figure 1.3*).

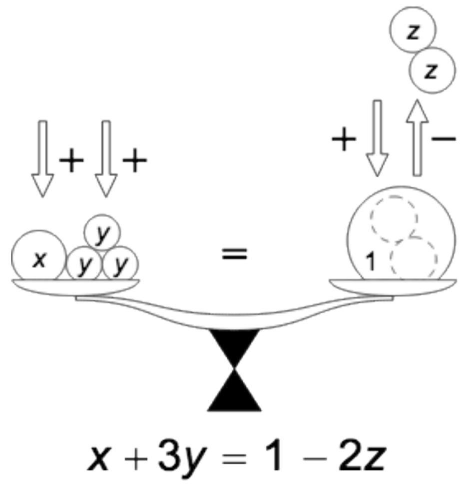


Figure 1.3
Equation as a Balance
Illustration of a simple equation as a balance. x, y, and z are real numbers, analogous to weights.

Equations often express relationships between given quantities—the knowns—and quantities yet to be determined—the **unknowns**. By convention, unknowns are denoted by letters at the end of the alphabet, x,y, z, w, ..., while knowns are denoted by letters at the beginning, a, b, c, d, ... The process of expressing the unknowns in terms of the knowns is called solving the equation. In an equation with a single unknown, a value of that unknown for which the equation is true is called a solution or root of the equation. In a set of simultaneous equations, or system of equations, multiple equations are given with multiple unknowns. A solution to the system is an assignment of values to all the unknowns so that all of the equations are true.

Inequalities

An **inequality** is a relation that holds between two values when they are different. The notation $a \neq b$ means that a is not equal to b.

It does not say that one is greater than the other, or even that they can be compared in size.

In either case, a is not equal to b. These relations are known as strict inequalities. To compare the size of the values, there are two types of relations:

- The notation $a < b$ means that a is less than b.
- The notation $a > b$ means that a is greater than b.

The notation $a < b$ may also be read as "a is strictly less than b".

In contrast to strict inequalities, there are two types of inequality relations that are not strict:

- The notation $a \le b$ means that a is less than or equal to b (or, equivalently, not greater than b, or at most b).
- The notation $a \ge b$ means that a is greater than or equal to b (or, equivalently, not less than b, or at least b)

Source: https://www.boundless.com/algebra/the-building-blocks-of-algebra/real-numbers/equations-inequalities-properties/

Introduction to Absolute Value

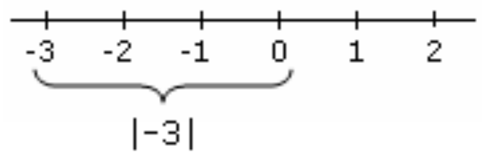
The absolute value may be thought of as the distance of a real number from zero (or the non-negative value without regard to its sign).

KEY POINTS

- The absolute value of a number may be thought of as its distance from zero along the real number line, and more generally, the absolute value of the difference of two real numbers is the distance between them.
- The absolute value $|a|$ of a real number a is the non-negative value of a without regard to its sign. Namely, $|a| = a$ for a positive a, $|a| = -a$ for a negative a, and $|0| = 0$.
- The absolute value of a is always either positive or zero but never negative.

The **absolute value** of a number may be thought of as its distance from zero (Figure 1.4). In mathematics, the absolute value (or **modulus**) $|a|$ of a real number a is the non-negative value of a

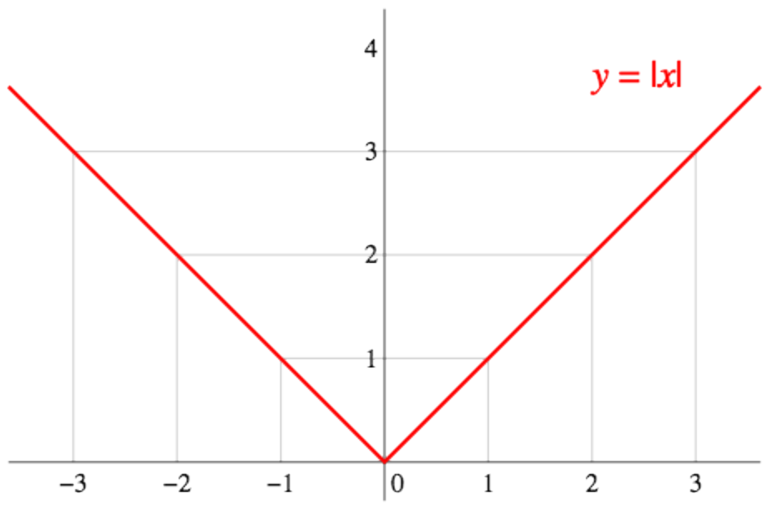
Figure 1.4 Absolute Value



The absolute value of a real number may be thought of as its distance from zero.

without regard to its sign (*Figure 1.5*). Namely, | a | = a for a positive a, | a | = −a for a negative a, and | 0 | = 0. For example, the absolute value of 3 is 3, and the absolute value of −3 is also 3. The absolute value is closely related to the notions of magnitude, distance, and norm in various mathematical and physical contexts.

