

*Detection, Estimation, and Modulation Theory, Part III:  
Radar–Sonar Signal Processing and Gaussian Signals in Noise.* Harry L. Van Trees  
Copyright © 2001 John Wiley & Sons, Inc.  
ISBNs: 0-471-10793-X (Paperback); 0-471-22109-4 (Electronic)

## **Detection, Estimation, and Modulation Theory**

---

*Detection, Estimation, and Modulation Theory, Part III:  
Radar–Sonar Signal Processing and Gaussian Signals in Noise.* Harry L. Van Trees  
Copyright © 2001 John Wiley & Sons, Inc.  
ISBNs: 0-471-10793-X (Paperback); 0-471-22109-4 (Electronic)

# **Detection, Estimation, and Modulation Theory**

## **Radar-Sonar Processing and Gaussian Signals in Noise**

---

**HARRY L. VAN TREES**  
George Mason University



A Wiley-Interscience Publication  
JOHN WILEY & SONS, INC.

New York • Chichester • Weinheim • Brisbane • Singapore • Toronto

This text is printed on acid-free paper. ☺

Copyright © 2001 by John Wiley & Sons, Inc. All rights reserved.

Published simultaneously in Canada.

No part of this publication may be reproduced, stored in a retrieval system or transmitted in any form or by any means, electronic, mechanical, photocopying, recording, scanning or otherwise, except as permitted under Section 107 or 108 of the 1976 United States Copyright Act, without either the prior written permission of the Publisher, or authorization through payment of the appropriate per-copy fee to the Copyright Clearance Center, 222 Rosewood Drive, Danvers, MA 01923, (978) 750-8400, fax (978) 750-4744. Requests to the Publisher for permission should be addressed to the Permissions Department, John Wiley & Sons, Inc., 605 Third Avenue, New York, NY 10158-0012, (212) 850-6011, fax (212) 850-6008, E-Mail: PERMREQ @ WILEY.COM.

For ordering and customer service, call 1-800-CALL-WILEY.

**ISBN 0-471-22109-4**

This title is also available in print as ISBN 0-471-10793-X.

*Library of Congress Cataloging in Publication Data is available.*

ISBN 0-471-10793-X

Printed in the United States of America

10 9 8 7 6 5 4 3 2 1

*To Diane*

and Stephen, Mark, Kathleen, Patricia,  
Eileen, Harry, and Julia

and the next generation—

Brittany, Erin, Thomas, Elizabeth, Emily,  
Dillon, Bryan, Julia, Robert, Margaret,  
Peter, Emma, Sarah, Harry, Rebecca,  
and Molly

## *Preface for Paperback Edition*

In 1968, Part I of *Detection, Estimation, and Modulation Theory* [VT68] was published. It turned out to be a reasonably successful book that has been widely used by several generations of engineers. There were thirty printings, but the last printing was in 1996. Volumes II and III ([VT71a], [VT71b]) were published in 1971 and focused on specific application areas such as analog modulation, Gaussian signals and noise, and the radar-sonar problem. Volume II had a short life span due to the shift from analog modulation to digital modulation. Volume III is still widely used as a reference and as a supplementary text. In a moment of youthful optimism, I indicated in the the Preface to Volume III and in Chapter III-14 that a short monograph on optimum array processing would be published in 1971. The bibliography lists it as a reference, *Optimum Array Processing*, Wiley, 1971, which has been subsequently cited by several authors. After a 30-year delay, *Optimum Array Processing*, Part IV of *Detection, Estimation, and Modulation Theory* will be published this year.

A few comments on my career may help explain the long delay. In 1972, MIT loaned me to the Defense Communication Agency in Washington, D.C. where I spent three years as the Chief Scientist and the Associate Director of Technology. At the end of the tour, I decided, for personal reasons, to stay in the Washington, D.C. area. I spent three years as an Assistant Vice-President at COMSAT where my group did the advanced planning for the INTELSAT satellites. In 1978, I became the Chief Scientist of the United States Air Force. In 1979, Dr. Gerald Dinneen, the former Director of Lincoln Laboratories, was serving as Assistant Secretary of Defense for C3I. He asked me to become his Principal Deputy and I spent two years in that position. In 1981, I joined M/A-COM Linkabit. Linkabit is the company that Irwin Jacobs and Andrew Viterbi had started in 1969 and sold to M/A-COM in 1979. I started an Eastern operation which grew to about 200 people in three years. After Irwin and Andy left M/A-COM and started Qualcomm, I was responsible for the government operations in San Diego as well as Washington, D.C. In 1988, M/A-COM sold the division. At that point I decided to return to the academic world.

