

# Model predictive control of nonlinear processes

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## 1. Historical background

Process control has become an integral part of process plants. An automatic controller must be able to facilitate the plant operation over a wide range of operating conditions. The proportional-integral (PI) or proportional-integral-derivative (PID) controllers are commonly used in many industrial control systems. These controllers are tuned with different tuning techniques to deliver satisfactory plant performance.

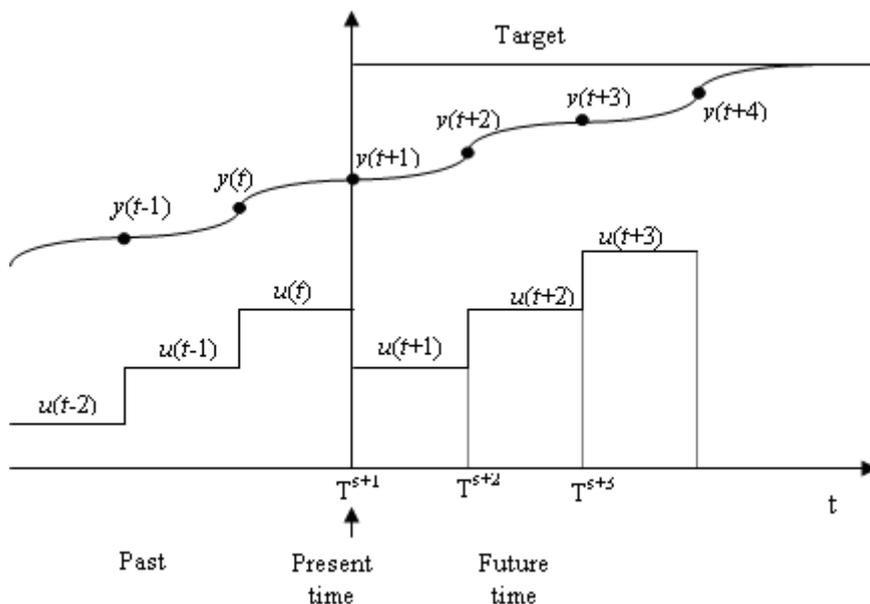


Fig. 1. MPC multi-step prediction scheme.

However, specific control problems associated with the plant operations severely limit the performance of conventional controllers. The increasing complexity of plant operations

together with tougher environmental regulations, rigorous safety codes and rapidly changing economic situations demand the need for more sophisticated process controllers. Model predictive control (MPC) is an important branch of automatic control theory. MPC refers to a class of control algorithms in which a process model is used to predict and optimize the process performance. MPC has been widely applied in industry (Qin and Badgwell, 1997). The idea of MPC is to calculate a control function for the future time in order to force the controlled system response to reach the reference value. Therefore, the future reference values are to be known and the system behavior must be predictable by an appropriate model. The controller determines a manipulated variable profile that optimizes some open-loop performance objective over a finite horizon extending from the current time into the future. This manipulated variable profile is implemented until a plant measurement becomes available. Feedback is incorporated by using the measurement to update the optimization problem for the next time step. Figure 1 explains the basic idea of MPC showing how the past input-output information is used to predict the future process behavior at the current time and how this information is extended to future to track the desired setpoint trajectory. The notation  $y$ ,  $u$  and  $T^s$  refer process output, control action and sample time, respectively.

## 2. Model predictive control scheme

Model predictive control (MPC) refers to a wide class of control algorithms that use an explicit process model to predict the behavior of a plant. The most significant feature that distinguishes MPC from other controllers is its long range prediction concept. This concept enables MPC to perform current computations to account the future dynamics, thus facilitating it to overcome the limitations of process dead time, non-minimum phase behavior and slow dynamics. In addition, MPC exhibits superior performance by systematically handling constraints violation.

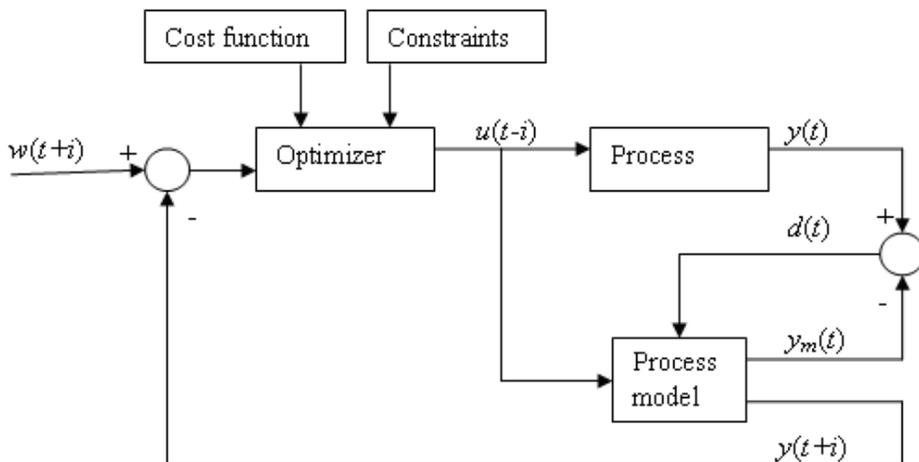


Fig. 2. MPC block diagram.

The fundamental framework of MPC algorithms is common for any kind of MPC schemes. The main differences in many MPC algorithms are the types models used to represent the plant dynamics and the cost function to be minimized. The multi-step model predictive control scheme shown in Figure 1 can be realized from the block diagram represented in Figure 2.