Figure 1.5 Absolute Value



The graph of y = |x|. The graph is symmetric for both negative and positive values of x.

Terminology and Notation

The term "absolute value" has been used in this sense since at least 1806 in French and 1857 in English. The notation | a | was introduced by Karl Weierstrass in 1841. Other names for absolute value include "the numerical value" and "the magnitude."

Definition and Properties

For any real number a, the absolute value or modulus of a is denoted by | a | (a vertical bar on each side of the quantity) and is defined as | a | = a for a greater than or equal to 0, and | a | = −a for a < 0. For instance, if a = -3, | -3 | = 3 = −(-3). The double negative yields a positive number. As can be seen from the above definition, the absolute value of a is always either positive or zero but never negative. From an analytic geometry point of view, the absolute value of a real number is that number's distance from zero along the real number line, and more generally, the absolute value of the difference of two real numbers is the distance between them.

Section 2

Exponents, Scientific Notation, Order of Operations

Integer Exponents

Scientific Notation

Order of Operations

https://www.boundless.com/algebra/the-building-blocks-of-algebra/exponents-scientific-notation-order-of-operations/

Integer Exponents

An exponent, written b^n, indicated multiplying b times itself n times, so b^3 is b·b·b.

KEY POINTS

- Exponentiation is a mathematical operation, written as b^n, involving two numbers, the base b and the exponent (or index or power) n. When n is a positive integer, exponentiation corresponds to repeated multiplication.
- The expression b^2 = b·b is called the square of b because the area of a square with side-length b is b^2. The expression b^3 = b·b·b is called the cube, because the volume of a cube with side-length b is b^3.
- Some observations may be made about exponents. Any number raised by the exponent 1 is the number itself. Any nonzero number raised by the exponent 0 is 1. These equations do not decide the value of 0^0. Raising 0 by a negative exponent would imply division by 0, so it is undefined.

Exponentiation is a mathematical **operation**, written as b^n, involving two numbers, the **base** b and the **exponent** (or index or power) n. When n is a positive integer, exponentiation corresponds to repeated multiplication; in other words, a product of n factors, each of which is equal to b (the product itself can also be called power) (*Figure 1.7*).

Figure 1.7 Exponent

$$b^n = \underbrace{b \times \cdots \times b}_{n}$$

Taking b to the n power, as shown, is equivalent to multiplying b times itself an n number of times.

Similarly, multiplication by a positive integer corresponds to repeated addition (*Figure 1.6*).

Figure 1.6 Multiplication

$$b \times n = \underbrace{b + \cdots + b}_{n}$$

Exponentiation is related to multiplication in that multiplication of b times n is equivalent to adding b together n number of times.

The exponent is usually shown as a superscript to the right of the base. The exponentiation b^n can be read as: b raised to the n-th power, b raised to the power of n, b raised by the exponent of n, or most briefly as b to the n. Some exponents have their own pronunciation. For example, b^2 is usually read as b squared and b^3 as b cubed. It is also often common to see b^n represented as b^n.

Exponentiation is used pervasively in many fields, including economics, biology, chemistry, physics, and computer science, with applications such as compound interest, population growth, chemical reaction kinetics, wave behavior, and public key cryptography.

Background and Terminology

The expression $b^2 = b \cdot b$ is called the square of b because the area of a square with side-length b is b^2.

The expression $b^3 = b \cdot b \cdot b$ is called the cube, because the volume of a cube with side-length b is b^3.

So 3^2 is pronounced "three squared", and 2^3 is "two cubed."

The exponent says how many copies of the base are multiplied together. For example, $3^5 = 3·3·3·3·3 = 243$. The base 3 appears 5 times in the repeated multiplication, because the exponent is 5. Here, 3 is the base, 5 is the exponent, and 243 is the power or, more specifically, the fifth power of 3, 3 raised to the fifth power, or 3 to the power of 5. The word "raised" is usually omitted, and very often "power" as well, so 3^5 is typically pronounced "three to the fifth" or "three to the five."

Integer Exponents

The exponentiation operation with integer exponents requires only elementary algebra.