I joined George Mason University in September of 1988. One of my priorities was to finish the book on optimum array processing. However, I found that I needed to build up a research center in order to attract young research-oriented faculty and

doctoral students. The process took about six years. The Center for Excellence in Command, Control, Communications, and Intelligence has been very successful and has generated over \$300 million in research funding during its existence. During this growth period, I spent some time on array processing but a concentrated effort was not possible. In 1995, I started a serious effort to write the Array Processing book.

Throughout the *Optimum Array Processing* text there are references to Parts I and III of *Detection, Estimation, and Modulation Theory*. The referenced material is available in several other books, but I am most familiar with my own work. Wiley agreed to publish Part I and III in paperback so the material will be readily available. In addition to providing background for Part IV, Part I is still useful as a text for a graduate course in Detection and Estimation Theory. Part III is suitable for a second level graduate course dealing with more specialized topics.

In the 30-year period, there has been a dramatic change in the signal processing area. Advances in computational capability have allowed the implementation of complex algorithms that were only of theoretical interest in the past. In many applications, algorithms can be implemented that reach the theoretical bounds.

The advances in computational capability have also changed how the material is taught. In Parts I and III, there is an emphasis on compact analytical solutions to problems. In Part IV, there is a much greater emphasis on efficient iterative solutions and simulations. All of the material in parts I and III is still relevant. The books use continuous time processes but the transition to discrete time processes is straightforward. Integrals that were difficult to do analytically can be done easily in Matlab<sup>®</sup>. The various detection and estimation algorithms can be simulated and their performance compared to the theoretical bounds. We still use most of the problems in the text but supplement them with problems that require Matlab<sup>®</sup> solutions.

We hope that a new generation of students and readers find these reprinted editions to be useful.

HARRY L. VAN TREES

Fairfax, Virginia  
June 2001

## *Preface*

In this book I continue the study of detection, estimation, and modulation theory begun in Part I [1]. I assume that the reader is familiar with the background of the overall project that was discussed in the preface of Part I. In the preface to Part II [2] I outlined the revised organization of the material. As I pointed out there, Part III can be read directly after Part I. Thus, some persons will be reading this volume without having seen Part II. Many of the comments in the preface to Part II are also appropriate here, so I shall repeat the pertinent ones.

At the time Part I was published, in January 1968, I had completed the “final” draft for Part II. During the spring term of 1968, I used this draft as a text for an advanced graduate course at M.I.T. and in the summer of 1968, I started to revise the manuscript to incorporate student comments and include some new research results. In September 1968, I became involved in a television project in the Center for Advanced Engineering Study at M.I.T. During this project, I made fifty hours of videotaped lectures on applied probability and random processes for distribution to industry and universities as part of a self-study package. The net result of this involvement was that the revision of the manuscript was not resumed until April 1969. In the intervening period, my students and I had obtained more research results that I felt should be included. As I began the final revision, two observations were apparent. The first observation was that the manuscript has become so large that it was economically impractical to publish it as a single volume. The second observation was that since I was treating four major topics in detail, it was unlikely that many readers would actually use all of the book. Because several of the topics can be studied independently, with only Part I as background, I decided to divide the material into three sections: Part II, Part III, and a short monograph on *Optimum Array Processing* [3]. This division involved some further editing, but I felt it was warranted in view of increased flexibility it gives both readers and instructors.

In Part II, I treated nonlinear modulation theory. In this part, I treat the random signal problem and radar/sonar. Finally, in the monograph, I discuss optimum array processing. The interdependence of the various parts is shown graphically in the following table. It can be seen that Part II is completely separate from Part III and *Optimum Array Processing*. The first half of *Optimum Array Processing* can be studied directly after Part I, but the second half requires some background from Part III. Although the division of the material has several advantages, it has one major disadvantage. One of my primary objectives is to present a unified treatment that enables the reader to solve problems from widely diverse physical situations. Unless the reader sees the widespread applicability of the basic ideas he may fail to appreciate their importance. Thus, I strongly encourage all serious students to read at least the more basic results in all three parts.

Prerequisites	
Part II	Chaps. I-5, I-6
Part III Chaps. III-1 to III-5 Chaps. III-6 to III-7 Chaps. III-8-end	Chaps. I-4, I-6 Chaps. I-4 Chaps. I-4, I-6, III-1 to III-7
Array Processing Chaps. IV-1, IV-2 Chaps. IV-3-end	Chaps. I-4 Chaps. III-1 to III-5, AP-1 to AP-2

The character of this book is appreciably different that that of Part I. It can perhaps be best described as a mixture of a research monograph and a graduate level text. It has the characteristics of a research monograph in that it studies particular questions in detail and develops a number of new research results in the course of this study. In many cases it explores topics which are still subjects of active research and is forced to leave some questions unanswered. It has the characteristics of a graduate level text in that it presents the material in an orderly fashion and develops almost all of the necessary results internally.