The basic elements in the block diagram are defined as follows. An appropriate model is used to predict the process outputs,  $y(t+i), i=1, \dots, N$  over a future time interval known as prediction horizon,  $N$ . A sequence of control actions,  $\Delta u(t+j), j=1, \dots, m$  over the control horizon  $m$  are calculated by minimizing some specified objective which is a function of predicted outputs,  $y(t+i)$ , set-point values,  $w(t+i)$  and control actions,  $\Delta u(t)$ . The first control move,  $u(t)$  of the sequence is implemented and the calculations are repeated for the subsequent sampling instants. In order to account the plant-model mismatch, a prediction error,  $d(t)$ , that is calculated based on plant measurement,  $y(t)$  and model prediction,  $y_m(t)$  is used to update the future predictions.

In MPC, the control law generates a control sequence, which forces the future system response to be equal to the reference values. The system response is based on future control actions, model parameters and the actual system states. Many methods for updating the optimization problem are possible, such as estimating model parameters and/or states, inferring about disturbances etc. MPC design considers different types of process models. These include first principle models, autoregressive moving average models, polynomial models, neural network models, fuzzy models etc. The attraction for MPC is due to its capability of handling various constraints directly in the formulation through on-line optimization. A variety of model predictive control techniques have been reported for controlling the processes of various complexities.

This chapter presents different linear and nonlinear model predictive controllers with case studies illustrating their application to real processes.

### 3. Linear model predictive control

Linear MPC (LMPC) algorithms employ linear or linearized models to obtain the predictive response of the controlled process. These algorithms include the Model Algorithmic Control (MAC) (Richalet et al., 1978), the Dynamic Matrix Control (DMC) (Cutler and Ramaker, 1980) and the Generalized Predictive Control (GPC) (Clarke et al., 1987). These algorithms are all similar in the sense that they rely on process models to predict the behavior of the process over some future time interval, and the control calculations are based on these model predictions. The models used for these predictions have usually been derived from linear approximations of the process or experimentally obtained step response data. A survey of theory and applications of such algorithms have been reported by Garcia et al. (1989).

#### 3.1 LMPC design

A classical autoregressive moving average (ARX) model structure that relates the plant output with the present and past plant input-output can be used to formulate a predictive model. The model parameters can be determined a priori by using the known input-output data to form a fixed predictive model or these parameters are updated at each sampling

time by an adaptive mechanism. The one step ahead predictive model can be recursively extended to obtain future predictions for the plant output. The minimization of a cost function based on future plant predictions and desired plant outputs generates an optimal control input sequence to act on the plant. The strategy is described as follows.

### *Predictive model*

The relation between the past input-output data and the predicted output can be expressed by an ARX model of the form

$$y(t+1) = a_1y(t) + \dots + a_{ny}y(t-ny+1) + b_1u(t) + \dots + b_{nu}u(t-nu+1) \quad (1)$$

where  $y(t)$  and  $u(t)$  are the process and controller outputs at time  $t$ ,  $y(t+1)$  is the one-step ahead model prediction at time  $t$ ,  $a$ 's and  $b$ 's represent the model coefficients and the  $nu$  and  $ny$  are input and output orders of the system.

### *Model identification*

The model output prediction can be expressed as

$$y_m(t+1) = \phi x_m(t) \quad (2)$$

where

$$\phi = [\alpha_1 \dots \alpha_{ny} \beta_1 \dots \beta_{nu}] \quad (3)$$

and

$$x_m(t) = [y(t) \dots y(t-ny+1) \ u(t) \dots u(t-nu+1)]^T \quad (4)$$

with  $\alpha$  and  $\beta$  as identified model parameters.

One of the most widely used estimators for model parameters and covariance is the popular recursive least squares (RLS) algorithm (Goodwin and Sin, 1984). The RLS algorithm provides the updated parameters of the ARX model in the operating space at each sampling instant or these parameters can be determined a priori using the known data of inputs and outputs for different operating conditions. The RLS algorithm is expressed as

$$\begin{aligned} \phi(t+1) &= \phi(t) + K(t) [y(t+1) - y_m(t+1)] \\ K(t) &= P(t) x_m(t+1) / [1 + x_m(t+1)^T P(t) x_m(t+1)] \\ P(t+1) &= 1/\lambda [P(t) - \{(P(t) x_m(t+1) x_m(t+1)^T P(t)) / (1 + x_m(t+1)^T P(t) x_m(t+1))\}] \end{aligned} \quad (5)$$

where  $\phi(t)$  represents the estimated parameter vector,  $\lambda$  is the forgetting factor,  $K(t)$  is the gain matrix and  $P(t)$  is the covariance matrix.

### *Controller formulation*

The  $N$  time steps ahead output prediction over a prediction horizon is given by

$$y_p(t+N) = \alpha_1 y(t+N-1) + \dots + \alpha_{ny} y(t-ny+N) + \beta_1 u(t+N-1) + \dots + \beta_{nu} u(t-nu+N) + err(t) \quad (6)$$

where  $y_p(t+N)$  represent the model predictions for  $N$  steps and  $err(t)$  is an estimate of the modeling error which is assumed as constant for the entire prediction horizon. If the control horizon is  $m$ , then the controller output,  $u$  after  $m$  time steps can be assumed to be constant.