Positive Integer Exponents

Formally, powers with positive integer exponents may be defined by the initial condition

$$b^1 = b$$

and the recurrence relation

$$b^{n+1} = b^n \cdot b$$

From the associativity of multiplication, it follows that for any positive integers m and n

$$b^{m+n} = b^m \cdot b^n$$

Arbitrary Integer Exponents

For non-zero b and positive n, the recurrence relation from the previous subsection can be rewritten as

$$b^n = \frac{b^{n+1}}{b}$$

By defining this relation as valid for all integer n and nonzero b, it follows that

$$b^0 = \frac{b^1}{b} = 1$$

$$b^{-1} = \frac{b^0}{b} = \frac{1}{b}$$

and more generally,

$$b^{-n} = \frac{1}{b^n}$$

for any nonzero b and any nonnegative integer n (and indeed any integer n).

The following observations may be made:

- Any number raised by the exponent 1 is the number itself.
- Any nonzero number raised by the exponent 0 is 1; one interpretation of these powers is as empty products.
- These equations do not decide the value of 0^0.
- Raising 0 by a negative exponent would imply division by 0, so it is left undefined.

The identity

$$b^{m+n} = b^m \cdot b^n$$

initially defined only for positive integers m and n, holds for arbitrary integers m and n, with the constraint that m and n must both be positive when b is zero.

Scientific Notation

Scientific notation expresses a number as a· 10^b, where a has one digit to the left of the decimal.

KEY POINTS

- Scientific notation is a way of writing numbers that are too big or too small to be conveniently written in decimal form.
- In normalized scientific notation, the exponent b is chosen so that the absolute value of a remains at least one but less than ten ($1 \le |a| < 10$). Following these rules, 350 would always be written as 3.5×10^2.
- Most calculators present very large and very small results in scientific notation. Because superscripted exponents like 10^7 cannot always be conveniently displayed, the letter E or e is often used to represent "times ten raised to the power of" (which would be written as "x 10^b ").

Standard Form to Scientific Form

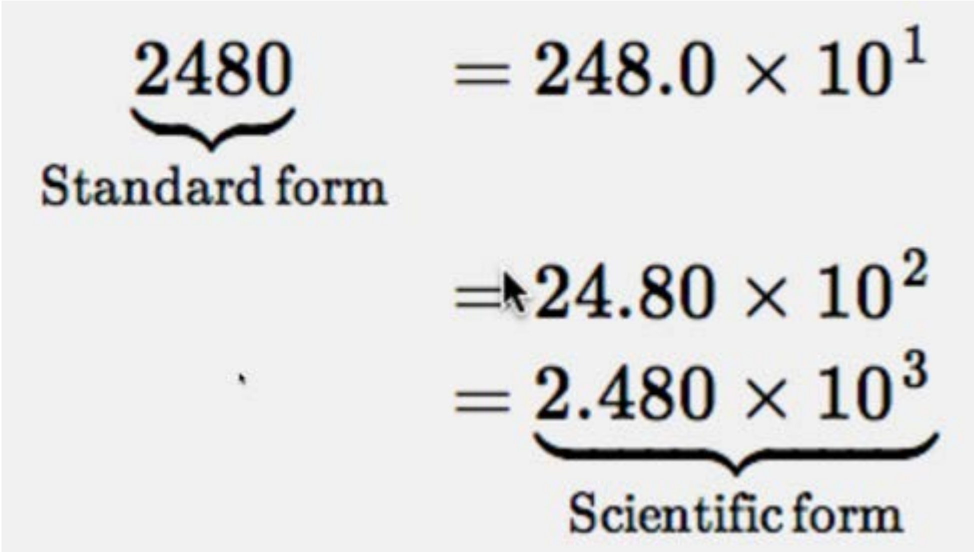
Very large numbers such as 43,000,000,000,000,000,000 (the number of different possible configurations of Rubik's cube) and very small numbers such as 0.000000000000000000000340 (the mass of the amino acid tryptophan) are extremely inconvenient to write and read. Such numbers can be expressed more conveniently by writing them as part of a power of 10.

To see how this is done, let us start with a somewhat smaller number such as 2480.

The last form in *Figure 1.8* is called the scientific form of the number. There is one nonzero digit to the left of the decimal point and the absolute value of the exponent on 10 records the number of places the original decimal point was moved to the left. If instead we have a very small number, such as 0.00059, we instead move the decimal place to the right, as in the following:

$$0.00059 = \frac{5.9}{10000} = \frac{5.9}{10^4} = 5.9 \cdot 10^{-4}$$

Figure 1.8 Standard Form Versus Scientific Form



In standard form, the number is written out as you are accustomed to, the ones digit to the farthest to the right (unless there is a decimal), then the tens digit to the left of the ones, and so on. In scientific notation, a number in standard notation with one nonzero digit to the left of the decimal is multiplied by ten to some power, as shown.