The book should appeal to three classes of readers. The first class consists of graduate students. The random signal problem, discussed in Chapters 2 to 7, is a logical extension of our earlier work with deterministic signals and completes the hierarchy of problems we set out to solve. The

last half of the book studies the radar/sonar problem and some facets of the digital communication problem in detail. It is a thorough study of how one applies statistical theory to an important problem area. I feel that it provides a useful educational experience, even for students who have no ultimate interest in radar, sonar, or communications, because it demonstrates system design techniques which will be useful in other fields.

The second class consists of researchers in this field. Within the areas studied, the results are close to the current research frontiers. In many places, specific research problems are suggested that are suitable for thesis or industrial research.

The third class consists of practicing engineers. In the course of the development, a number of problems of system design and analysis are carried out. The techniques used and results obtained are directly applicable to many current problems. The material is in a form that is suitable for presentation in a short course or industrial course for practicing engineers. I have used preliminary versions in such courses for several years.

The problems deserve some mention. As in Part I, there are a large number of problems because I feel that problem solving is an essential part of the learning process. The problems cover a wide range of difficulty and are designed to both augment and extend the discussion in the text. Some of the problems require outside reading, or require the use of engineering judgement to make approximations or ask for discussion of some issues. These problems are sometimes frustrating to the student but I feel that they serve a useful purpose. In a few of the problems I had to use numerical calculations to get the answer. I strongly urge instructors to work a particular problem before assigning it. Solutions to the problems will be available in the near future.

As in Part I, I have tried to make the notation mnemonic. All of the notation is summarized in the glossary at the end of the book. I have tried to make my list of references as complete as possible and acknowledge any ideas due to other people.

Several people have contributed to the development of this book. Professors Arthur Baggeroer, Estil Hoversten, and Donald Snyder of the M.I.T. faculty, and Lewis Collins of Lincoln Laboratory, carefully read and criticized the entire book. Their suggestions were invaluable. R. R. Kurth read several chapters and offered useful suggestions. A number of graduate students offered comments which improved the text. My secretary, Miss Camille Tortorici, typed the entire manuscript several times.

My research at M.I.T. was partly supported by the Joint Services and by the National Aeronautics and Space Administration under the auspices of the Research Laboratory of Electronics. I did the final editing

while on Sabbatical Leave at Trinity College, Dublin. Professor Brendan Scaife of the Engineering School provided me office facilities during this period, and M.I.T. provided financial assistance. I am thankful for all of the above support.

Harry L. Van Trees

Dublin, Ireland,

## REFERENCES

- [1] Harry L. Van Trees, *Detection, Estimation, and Modulation Theory, Pt. I*, Wiley, New York, 1968.
- [2] Harry L. Van Trees, *Detection, Estimation, and Modulation Theory, Pt. II*, Wiley, New York, 1971.
- [3] Harry L. Van Trees, *Optimum Array Processing*, Wiley, New York, 1971.

# *Contents*

<b>1</b>	<b><i>Introduction</i></b>	<b>1</b>
1.1	Review of Parts I and II	1
1.2	Random Signals in Noise	3
1.3	Signal Processing in Radar-Sonar Systems	6
	<i>References</i>	7
<b>2</b>	<b><i>Detection of Gaussian Signals in White Gaussian Noise</i></b>	<b>8</b>
2.1	Optimum Receivers	9
2.1.1	Canonical Realization No. 1: Estimator-Correlator	15
2.1.2	Canonical Realization No. 2: Filter-Correlator Receiver	16
2.1.3	Canonical Realization No. 3: Filter-Squarer-Integrator (FSI) Receiver	17
2.1.4	Canonical Realization No. 4: Optimum Realizable Filter Receiver	19
2.1.5	Canonical Realization No. 4S: State-variable Realization	23
2.1.6	Summary: Receiver Structures	31
2.2	Performance	32
2.2.1	Closed-form Expression for $\mu(s)$	35
2.2.2	Approximate Error Expressions	38
2.2.3	An Alternative Expression for $\mu_R(s)$	42
2.2.4	Performance for a Typical System	44
2.3	Summary: Simple Binary Detection	46
2.4	Problems	48
	<i>References</i>	54