An internal model is used to eliminate the discrepancy between model and process outputs,  $error(t)$ , at each sampling instant

$$error(t) = y(t) - y_m(t) \tag{7}$$

where  $y_m(t)$  is the one-step ahead model prediction at time  $(t-1)$ . The estimate of the error is then filtered to produce  $err(t)$  which minimizes the instability introduced by the modeling error feedback. The filter error is given by

$$err(t) = (1-K_f) err(t-1) + K_f error(t) \tag{8}$$

where  $K_f$  is the feedback filter gain which has to be tuned heuristically.

Back substitutions transform the prediction model equations into the following form

$$\begin{aligned} y_p(t+N) = & f_{N,1}y(t) + \dots + \\ & f_{N,ny}y(t-ny+1) + f_{N,ny+1}u(t-1) + \\ & \dots + f_{N,ny+nu-1}u(t-nu+1) + \\ & g_{N,1}u(t) + \dots + g_{N,m}u(t+m-1) \\ & + e_N err(t) \end{aligned} \tag{9}$$

The elements  $f$ ,  $g$  and  $e$  are recursively calculated using the parameters  $\alpha$  and  $\beta$  of Eq. (3). The above equations can be written in a condensed form as

$$Y(t) = F X(t) + G U(t) + E err(t) \tag{10}$$

where

$$Y(t) = [y_p(t+1) \dots y_p(t+N)]^T \tag{11}$$

$$X(t) = [y(t) \quad y(t-1) \dots y(t-ny+1) \quad u(t-1) \dots u(t-nu+1)]^T \tag{12}$$

$$U(t) = [u(t) \dots u(t+m-1)]^T \tag{13}$$

$$\begin{aligned} F = & \begin{bmatrix} f_{11} & f_{12} & \dots & f_{1(ny+nu-1)} \\ f_{21} & f_{22} & \dots & f_{2(ny+nu-1)} \\ \vdots & & & \\ \vdots & & & \\ f_{N1} & f_{N12} & \dots & f_{N(ny+nu-1)} \end{bmatrix} \\ G = & \begin{bmatrix} g_{11} & 0 & 0 & \dots & 0 \\ g_{21} & g_{21} & 0 & \dots & 0 \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ g_{m1} & g_{m2} & g_{m3} & \dots & g_{mm} \\ \cdot & \cdot & \cdot & \dots & \cdot \\ \cdot & \cdot & \cdot & \dots & \cdot \\ \cdot & \cdot & \cdot & \dots & \cdot \\ g_{N1} & g_{N2} & g_{N3} & \dots & g_{Nm} \end{bmatrix} \\ E = & [e_1 \dots e_N]^T \end{aligned}$$

In the above,  $Y(t)$  represents the model predictions over the prediction horizon,  $X(t)$  is a vector of past plant and controller outputs and  $U(t)$  is a vector of future controller outputs. If the coefficients of  $F$ ,  $G$  and  $E$  are determined then the transformation can be completed. The number of columns in  $F$  is determined by the ARX model structure used to represent the system, where as the number of columns in  $G$  is determined by the length of the control horizon. The number of rows is fixed by the length of the prediction horizon.

Consider a cost function of the form

$$J = \sum_{i=1}^N [y_p(t+i) - w(t+i)]^2 + \sum_{i=1}^m \gamma [u(t+i-1)]^2$$

$$= \sum_{i=1}^N [Y(t) - W(t)]^T [Y(t) - W(t)] + \sum_{i=1}^m \gamma U(t)^T U(t) \quad (14)$$

where  $W(t)$  is a setpoint vector over the prediction horizon

$$W(t) = [w(t+1) \dots w(t+N)]^T \quad (15)$$

The minimization of the cost function,  $J$  gives optimal controller output sequence

$$U(t) = [G^T G + \gamma I]^{-1} G^T [W(t) - FX(t) - Eerr(t)] \quad (16)$$

The vector  $U(t)$  generates control sequence over the entire control horizon. But, the first component of  $U(t)$  is actually implemented and the whole procedure is repeated again at the next sampling instant using latest measured information.

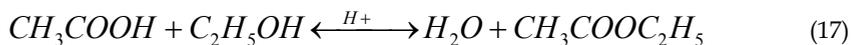
Linear model predictive control involving input-output models in classical, adaptive or fuzzy forms is proved useful for controlling processes that exhibit even some degree of nonlinear behavior (Eaton and Rawlings, 1992; Venkateswarlu and Gangiah, 1997 ; Venkateswarlu and Naidu, 2001).

### 3.2 Case study: linear model predictive control of a reactive distillation column

In this study, a multistep linear model predictive control (LMPC) strategy based on autoregressive moving average (ARX) model structure is presented for the control of a reactive distillation column. Although MPC has been proved useful for a variety of chemical and biochemical processes (Garcia et al., 1989 ; Eaton and Rawlings, 1992), its application to a complex dynamic system like reactive distillation is more interesting.

#### The process and the model

Ethyl acetate is produced through an esterification reaction between acetic acid and ethyl alcohol



The achievable conversion in this reversible reaction is limited by the equilibrium conversion. This quaternary system is highly non-ideal and forms binary and ternary

azeotropes, which introduce complexity to the separation by conventional distillation. Reactive distillation can provide a means of breaking the azeotropes by altering or eliminating the conditions for azeotrope formation. Thus reactive distillation becomes attractive alternative for the production of ethyl acetate.

The rate equation of this reversible reaction in the presence of a homogeneous acid catalyst is given by (Alejski and Duprat, 1996)

$$r_1 = k_1 C_1 C_2 - \frac{k_1}{K_c} C_3 C_4$$

$$k_1 = (4.195 C_k + 0.08815) \exp(-6500.1/T) \quad (18)$$

$$K_c = 7.558 - 0.012T$$

Vora and Daoutidis (2001) have presented a two feed column configuration for ethyl acetate reactive distillation and found that by feeding the two reactants, ethanol and acetic acid, on different trays counter currently allows to enhance the forward reaction on trays and results in higher conversion and purity over the conventional column configuration of feeding the reactants on a single tray. All plates in the column are considered to be reactive. The column consists of 13 stages including the reboiler and the condenser. The less volatile acetic acid enters the 3 rd tray and the more volatile ethanol enters the 10 th tray. The steady state operating conditions of the column are shown in Table 1.

Acetic acid feed flow rate, $F_{Ac}$	6.9 mol/s
Ethanol flow rate, $F_{Eth}$	6.865 mol/s
Reflux flow rate, $L_o$	13.51 mol/s
Distillate flow rate, $D$	6.68 mol/s
Bottoms flow rate, $B$	7.085 mol/s
Reboiler heat duty, $Q_r$	$5.88 \times 10^5$ J/mol
Boiling points, °K (Acetic acid, ethanol, water, ethyl acetate)	391.05, 351.45, 373.15, 350.25
Distillate composition (Acetic acid, ethanol, water, ethyl acetate)	0.0842, 0.1349, 0.0982, 0.6827
Bottoms composition (Acetic acid, ethanol, water, ethyl acetate)	0.1650, 0.1575, 0.5470, 0.1306

Table 1. Design conditions for ethyl acetate reactive distillation column

The dynamic model representing the process operation involves mass and component balance equations with reaction terms, along with energy equations supported by vapor-liquid equilibrium and physical properties (Alejski & Duprat, 1996). The assumptions made in the formulation of the model include adiabatic column operation, negligible heat of reaction, negligible vapor holdup, liquid phase reaction, physical equilibrium in streams leaving each stage, negligible down comer dynamics and negligible weeping of liquid through the openings on the tray surface. The equations representing the process are given as follows.

**Total mass balance**

Total condenser:

$$\frac{dM_1}{dt} = V_2 -(L_1 + D) + \Delta R_1 \quad (19)$$

Plate j:

$$\frac{dM_j}{dt} = FV_j + V_{j+1} + FL_j + L_{j-1} - V_j - L_j + \Delta R_j \quad (20)$$

Reboiler :

$$\frac{dM_n}{dt} = L_{n-1} - V_n - L_n + \Delta R_n \quad (21)$$

**Component mass balance**

Total condenser :

$$\frac{d(M_1 x_{i,1})}{dt} = V_2 y_{i,2} - (L_1 + D)x_{i,1} + R_{i,1} \quad (22)$$

Plate j:

$$\frac{d(M_j x_{i,j})}{dt} = FV_j y_{i,j} + V_{j+1} y_{i,j+1} + FL_j x_{i,j} + L_{j-1} x_{i,j-1} - V_j y_{i,j} - L_j x_{i,j} + R_{i,j} \quad (23)$$

Reboiler:

$$\frac{d(M_n x_{i,n})}{dt} = L_{n-1} x_{i,n-1} - V_n y_{i,n} - L_n x_{i,n} + R_{i,n} \quad (24)$$

**Total energy balance**

Total condenser :

$$\frac{dE_1}{dt} = V_2 H_2 - (L_1 + D)h_1 + Q_1 \quad (25)$$

Plate j:

$$\frac{dE_j}{dt} = FV_j Hf_j + V_{j+1} H_{j+1} + FL_j hf_j + L_{j-1} h_{j-1} - V_j H_j - L_j h_j + Q_j \quad (26)$$

Reboiler:

$$\frac{dE_n}{dt} = L_{n-1} h_{n-1} - V_n H_n - L_n h_n + Q_n \quad (27)$$

Level of liquid on the tray

$$L^{liq} = \frac{M_n MW_{av}}{A_{tray} \rho_{av}} \quad (28)$$

Flow of liquid over the weir

$$\text{If } (L^{liq} < h_{weir}) \text{ then } L_n = 0 \quad (29)$$

else

$$L_n = 1.84 \frac{L_{weir} \rho_{av}}{MW_{av}} (L^{liq} - h_{weir})^{3/2} \quad (30)$$

*Mole fraction normalization*

$$\sum_{i=1}^{NC} x_i = \sum_{i=1}^{NC} y_i = 1 \quad (31)$$

### *VLE calculations*

For the column operation under moderate pressures, the VLE equation assumes the ideal gas model for the vapor phase, thus making the vapor phase activity coefficient equal to unity. The VLE relation is given by

$$y_i P = x_i \gamma_i P_i^{sat} \quad (i = 1, 2, \dots, NC) \quad (32)$$

The liquid phase activity coefficients are calculated using UNIFAC method (Smith et al., 1996).

### *Enthalpies Calculation*

The relations for the liquid enthalpy  $h$ , the vapor enthalpy  $H$  and the liquid density  $\rho$  are:

$$\begin{aligned} h &= h(P, T, x) \\ H &= H(P, T, y) \\ \rho^{liq} &= \rho^{liq}(P, T, x) \end{aligned} \quad (33)$$

### *Control scheme*

The design and implementation of the control strategy is studied for the single input-single output (SISO) control of the ethyl acetate reactive distillation column with its double feed configuration. The objective is to control the desired product purity in the distillate stream inspite disturbances in column operation. This becomes the main control loop. Since reboiler and condenser holdups act as pure integrators, they also need to be controlled. These become the auxiliary control loops. Reflux flow rate is used as a manipulated variable to control the purity of the ethyl acetate. Distillate flow rate is used as a manipulated variable to control the condenser holdup, while bottom flow rate is used to control the reboiler holdup. In this work, it is proposed to apply a multistep model predictive controller for the main loop and conventional PI controllers for the auxiliary control loops. This control scheme is shown in the Figure 3.