<b>3</b>	<b><i>General Binary Detection: Gaussian Processes</i></b>	<b>56</b>
3.1	Model and Problem Classification	56
3.2	Receiver Structures	59
3.2.1	Whitening Approach	59
3.2.2	Various Implementations of the Likelihood Ratio Test	61
3.2.3	Summary: Receiver Structures	65
3.3	Performance	66
3.4	Four Special Situations	68
3.4.1	Binary Symmetric Case	69
3.4.2	Non-zero Means	72
3.4.3	Stationary “Carrier-symmetric” Bandpass Problems	74
3.4.4	Error Probability for the Binary Symmetric Bandpass Problem	77
3.5	General Binary Case: White Noise Not Necessarily Present: Singular Tests	79
3.5.1	Receiver Derivation	80
3.5.2	Performance: General Binary Case	82
3.5.3	Singularity	83
3.6	Summary: General Binary Problem	88
3.7	Problems	90
	<i>References</i>	97
<b>4</b>	<b><i>Special Categories of Detection Problems</i></b>	<b>99</b>
4.1	Stationary Processes: Long Observation Time	99
4.1.1	Simple Binary Problem	100
4.1.2	General Binary Problem	110
4.1.3	Summary: SPLOT Problem	119
4.2	Separable Kernels	119
4.2.1	Separable Kernel Model	120
4.2.2	Time Diversity	122
4.2.3	Frequency Diversity	126
4.2.4	Summary: Separable Kernels	130
4.3	Low-Energy-Coherence (LEC) Case	131
4.4	Summary	137
4.5	Problems	137
	<i>References</i>	145

5	<i>Discussion: Detection of Gaussian Signals</i>	147
5.1	Related Topics	147
5.1.1	$M$ -ary Detection: Gaussian Signals in Noise	147
5.1.2	Suboptimum Receivers	151
5.1.3	Adaptive Receivers	155
5.1.4	Non-Gaussian Processes	156
5.1.5	Vector Gaussian Processes	157
5.2	Summary of Detection Theory	157
5.3	Problems	159
	<i>References</i>	164
6	<i>Estimation of the Parameters of a Random Process</i>	167
6.1	Parameter Estimation Model	168
6.2	Estimator Structure	170
6.2.1	Derivation of the Likelihood Function	170
6.2.2	Maximum Likelihood and Maximum A-Posteriori Probability Equations	175
6.3	Performance Analysis	177
6.3.1	A Lower Bound on the Variance	177
6.3.2	Calculation of $J^{(2)}(A)$	179
6.3.3	Lower Bound on the Mean-Square Error	183
6.3.4	Improved Performance Bounds	183
6.4	Summary	184
6.5	Problems	185
	<i>References</i>	186
7	<i>Special Categories of Estimation Problems</i>	188
7.1	Stationary Processes: Long Observation Time	188
7.1.1	General Results	189
7.1.2	Performance of Truncated Estimates	194
7.1.3	Suboptimum Receivers	205
7.1.4	Summary	208
7.2	Finite-State Processes	209
7.3	Separable Kernels	211
7.4	Low-Energy-Coherence Case	213

7.5	Related Topics	217
7.5.1	Multiple-Parameter Estimation	217
7.5.2	Composite-Hypothesis Tests	219
7.6	Summary of Estimation Theory	220
7.7	Problems	221
	<i>References</i>	232
8	<i>The Radar-sonar Problem</i>	234
	<i>References</i>	237
9	<i>Detection of Slowly Fluctuating Point Targets</i>	238
9.1	Model of a Slowly Fluctuating Point Target	238
9.2	White Bandpass Noise	244
9.3	Colored Bandpass Noise	247
9.4	Colored Noise with a Finite State Representation	251
9.4.1	Differential-equation Representation of the Optimum Receiver and Its Performance: I	252
9.4.2	Differential-equation Representation of the Optimum Receiver and Its Performance: II	253
9.5	Optimal Signal Design	258
9.6	Summary and Related Issues	260
9.7	Problems	263
	<i>References</i>	273
10	<i>Parameter Estimation: Slowly Fluctuating Point Targets</i>	275
10.1	Receiver Derivation and Signal Design	275
10.2	Performance of the Optimum Estimator	294
10.2.1	Local Accuracy	294
10.2.2	Global Accuracy (or Ambiguity)	302
10.2.3	Summary	307
10.3	Properties of Time-Frequency Autocorrelation Functions and Ambiguity Functions	308