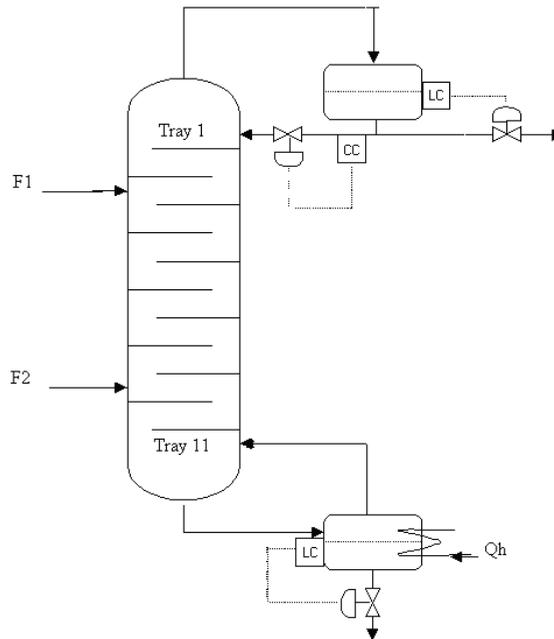


Fig. 3. Control structure of two feed ethyl acetate reactive distillation column.

### Analysis of Results

The performance of the multistep linear model predictive controller (LMPC) is evaluated through simulation. The product composition measurements are obtained by solving the model equations using Euler's integration with sampling time of 0.01 s. The input and output orders of the predictive model are considered as  $n_u = 2$  and  $n_y = 2$ . The diagonal elements of the initial covariance matrix,  $P(0)$  in the RLS algorithm are selected as 10.0, 1.0, 0.01, 0.01, respectively. The forgetting factor,  $\lambda$  used in recursive least squares is chosen as 5.0. The feedback controller gain  $K_f$  is assigned as 0.65. The tuning parameter  $\gamma$  in the control law is set as  $0.115 \times 10^{-6}$ . The PI controller parameters of ethyl acetate composition are evaluated by using the continuous cycling method of Ziegler and Nichols. The tuned controller settings are  $k_c = 11.15$  and  $\tau_I = 1.61 \times 10^4$  s. The PI controller parameters used for reflux drum and reboiler holdups are  $k_c = -0.001$  and  $\tau_I = 5.5$  h, and  $k_c = -0.001$  and  $\tau_I = 5.5$  h, respectively (Vora and Daoutidis, 2001).

The LMPC is implemented by adaptively updating the prediction model using recursive least squares. On evaluating the effect of different prediction and control horizons, it is observed that the LMPC with a prediction horizon of around 5 and a control horizon of 2 has shown reasonably better control performance. The LMPC is also referred here as MPC. Figure 4 shows the results of MPC and PI controller when they are applied for tracking series of step changes in ethyl acetate composition. The regulatory control performance of MPC and PI controller for 20% decrease in feed rate of acetic acid is shown in Figure 5. The results thus show the effectiveness of the multistep linear model predictive control strategy for the control of highly nonlinear reactive distillation column.

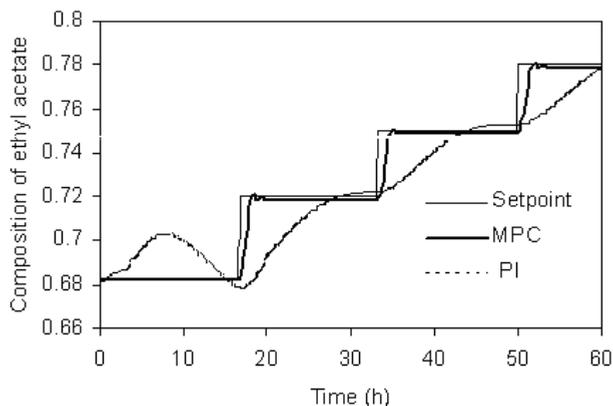


Fig. 4. Performance of MPC and PI controller for tracking series of step changes in distillate composition.

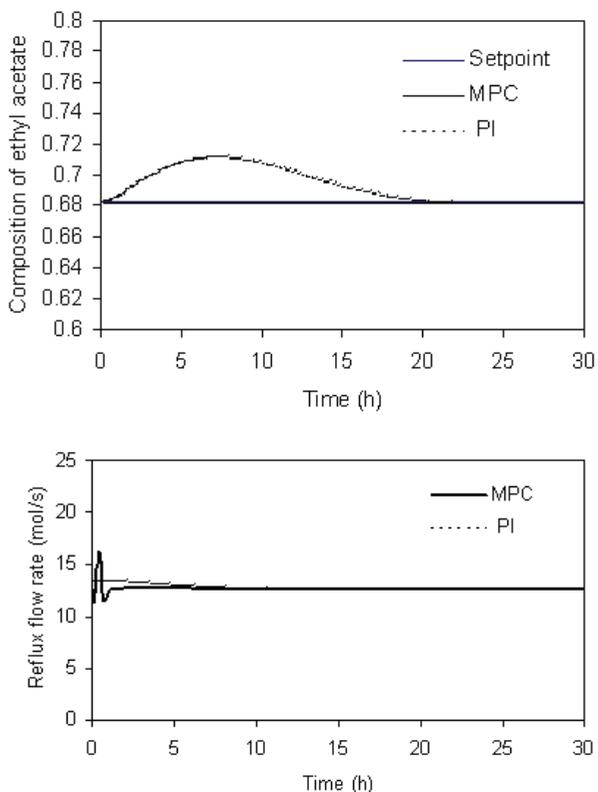


Fig.5. Output and input profiles for MPC and PI controller for 20% decrease in the feed rate of acetic acid.

## 4. Generalized predictive control

The generalized predictive control (GPC) is a general purpose multi-step predictive control algorithm (Clarke et al., 1987) for stable control of processes with variable parameters, variable dead time and a model order which changes instantaneously. GPC adopts an integrator as a natural consequence of its assumption about the basic plant model. Although GPC is capable of controlling such systems, the control performance of GPC needs to be ascertained if the process constraints are to be encountered in nonlinear processes. Camacho (1993) proposed a constrained generalized predictive controller (CGPC) for linear systems with constrained input and output signals. By this strategy, the optimum values of the future control signals are obtained by transforming the quadratic optimization problem into a linear complementarity problem. Camacho demonstrated the results of the CGPC strategy by carrying out a simulation study on a linear system with pure delay. Clarke et al. (1987) have applied the GPC to open-loop stable unconstrained linear systems. Camacho applied the CGPC to constrained open-loop stable linear system. However, most of the real processes are nonlinear and some processes change behavior over a period of time. Exploring the application of GPC to nonlinear process control will be more interesting. In this study, a constrained generalized predictive control (CGPC) strategy is presented and applied for the control of highly nonlinear and open-loop unstable processes with multiple steady states. Model parameters are updated at each sampling time by an adaptive mechanism.

### 4.1 GPC design

A nonlinear plant generally admits a local-linearized model when considering regulation about a particular operating point. A single-input single-output (SISO) plant on linearization can be described by a Controlled Autoregressive Integrated Moving Average (CARIMA) model of the form

$$A(q^{-1})y(t) = B(q^{-1})q^{-d}u(t) + C(q^{-1})e(t)/\Delta \quad (34)$$

where  $A$ ,  $B$  and  $C$  are polynomials in the backward shift operator  $q^{-1}$ . The  $y(t)$  is the measured plant output,  $u(t)$  is the controller output,  $e(t)$  is the zero mean random Gaussian noise,  $d$  is the delay time of the system and  $\Delta$  is the differencing operator  $1-q^{-1}$ .

The control law of GPC is based on the minimization of a multi-step quadratic cost function defined in terms of the sum of squares of the errors between predicted and desired output trajectories with an additional term weighted by projected control increments as given by

$$J(N_1, N_2, N_3) = E \left\{ \sum_{j=N_1}^{N_2} \{ [y(t+j|t) - w(t+j)]^2 + \sum_{j=1}^{N_3} \lambda [\Delta u(t+j-1)]^2 \} \right\} \quad (35)$$

where  $E\{\cdot\}$  is the expectation operator,  $y(t+j|t)$  is a sequence of predicted outputs,  $w(t+j)$  is a sequence of future setpoints,  $\Delta u(t+j-1)$  is a sequence of predicted control increments and  $\lambda$  is the control weighting factor. The  $N_1$ ,  $N_2$  and  $N_3$  are the minimum costing horizon, the maximum costing horizon and the control horizon, respectively. The values of  $N_1$ ,  $N_2$  and  $N_3$  of Eq. (35) can be defined by  $N_1 = d + 1$ ,  $N_2 = d + N$ , and  $N_3 = N$ , respectively.

Predicting the output response over a finite horizon beyond the dead-time of the process enables the controller to compensate for constant or variable time delays. The recursion of the Diophantine equation is a computationally efficient approach for modifying the predicted output trajectory. An optimum  $j$ -step a head prediction output is given by

$$y(t + j | t) = G_j(q^{-1}) \Delta u(t + j - d - 1) + F_j(q^{-1})y(t) \tag{36}$$

where  $G_j(q^{-1}) = E_j(q^{-1})B(q^{-1})$ , and  $E_j$  and  $F_j$  are polynomials obtained recursively solving the Diophantine equation,

$$1 = E_j(q^{-1})A\Delta + q^{-j}F_j(q^{-1}) \tag{37}$$

The  $j$ -step ahead optimal predictions of  $y$  for  $j = 1, \dots, N_2$  can be written in condensed form

$$Y = Gu + f \tag{38}$$

where  $f$  contains predictions based on present and past outputs up to time  $t$  and past inputs and referred to free response of the system, i.e.,  $f = [f_1, f_2, \dots, f_N]$ . The vector  $u$  corresponds to the present and future increments of the control signal, i.e.,  $u = [\Delta u(t), \Delta u(t+1), \dots, \Delta u(t+N-1)]^T$ . Eq. (35) can be written as

$$J = (Gu + f - w)^T (Gu + f - w) + \lambda u^T u \tag{39}$$

The minimization of  $J$  gives unconstrained solution to the projected control vector

$$u = (G^T G + \lambda I)^{-1} G^T (w - f) \tag{40}$$

The first component of the vector  $u$  is considered as the current control increment  $\Delta u(t)$ , which is applied to the process and the calculations are repeated at the next sampling instant. The schematic of GPC control law is shown in Figure 6, where  $K$  is the first row of the matrix  $(G^T G + \lambda I)^{-1} G^T$ .