10.4	Coded Pulse Sequences	313
10.4.1	On-off Sequences	313
10.4.2	Constant Power, Amplitude-modulated Waveforms	314
10.4.3	Other Coded Sequences	323
10.5	Resolution	323
10.5.1	Resolution in a Discrete Environment: Model	324
10.5.2	Conventional Receivers	326
10.5.3	Optimum Receiver: Discrete Resolution Problem	329
10.5.4	Summary of Resolution Results	335
10.6	Summary and Related Topics	336
10.6.1	Summary	336
10.6.2	Related Topics	337
10.7	Problems	340
	<i>References</i>	352
<b>11</b>	<b><i>Doppler-Spread Targets and Channels</i></b>	<b>357</b>
11.1	Model for Doppler-Spread Target (or Channel)	360
11.2	Detection of Doppler-Spread Targets	365
11.2.1	Likelihood Ratio Test	366
11.2.2	Canonical Receiver Realizations	367
11.2.3	Performance of the Optimum Receiver	370
11.2.4	Classes of Processes	372
11.2.5	Summary	375
11.3	Communication Over Doppler-Spread Channels	375
11.3.1	Binary Communications Systems: Optimum Receiver and Performance	376
11.3.2	Performance Bounds for Optimized Binary Systems	378
11.3.3	Suboptimum Receivers	385
11.3.4	$M$ -ary Systems	396
11.3.5	Summary: Communication over Doppler-spread Channels	397
11.4	Parameter Estimation: Doppler-Spread Targets	398
11.5	Summary: Doppler-Spread Targets and Channels	401
11.6	Problems	402
	<i>References</i>	411

<b>12</b>	<b><i>Range-Spread Targets and Channels</i></b>	<b>413</b>
12.1	Model and Intuitive Discussion	415
12.2	Detection of Range-Spread Targets	419
12.3	Time-Frequency Duality	421
12.3.1	Basic Duality Concepts	422
12.3.2	Dual Targets and Channels	424
12.3.3	Applications	427
12.4	Summary: Range-Spread Targets	437
12.5	Problems	438
	<i>References</i>	443
<b>13</b>	<b><i>Doubly-Spread Targets and Channels</i></b>	<b>444</b>
13.1	Model for a Doubly-Spread Target	446
13.1.1	Basic Model	446
13.1.2	Differential-Equation Model for a Doubly-Spread Target (or Channel)	454
13.1.3	Model Summary	459
13.2	Detection in the Presence of Reverberation or Clutter (Resolution in a Dense Environment)	459
13.2.1	Conventional Receiver	461
13.2.2	Optimum Receivers	472
13.2.3	Summary of the Reverberation Problem	480
13.3	Detection of Doubly-Spread Targets and Communication over Doubly-Spread Channels	482
13.3.1	Problem Formulation	482
13.3.2	Approximate Models for Doubly-Spread Targets and Doubly-Spread Channels	487
13.3.3	Binary Communication over Doubly-Spread Channels	502
13.3.4	Detection under LEC Conditions	516
13.3.5	Related Topics	521
13.3.6	Summary of Detection of Doubly-Spread Signals	525
13.4	Parameter Estimation for Doubly-Spread Targets	525
13.4.1	Estimation under LEC Conditions	527
13.4.2	Amplitude Estimation	530
13.4.3	Estimation of Mean Range and Doppler	533
13.4.4	Summary	536

13.5 Summary of Doubly-Spread Targets and Channels	536
13.6 Problems	538
<i>References</i>	553
<b>14 Discussion</b>	<b>558</b>
14.1 Summary: Signal Processing in Radar and Sonar Systems	558
14.2 Optimum Array Processing	563
14.3 Epilogue	564
<i>References</i>	564
<i>Appendix: Complex Representation of Bandpass Signals, Systems, and Processes</i>	565
A.1 Deterministic Signals	566
A.2 Bandpass Linear Systems	572
A.2.1 Time-Invariant Systems	572
A.2.2 Time-Varying Systems	574
A.2.3 State-Variable Systems	574
A.3 Bandpass Random Processes	576
A.3.1 Stationary Processes	576
A.3.2 Nonstationary Processes	584
A.3.3 Complex Finite-State Processes	589
A.4 Summary	598
A.5 Problems	598
<i>References</i>	603
<i>Glossary</i>	605
<i>Author Index</i>	619
<i>Subject Index</i>	623

## Thank You for previewing this eBook

You can read the full version of this eBook in different formats:

- HTML (Free /Available to everyone)
- PDF / TXT (Available to V.I.P. members. Free Standard members can access up to 5 PDF/TXT eBooks per month each month)
- Epub & Mobipocket (Exclusive to V.I.P. members)

To download this full book, simply select the format you desire below