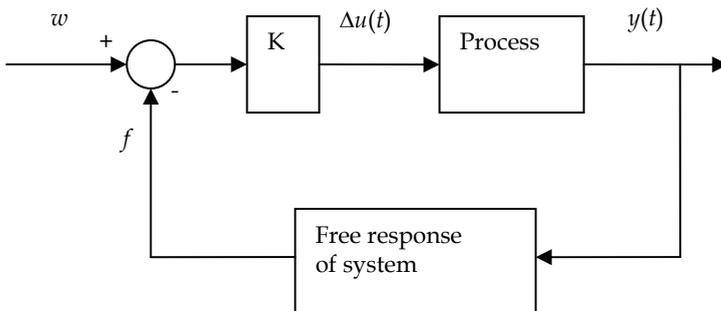


Fig. 6. The GPC control law

#### 4.2 Constrained GPC design

In practice, all processes are subject to constraints. Control valves are limited by fully closed and fully open positions and maximum slew rates. Constructive and safety reasons as well as sensor ranges cause limits in process variables. Moreover, the operating points of plants are determined in order to satisfy economic goals and usually lie at the intersection of certain constraints. Thus, the constraints acting on a process can be manipulated variable limits ( $u_{\min}$ ,  $u_{\max}$ ), slew rate limits of the actuator ( $du_{\min}$ ,  $du_{\max}$ ), and limits on the output signal ( $y_{\min}$ ,  $y_{\max}$ ) as given by

$$\begin{aligned} u_{\min} &\leq u(t) \leq u_{\max} \\ du_{\min} &\leq u(t) - u(t-1) \leq du_{\max} \\ y_{\min} &\leq y(t) \leq y_{\max} \end{aligned} \quad (41)$$

These constraints can be expressed as

$$\begin{aligned} lu_{\min} &\leq Tu + u(t-1)l \leq lu_{\max} \\ ldu_{\min} &\leq u \leq ldu_{\max} \\ ly_{\min} &\leq Gu + f \leq ly_{\max} \end{aligned} \quad (42)$$

where  $l$  is an  $N$  vector containing ones, and  $T$  is an  $N \times N$  lower triangular matrix containing ones. By defining a new vector  $x = u - ldu_{\min}$ , the constrained equations can be transformed as

$$\begin{aligned} x &\geq 0 \\ Rx &\leq c \end{aligned} \quad (43)$$

with

$$R = \begin{bmatrix} I_{N \times N} \\ T \\ -T \\ G \\ -G \end{bmatrix} ; \quad c = \begin{bmatrix} l(du_{\max} - du_{\min}) \\ lu_{\max} - Tldu_{\min} - u(t-1)l \\ -lu_{\min} + Tldu_{\min} + u(t-1)l \\ ly_{\max} - f - Gldu_{\min} \\ -ly_{\min} + f + Gldu_{\min} \end{bmatrix} \quad (44)$$

Eq. (39) can be expressed as

$$J = \frac{1}{2}u^T Hu + bu + f_o \quad (45)$$

where

$$\begin{aligned}
 H &= 2(G^T G + \lambda I) \\
 b &= G^T (f - w) + (f - w)^T G \\
 f_o &= (f - w)^T (f - w)
 \end{aligned}$$

The minimization of  $J$  with no constraints on the control signal gives

$$u = -H^{-1}b \tag{46}$$

Eq. (45) in terms of the newly defined vector  $x$  becomes

$$J = \frac{1}{2} x^T Hx + ax + f_1 \tag{47}$$

where

$$\begin{aligned}
 a &= b + u_{\min} l^T H \\
 f_1 &= f_o + \frac{1}{2} du_{\min}^2 l^T Hl + bldu_{\min}
 \end{aligned}$$

The solution of the problem can be obtained by minimization of Eq. (47) subject to the constraints of Eq. (43). By using the Lagrangian multiplier vectors  $v_1$  and  $v$  for the constraints,  $x \geq 0$  and  $Rx \leq c$ , respectively, and introducing the slack variable vector  $v_2$ , the Kuhn-Tucker conditions can be expressed as

$$\begin{aligned}
 Rx + v_2 &= c \\
 -Hx - R^T v + v_1 &= a \\
 x^T v_1 &= 0 \\
 v^T v_2 &= 0 \\
 x, v, v_1, v_2 &\geq 0
 \end{aligned} \tag{48}$$

Camacho (1993) has proposed the solution of this problem with the help of Lemke’s algorithm (Bazaraa and Shetty, 1979) by expressing the Kuhn-Tucker conditions as a linear complementarity problem starting with the following tableau

	$v_2$	$x$	$v$	$v_1$	$z_0$	
$v_2$	$I_{m \times n}$	$O_{m \times N}$	$-RH^{-1}R^T$	$RH^{-1}$	$-1$	$v_{2 \min}$
$x$	$O_{N \times m}$	$I_{N \times N}$	$H^{-1}R^T$	$-H^{-1}$	$-1$	$x_{\min}$

(49)

Here,  $z_0$  is the artificial variable which will be driven to zero iteratively.

In this study, the above stated constrained generalized predictive linear control of Camacho (1993) is extended to open-loop unstable constrained control of nonlinear processes. In this

strategy, process nonlinearities are accounted through adaptation of model parameters while taking care of input and output constraints acting on the process. The following recursive least squares formula (Hsia, 1977) is used for on-line estimation of parameters and the covariance matrix after each new sample:

$$\begin{aligned}\theta(k+1) &= \theta(k) + \gamma(k+1)P(k)v(k+1)[y(k+1) - v^T(k+1)\theta(k)] \\ P(k+1) &= \frac{1}{\lambda} \left[ P(k) - \gamma(k+1)P(k)v(k+1)v^T(k+1)P(k) \right] \\ \gamma(k+1) &= \frac{1}{\left[ 1 + v^T(k+1)P(k)v(k+1) \right]}\end{aligned}\quad (50)$$

where  $\theta$  is the parameter vector,  $\gamma$  is the intermediate estimation variable,  $P$  is the covariance matrix,  $v$  is the vector of input-output variables,  $y$  is the output variable, and  $0 < \lambda < 1$  is the forgetting factor. The initial covariance matrix and exponential forgetting factor are selected based on various trials so as to provide reasonable convergence in parameter estimates.

The CGPC strategy of nonlinear processes is described in the following steps:

1. Specify the controller design parameters  $N_1$ ,  $N_2$ ,  $N_3$  and also the initial parameter estimates and covariance matrix for recursive identification of model parameters.
2. Update the model parameters using recursive least squares method.
3. Initialize the polynomials  $E_1$  and  $F_1$  of Diophantine identity, Eq. (37), using the estimated parameters. Further initialize  $G_1$  as  $E_1 B$ .
4. Compute the polynomials  $E_j$ ,  $F_j$  and  $G_j$  over the prediction horizon and control horizon using the recursion of Diophantine.
5. Compute matrices  $H$ ,  $R$ , and  $G$ , and vectors  $f$  and  $c$  using the polynomials determined in step 4.
6. Compute the unconstrained solution  $x_{min} = -H^{-1} a$ .
7. Compute  $v_{2min} = c - R x_{min}$ . If  $x_{min}$  and  $v_{2min}$  are nonnegative, then go to step 10.
8. Start Lemke's algorithm with  $x$  and  $v_2$  in the basis with the tableau, Eq. (49).
9. If  $x_1$  is not in the first column of the tableau, make it zero; otherwise, assign it the corresponding value.
10. Compute  $u(t) = x_1 + d u_{min} + u(t-1)$ .
11. Implement the control action, then shift to the next sampling instant and go to step 2.

### 4.3 Case study: constrained generalized predictive control (CGPC) of open-loop unstable CSTR

The design and implementation of the CGPC strategy is studied by applying it for the control of a nonlinear open-loop unstable chemical reactor (Venkateswarlu and Gangiah, 1997).

#### Reactor

A continuous stirred tank reactor (CSTR) in which a first order exothermic irreversible reaction occurs is considered as an example of an unstable nonlinear process. The dynamic equations describing the process can be written as

$$V \frac{dC_A}{dt'} = F(C_{Af} - C_A) - V k_0 \exp\left[-E / R_g T_r\right] C_A \quad (51)$$

$$V \rho C_p \frac{dT_r}{dt'} = \rho C_p F(T_f - T_r) + V(-\Delta H) k_0 \exp\left[-E / R_g T_r\right] C_A - UA_h(T_r - T_c) \quad (52)$$

where  $C_A$  and  $T_r$  are reactant concentration and temperature, respectively. The coolant temperature  $T_c$  is assumed to be the manipulated variable. Following the analysis of Uppal et al. (1974), the model is made dimensionless by introducing the parameters as

$$\phi = \frac{E}{R_g T_{fo}}, \quad B_h = \frac{(-\Delta H) C_{Afo}}{\rho C_p T_{fo}}, \quad D_a = \frac{k_o e^{-\phi} V}{F_o}, \quad \beta = \frac{UA_h}{F_o \rho C_p} \quad (53)$$

where  $F_o$ ,  $C_{Afo}$  and  $T_{fo}$  are the nominal characteristic values of volumetric flow rate, feed composition and feed temperature, respectively. The corresponding dimensionless variables are defined by

$$t = \frac{t' F_o}{V}, \quad x_1 = \frac{C_{Afo} - C_A}{C_{Afo}}, \quad x_2 = \frac{T_r - T_{fo}}{T_{fo}}, \quad u = \frac{T_c - T_{co}}{T_{fo}} \phi \quad (54)$$

where  $T_{co}$  is some reference value for the coolant temperature.

The modeling equations can be written in dimensionless form (Calvet and Arkun, 1988; Hernandez and Arkun, 1992) as

$$\begin{aligned} \dot{x}_1 &= -x_1 + Da(1 - x_1) \exp\left(\frac{x_2}{1 + x_2 / \phi}\right) \\ \dot{x}_2 &= -x_2 + B_h Da(1 - x_1) \exp\left(\frac{x_2}{1 + x_2 / \phi}\right) + \beta(u - x_2) \\ y &= x_1 \end{aligned} \quad (55)$$

where  $x_1$  and  $x_2$  are the dimensionless reactant concentration and temperature, respectively. The input  $u$  is the cooling jacket temperature,  $D_a$  is the Damkohler number,  $\phi$  is the dimensionless activation energy,  $B_h$  is the heat of reaction and  $\beta$  is the heat transfer coefficient. If the physical parameters chosen are  $D_a = 0.072$ ,  $\phi = 20.0$ ,  $B_h = 8.0$ , and  $\beta = 0.3$ , then the system can exhibit up to three steady states, one of which is unstable as shown in Figure 7. Here the task is to control the reactor at and around the unstable operating point. The cooling water temperature is the input  $u$ , which is the manipulated variable to control the reactant concentration,  $x_1$ .

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